PRINCIPLES OF DIRECT-CURRENT MACHINES

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ELECTRICAL ENGINEERING TEXTS

PRINCIPLES

OF

DIRECT-CURRENT MACHINES

BY

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TO MY MOTHER

SARAH SUSS LANGSDORF

PREFACE TO THE THIRD EDITION

While, since the appearance of the second edition of this book, there have been no developments in the field it aims to cover of sufficient importance to call for radical changes in content or treatment, some alterations and additions have been introduced for the purpose of avoiding possible misunderstanding or incorrect interpretation; and the problem material has been sufficiently changed to give it the element of freshness.

The preface to the second edition called attention to the fact that certain material useful from the point of view of design had been placed where its omission would not interfere with continuity of treatment in a course of study limited to general considerations concerning operational characteristics; this feature has now been emphasized by the insertion of foot-notes designating the sections in question. They have not been eliminated because, while the book does not aim to be a text on design, it is unquestionably in the interest of thoroughness to place before the student specific illustrations of applications of the general principles of the subject, and which may hold his interest and enlarge his horizon even though the field of design may not attract him as a life work. The author is unable to appreciate the view apparently entertained by some teachers that the mind of the average undergraduate student must on no account be "confused" by any descent from the rarified atmosphere of general principles; rather does it seem to him that such "confusion," if detected, is a clear indication that the full import of these fundamentals has not been grasped, and, therefore, that the teacher's job has not been finished. Further, it cannot be doubted that any engineer who is called upon to operate, or test, or pass upon the suitability of, a particular machine will be better equipped for such work if he is able to bring to bear considerations of the factors that influenced the design of the machine.

The analytical theory of the third-brush generator given in Chapter XI has been modified to take into account the possible

adjustment of the third brush away from the mid-position assumed in the original theory; this new treatment is based on the work of Paul W. Baker, one of the author's former students, who undertook this development of the original treatment as a thesis in partial fulfillment of the requirements for the professional degree of Electrical Engineer.

ALEXANDER S. LANGSDORF.

WASHINGTON UNIVERSITY,
ST. LOUIS, MO.,
June, 1923.

PREFACE TO THE SECOND EDITION

During the four years that have elapsed since the original appearance of this book, the author has been fortunate in receiving from time to time, from fellow-teachers in other institutions, a number of constructive criticisms and suggestions based upon their use of the text in regular class-room work. Most of these suggested changes have been incorporated in the present edition, and at the same time there have been included certain alterations and new material which personal experience has shown to be desirable. In general, the changes have been in the nature of amplification of certain explanations which were originally somewhat too condensed, and the new material has been selected to fill in gaps and make the presentation as self-contained as possible. At the same time the illustrative problems at the ends of the various chapters have been replaced by new sets. In a few cases where the original symbols did not conform to the standards recommended by the American Institute of Electrical Engineers, corrections have been made.

It was expected that the original edition might be criticized on the ground that some of the material emphasized the designer's viewpoint, though it was pointed out in the preface to the first edition that these portions could be omitted, at the discretion of the instructor, without interfering with continuity of treatment. Some of the rearrangements have been made with the idea of separating design material from the main context. It has not been eliminated for the reason that the fundamental idea of the book is to present the subject in such manner as to satisfy the student who wants to know why, rather than merely how, things are done.

The author desires to express his cordial thanks to all those fellow teachers whose friendly criticisms have greatly assisted the work of preparing this revision.

ALEXANDER S. LANGSDORF.

WASHINGTON UNIVERSITY,
ST. LOUIS, MO.
August, 1919.

PREFACE TO FIRST EDITION

This book has been prepared with the object of placing before junior and senior students of electrical engineering a reasonably complete treatment of the fundamental principles that underly the design and operation of all types of direct-current machinery. Instead of attempting to touch the "high spots" in the whole field of direct-current engineering, attention has been concentrated upon certain important features that are ordinarily dismissed with little more than passing mention, but which, in the opinion of the author, are vital to a thorough grasp of the subject. For example, the book will be found to contain in Chapter III a full derivation of the rules covering armature windings (following Professor Arnold), in addition to the usual description of typical windings; Chapters VI and VII include a considerable amount of new material concerning the operating characteristics of generators and motors, the treatment being largely graphical and including the use of three-dimensional diagrams for depicting the mutual relationships among all of the variables; and in Chapters VIII and IX there has been developed a much more extensive treatment of the important subject of commutation than has been heretofore easily accessible to students of the type for whom the book is intended. In the selection and arrangement of the material dealing with commutation, care has been exercised to eliminate those minute details and excessive refinements that are more likely to confuse than to clarify.

Although the methods of the calculus have been freely used throughout the book, a conscious effort has been made to give special prominence to the physical concepts of which the equations are merely the short-hand expressions; to this end, the mathematical analysis has been preceded, wherever possible, by a full and copiously illustrated discussion of the physical facts of the problem and their relations to one another. This has been done to counteract the tendency, manifested by many students, to look upon a mathematical solution of a problem as an end complete in itself, apparently without a due realization

that the first essential is a clearly thought out analysis of physical realities. As an example of this procedure, attention is directed to the new material of Article 210 of Chapter XI.

The illustrative problems at the end of each of the first ten chapters have, for the most part, been designed to prevent the practice of feeding figures into one end of a formula and extracting the result (painlessly) from the other end. No attempt has been made to include as complete a set of problems as is desirable in studying the subject, for the reason that each instructor will naturally prepare a set to meet his own needs. Some of the problems at the end of Chapters VI and VII will be found to tax the reasoning powers of the best students, but all of them have been successfully solved in the author's classes. Answers have not been given in the text, but will be supplied upon request to those instructors who ask for them.

It is not to be expected that a new book on direct currents can avoid including much material common to the large number of existing texts on the subject. Such originality as has been brought to bear, aside from that represented by the new matter already referred to, has been exercised in selecting from the vast amount of available material those parts that seem most essential to an orderly presentation of the subject. Numerous well-known texts have been freely drawn upon, with suitable acknowledgment in all essential cases.

That part of Chapter IV which deals with details of the calculation of the magnetization curve and of magnetic leakage, and the part of Chapter VIII in which the formulas for armature inductance are developed, may be omitted without interfering with the continuity of treatment, in case design is taught as a separate course.

In conclusion, the author desires to express his sincere thanks to Professor H. E. Clifford, of Harvard University, who made helpful criticisms and suggestions after reading the original manuscript, and who also assisted in the proof reading; and to the various manufacturers who have kindly contributed illustrations.

Alexander S. Langsdorf.

Washington University,
St. Louis, Mo.
August, 1915.

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TABLE OF SYMBOLS

(The figures refer to the page on which the symbol is first introduced; symbols formed from those given below by the mere addition of subscripts or primes for the purpose of distinguishing between quantities having the same general meaning, are not separately listed. Unless otherwise indicated, metric units are implied. Inch units are distinguished in the text by the use of the double prime (″).

In some instances the same symbol has been used to represent more than one quantity, though such cases are widely separated in the text, and the meaning may readily be determined from the lettering of the accompanying illustrations or from the context.)

A

		Page
a	cross-section of conductor	23
	constant length	63
	number of armature circuits	78
	constant in Froelich's equation	153
	radiating surface	410
a_x	cross-section of tube of flux	194
at	amp.-turns per cm.	155
A	area, cross-section, sq. cm.	8
AT	amp.-turns per pair of poles	154
$(AT)_d$	demagnetizing amp.-turns per pair of poles	175

B

		Page
b	pole arc	157
	constant in Froelich's equation	153
	brush width	295
b'	corrected pole arc	157
b_c	peripheral width of pole core	168
b'_i	corrected arc of commutating pole	361
b_s	width of slot	159
b_t	width of tooth at tip	163
b'_t	width of tooth at root	388
b_v	width of ventilating duct	160
b_o	width of slot opening	338
B	flux density	8
B'	amplitude of flux pulsations at pole face	394
B_e	flux density in axis of commutation, no load	330

C

		Page
c	constant	390
C	constant	59

D

d	diameter	71
	diameter of armature	183
d_c	diameter of pole core	107
D	distance or length	38

E

e	e.m.f. in volts	60
e_{max}	maximum reactance voltage	303
e_r	average e.m.f. of self-induction	302
E	e.m.f. in volts	22
	modulus of elasticity	18
$\overline{E}$	e.m.f. in abvolts	20
E_a	counter e.m.f.	104
E_c	commutating e.m.f.	296
E_{co}	commutating e.m.f., no load	330
E_{cl}	commutating e.m.f., load	331
E_T	commutating e.m.f. at end of period of commutation	299

F

f	force in dynes	5
	field step in terms of single pole pitch	132
	frequency of magnetic reversals	387
	coefficient of friction	395
F	force in dynes	63

G

g	conductance	29
G	conductance	29

H

h	radial depth of armature core under teeth	166
	circular mils per ampere	405
h_c	radial length of pole core	168
h_s	radial length of pole shoe	167
h_1	depth of armature coil	337
h_2	depth of straight part of slot above lower coil	338
h'_2	depth of straight part of slot above upper coil	339
h_3	depth of inclined part of tooth-tip	338
h_4	depth of tooth overhang	338
H	field intensity	5

I

		Page
r_f	resistance of series field winding	101
	resistance of separately excited field winding	200
r_s	resistance of shunt field winding	103
R	resistance in ohms	23
R_a	total resistance of all wire on armature	83
R_b	resistance of entire brush contact	298
R_c	resistance of short-circuited coil	298
R_l	resistance of commutator lead	298

S

s	distance, length	20
	number of winding sections	80
	diagonal of rectangular coil section	340
	specific heat	406
s_a	section of armature conductor	385
S	number of commutator segments	128

T

t	time in seconds	19
	temperature, deg. C.	23
	tooth pitch	159
	thickness of laminations	390
t_v	distance center to center of ventilating ducts	160
T	period of commutation, seconds	173
	torque	245

U

U	potential energy	34

V

v	velocity	20
	potential difference	29
	peripheral velocity of armature	334
v_c	peripheral velocity of commutator	318
v_r	voltage consumed in regulating rheostat	106
V	potential difference	28
	volume of core	387
V_t	volume of tooth	389

W

w	beam loading per unit length	18
	watts radiated	410
W	work, energy	20
	weight of core	388
W_c	loss at brush contact	305
W_R	loss in rheostat	281

X

Y

w

α

β

γ

δ

ε

ψ

ω

PRINCIPLES OF DIRECT-CURRENT MACHINES

CHAPTER I

GENERAL LAWS AND DEFINITIONS

1. Introductory.—A clear conception of the theory underlying the design and the operating characteristics of electrical machinery depends upon a thorough understanding of a few fundamental physical facts concerning the properties of electricity and magnetism and of the formulation of these facts as laws or definitions. The object of this chapter is to present in condensed form those facts, laws and definitions which are immediately applicable to the theory of direct-current machines. For a more extended treatment of these basic principles the student is referred to the numerous texts in which the subject is treated in detail.

2. Magnets. Magnetic Field.—The term magnet was originally applied to the lodestone, an oxide of iron (Fe_3O_4), now called magnetite, which from the earliest times has been known to have the property of attracting bits of iron or steel. It was also known at a very early date that this property could be imparted to pieces of iron or steel by rubbing them with a lodestone, but it was not until the nineteenth century that it was discovered that the best way to impart these properties to iron or steel bars is to surround them by a coil of insulated wire in which a current of electricity is made to flow. Certain alloys of steel, such as chrome-steel, retain this property indefinitely and such magnets are therefore called permanent magnets; pure soft iron, on the other hand, even though highly magnetized by a current through a surrounding coil, will quickly lose its magnetism if the current is cut off, or if the iron is removed from

the coil. In general, the permanence of the magnetized condition becomes more pronounced as the iron or steel becomes harder. The combination of an iron core and a current-carrying coil surrounding it is called an electromagnet to distinguish it from the permanent magnet or, more simply, the magnet. The force exerted by a magnet on pieces of iron which are not themselves magnets is always a force of attraction.

The simplest form of magnet is a straight rod, in which form it is called a bar magnet. If such a magnet is dipped into a mass of soft iron filings the latter will cling to the two ends of the magnet in the form of tufts, while the middle portion will be left comparatively free, thereby giving rise to the notion that the magnetic properties apparently reside at or near the ends, which are called the poles of the magnet; but the magnetized condition is in reality distributed fairly uniformly throughout the mass of the magnet, provided it is homogeneous, as may be shown by cutting the magnet into short lengths. No matter how far the subdivision is carried, each part remains a complete magnet with two equal poles.

A bar magnet freely suspended at its center of gravity will turn until its axis lies in a plane tangent to the magnetic meridian through the point of suspension. In general, in northern latitudes, the northern end will dip below the horizontal and the southern end will be correspondingly elevated. By counterweighting the bar, it may be made to assume a horizontal position. In this form the bar magnet is a compass. If such a magnet is disturbed, it will oscillate about a mean position but will ultimately come to rest, the same end always pointing in the same direction.

If two suspended bar magnets are brought close together, it is found that the two north-seeking poles invariably repel each other, and that the two south-seeking poles likewise repel each other; but a north-seeking pole of one magnet will attract the south-seeking pole of the other. These facts are summarized in the statement that like poles repel, and unlike poles attract, each other.

The directive effect of the earth upon a compass is accounted for by the fact that the earth is itself a huge magnet whose magnetic poles are situated near the geographical poles. Since

the north-seeking pole of the compass is attracted by the north magnetic pole of the earth, the former is in reality the "true south" pole of the compass; but it is called the north-seeking end, and generally for the sake of brevity merely the north pole. The opposite characteristics of the two poles of the magnet give rise to the terms *positive pole* and *negative pole*, the positive pole being the north-seeking pole, the negative pole the south-seeking pole.

When a magnet is brought near a mass of soft iron filings, each particle of the latter becomes *magnetized by induction* and develops two poles of opposite sign. The induced pole adjacent to the nearest inducing pole has a polarity opposite in sign to that of the inducing pole, thereby giving rise to a force of attraction; the other induced pole is repelled, but being farther away than the first pole, the resultant force on the induced magnet as a whole is one of attraction.

The region surrounding a magnet, and within which occur the phenomena described above, is called the *magnetic field* of the magnet. Thus far the only features that have been considered are the forces exerted by one magnet upon another, and the induced magnetism set up by a magnet in a previously unmagnetized piece of iron or steel. But it was discovered by Oersted, about 1820, that an electric current will exert a force upon a compass needle, so that a magnetic field likewise exists in the space surrounding the conductor through which the current is flowing; this explains the fact mentioned above, that a piece of iron or steel may be magnetized (by induction) when inserted in a coil carrying a current. Moreover, if a *closed* conductor is moved in a magnetic field in such a way that the flux linked with it changes in value from moment to moment, no matter whether that field is produced by a magnet or by a current in a conductor, there will in general be induced a flow of current in the moving conductor; this phenomenon was discovered by Faraday in 1831 and is known as *electromagnetic induction*. And further, a conductor carrying a current, when placed in a magnetic field produced by a magnet or by another electric current, will in general be acted upon by a force. Quite generally, therefore, a magnetic field may be defined as a region within which (*a*) magnets are acted upon by a force; (*b*) magnetic substances may become mag-

netized by induction; (*c*) moving conductors, if closed upon themselves, may become the seat of an induced current; (*d*) conductors carrying electric currents may be acted upon by forces.

Although iron and steel possess magnetic properties to a much greater degree than other substances, there are materials, of which nickel and cobalt are examples, which have these properties to a lesser extent. Substances in this group are attracted by a magnet, and are called *paramagnetic;* they tend to orient themselves in a magnetic field with their longer axes in the direction of the field. But there are substances like bismuth, phosphorus and zinc which are repelled by both poles of a magnet, and tend to set themselves with their longer axes across the field; they are called *diamagnetic* substances. Since iron and its alloys are the most prominent examples of the paramagnetic group, substances belonging to this group are frequently called ferromagnetic, or for the sake of brevity, merely magnetic. It is interesting to note that certain alloys, called the Heusler alloys, which contain no iron at all, have been found to have marked magnetic properties equivalent to that of a low grade of cast iron; these alloys are made of copper, aluminum and manganese, no one of which is itself magnetic, and the magnetic properties are best when the proportions of the three constituents are relatively about the same as their atomic weights.

3. Unit Magnet Pole.—Every magnetized body exhibits the phenomenon of polarity, that is, the simultaneous existence of poles of opposite sign. One polarity cannot exist without the other. The magnetized condition obtains throughout the entire mass of the magnet, but its intensity generally varies from point to point. In speaking of the pole of a magnet it should be understood that there is no one point at which the magnetism is actually concentrated, but the conception of concentrated point poles is useful for purposes of computation even though the idea is artificial. In the case of a long, slim magnet, like a knitting needle, the magnetism acts as though it were mostly concentrated at or near the ends, so that such a magnet approximates fairly well the condition of concentrated point poles. In particular, if one pole of such a magnet is placed in a magnetic field, its other pole being so far removed as to be acted upon with little or no force,

the magnet will behave as though it consisted of a single isolated pole, and the forces acting upon it can then be studied.

For purposes of quantitative measurement, a *unit magnet pole* is defined as a *point pole of such strength that it will exert a force of* 1 *dyne upon an equal pole at a distance of* 1 *cm.*, both poles being in air. The force will be a repulsion if the two unit poles are of the same sign; it will be an attraction if they are of opposite sign. Unless otherwise stated, the unit pole is always assumed to be a north-seeking or positive pole.

If a unit pole is placed 1 cm. away from a pole of unknown strength, the surrounding medium being air, and the force between them is found to be m dynes, it is assumed that the second pole has a strength of m units. In other words, the strength of a magnet pole is measured by the force in dynes with which it acts upon (or is acted upon by) a unit pole at a distance of 1 cm., in air. Two magnet poles of strength m and m', respectively, placed 1 cm. apart in air, will then act upon each other with a force of mm' dynes, in accordance with this definition.

In 1800 Coulomb discovered the fact that the force of attraction or repulsion between two magnet poles is inversely proportional to the square of the distance between them. In general, the force between two poles m and m' separated by a distance r is then

$$f = k\frac{mm'}{r^2} \tag{1}$$

If force is measured in dynes, distance in centimeters, and pole strength in terms of the unit defined above, then when m, m', and r are all equal to unity, f is equal to unity by definition, hence $k = 1$, or

$$f = \frac{mm'}{r^2} \text{ dynes} \tag{2}$$

4. Field Intensity. Uniform and Non-uniform Fields.—The *intensity* of a magnetic field at a given point in air is measured by the force in dynes which would act upon a unit magnet pole placed at that point, provided the introduction of the test pole into the field does not alter its original intensity or distribution. Field intensity is represented by the symbol H. Thus, a field is said to have unit intensity at a particular point in air if it acts

upon a unit magnet pole placed at that point with a force of one dyne. This unit field intensity is called the *gilbert per centimeter*, though the term *gauss* (see Art. 5) is also justifiable. A field of intensity H gilberts per cm. (or gausses) then means a field which will act upon a unit magnet pole in air with a force of H dynes, or with a force of mH dynes upon a point pole of strength m units.

If a magnetic field is so distributed that a test (point) pole is everywhere acted upon by the same force in the same direction, the field is said to be a *uniform field*. It is possible to have magnetic fields in which the intensity is uniform within definite boundaries, but in which the direction varies from point to point. Such fields have *uniform intensity*. In general, however, the intensity of the field will vary both in magnitude and direction from point to point, in which case the field is non-uniform.

5. Lines and Tubes of Force.—If a unit magnet pole is moved about in a magnetic field in air, the force acting upon it will in general vary in magnitude and direction from point to point. At each point in the field the force can be represented by a line whose length is proportional to the magnitude of the force and whose direction coincides with that of the force; in other words, the field intensity at each point can be represented by a vector. If curves are now drawn in such a way that their tangents are at each point in the direction of the intensity vector at that point, the curves will be *lines of magnetic force*. Obviously there will be an infinite number of such lines in any magnetic field, since there is an infinite number of points which do not lie on one and the same line of force, and through each of which a line may be drawn. It is also clear that lines of force cannot intersect, for if they did, each of the intersecting lines would have a different tangent at the point of intersection, therefore implying that a magnetic point pole placed at that point would simultaneously experience more than one force—a condition that is clearly impossible.

If a current is passed through a solenoid having an air core, as shown in Fig. 1, the distribution of the lines of force in a plane passing through the axis of the solenoid will have the general form indicated in the sketch, as may be proved experimentally by sprinkling iron filings on a sheet of paper through which the turns of the solenoid are wound. There is a similar distribution of

lines of force in every other plane through the axis of the coil. The outstanding fact observable in the diagram is that *the lines of force are closed curves,* and that each of the closed loops is linked with one or more turns of the coil in which the current is flowing.

There is a definite relation between the direction of flow of the current and the positive direction of the magnetic field due to the current (the positive direction of a line of force being the direction in which a free positive pole would be acted upon when on that line). The positive direction of the lines of force, where the lines pass *through* the turns of the conductor, is the same as the

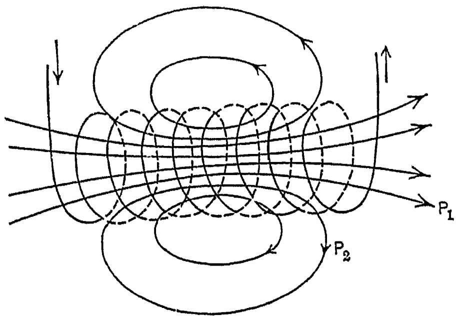


FIG. 1.—Lines of force due to a current in a solenoid.

direction of motion of the tip of a *right-handed* screw the head of which is turned in the direction of the flow of current. Other rules for determining the relation between direction of field and direction of current are as follows: (1) If the palm of the right hand is imagined placed on the coil with the fingers pointing in the direction of the flow of current, the extended thumb will point in the positive direction of the lines of force where they pass through the coil; (2) if the wire is imagined to be grasped in the right hand in such a manner that the thumb extends along the wire in the direction of the flow of the current, the fingers will link the wire in the positive direction of the lines of force.

In Fig. 1 only a few lines of force have been drawn, but it will be observed that inside the solenoid they are more closely compacted than they are at the ends and in the surrounding air. Experiment shows that the field intensity is likewise greater inside the solenoid than at the ends or outside, so that we are led

to the conception that the density of the lines, in air, may be used as a measure of the field intensity. It has already been pointed out that there is an infinite number of lines of force in any magnetic field, but in order to make use of the above conception it is convenient to picture the field by means of a finite number of "conventionalized" lines of force, these lines being selected so that the number per unit area, taken at right angles to the direction of the field, shall be numerically equal to the field intensity.

The symbol used to represent the number of (conventionalized) lines of force per unit area (1 sq. cm.) is B, while the symbol for field intensity is H. Therefore, *in air,*

$$B = H$$

If a magnetic field in air is uniform, the number of lines per unit area being equal to B, the total number of lines crossing an area A sq. cm. at right angles to the direction of the field will be BA, and this product is called the *magnetic flux* across the area. Flux is represented by the symbol Φ, so that

$$\Phi = BA$$

or

$$B = \frac{\Phi}{A}, \tag{3}$$

whence B is called the *flux density,* meaning the number of lines per square centimeter. The unit flux density is called the *gauss.* If the field is non-uniform, so that B varies from point to point, the flux across an area A which is everywhere perpendicular to the lines of force will be

$$\Phi = \int B dA,$$

the integration being extended over the entire area in question. If the area is not at right angles to the field, the flux crossing it is

$$\Phi = \int B \cos\alpha dA \tag{4}$$

where α is the angle between the direction of the field and the normal to the elementary area dA.

A bundle of lines of force threading through a given area will converge or diverge as the field intensity increases or decreases,

respectively. The lateral walls of such a bundle constitute a tubular surface the elements of which are lines of force, as shown in Fig. 2, and such a bundle is called a *tube of force.* If such a tube is in free space and does not contain any magnetic material, the total flux across all sections of the tube is constant, or

$$\Phi = \int B_1 \cos \alpha_1 dA_1 = \int B_2 \cos \alpha_2 dA_2, \tag{5}$$

for by hypothesis the longitudinal walls of the tube are made up of lines of force, and since lines of force cannot intersect, no flux can cross the walls of the tube, hence lines within the tube at one cross-section must be within the tube throughout its length. This fact is sometimes stated by saying that the flux across the walls of a tube of force is zero.

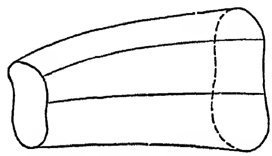


FIG. 2.—Tube of force.

6. Permeability.—Let an iron core be placed within the solenoid of Fig. 1 in the manner indicated in Fig. 3. With a given current flowing in the coil, it is found by experiment that the field intensity at the ends of the electromagnet is very much greater than when the iron was not present. The iron becomes magnetized by induction, poles of strength $+m$ and $-m$ developing at the ends, and the flux due to these poles is superimposed

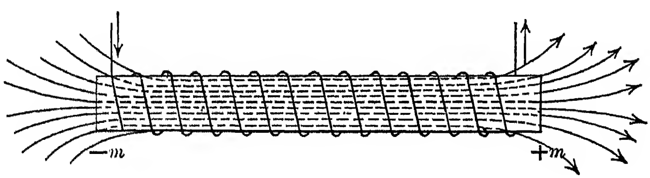


FIG. 3.—Solenoid with iron core.

upon the original flux. The new lines of force emanate from the induced north pole and return by way of the surrounding medium to the south pole, whence they complete their closed paths back to the north pole through the iron core. Within the core, therefore, the number of lines per unit area of cross-section is greater than when the core was not present, even though the field intensity H is maintained the same as it was in the absence of the core. In other words, within the iron of the core B is no longer equal to H, as was the case with an air core, but B is now very

much greater than H; as will be seen later, B may be as much as 2000 to 3000 times greater than H with certain grades of iron or steel.

The increased value of the flux when the original air core is replaced by an iron core may be thought of in terms of the following analogy: If water, or other fluid, is forced through a semiporous material like a layer of sand, the flow will depend upon the pressure, or head, and also upon the degree of porosity; but if there are cavities within the layer of sand, the stream lines of the flow will converge upon such cavities, and the resultant flow will be increased even though the pressure remains the same; the cavities offer a path of less resistance, or are more permeable to the flow than the denser parts of the layer In very much the same manner the iron core may be looked upon as a "magnetic cavity" in the sense that it is more permeable to the magnetic flux than the air it replaces. Accordingly, the ratio

$$\mu = \frac{B}{H} \tag{6}$$

is called the *permeability*, which is therefore a measure of the magnetic "conductivity" of the material. The permeability of air (or more strictly of a vacuum) is unity, since in that case $B = H$. In diamagnetic substances, which are repelled by both poles of a magnet, lines of force are conducted less readily than in air, so that their permeability is less than unity. There is no substance whose permeability is zero; if there were, such a substance would be a magnetic insulator.

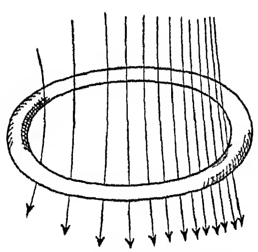


FIG. 4.—Conducting circuit in magnetic field.

7. Induced Currents and E.M.F.—It was discovered by Faraday in 1831 that a closed conductor which is threaded by, or linked with, a magnetic field will have a current induced in it when the strength of the field is altered. This phenomenon is called *electromagnetic induction*, and may be demonstrated in a number of different ways. Thus, if the stationary closed conducting ring of Fig. 4 is threaded by lines of magnetic force whose number changes from instant to instant, a current will

flow in the ring; again, there will be a current flow if the field is steady and the ring is rotated around a diameter so as alternately to include and exclude the magnetic flux; and again if the field is steady and the ring is given a motion of translation parallel to itself from a region where the field has a certain intensity to a region where the intensity is different; but if the ring be given a simple motion parallel to itself in a field of uniform intensity there will be no induced current.

It should be borne in mind that the flow of current in the various cases mentioned above is dependent upon the condition that the circuit be closed. The induced current is a secondary phenomenon, the primary effect of the changing magnetic field being to induce an *e.m.f.* which in turn produces the current.[1] For instance, let the wire *ab* move to the right along the rails SS', Fig. 5, and let Φ, Φ represent magnetic lines of force at right angles to the plane of the rails. There will result a displacement of electricity along the wire *ab*, a positive charge appearing at *b* and a negative charge at *a*. This means that there will be a difference of electrical potential (Art. 14) between *a* and *b*; if then the rails are joined by a conductor *cd*, a current will flow around the closed circuit *abcd*, but the difference of electrical potential may exist independently of the current, as for instance, when the circuit is open.

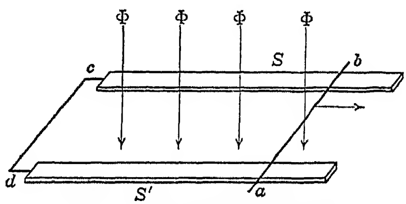


FIG. 5.—Development of e.m.f.

8. Direction of Induced E.M.F. (*a*) *Fleming's Right-hand Rule.*—A convenient method of determining the direction of the

[1] If an electrical circuit that is not closed upon itself is "cut" by lines of magnetic force, as described above, what actually occurs is a displacement of electricity along the conductor in a direction mutually perpendicular to the direction of the field and to the direction of motion of the conductor. While the electricity (consisting of electrons) is in process of displacement, its movement constitutes a true current, but currents of this sort are called displacement currents to distinguish them from the dynamic currents ordinarily dealt with in direct-current circuits. Strictly, therefore, the primary effect of a changing magnetic flux upon a conducting electrical circuit is to produce a displacement current, which in turn gives rise to a difference of electrical potential between the terminals of the circuit, and a (dynamic) current if the circuit be closed.

induced e.m.f. and of the resulting current if the circuit is closed is known as Fleming's rule, which is as follows: Hold the thumb, forefinger and middle finger of the *right hand* mutually perpendicular to one another, like the three axes of space coördinates, as illustrated in Fig. 6; point the forefinger in the direction of the lines of force, the thumb in the direction of motion of the wire relatively to the field, then the middle finger will point in the direction of the induced e.m.f.

Thus, referring to Fig. 5, if the forefinger of the right hand is pointed downward and the thumb to the right, the middle finger will point in the direction from *a* to *b*, so that the direction of flow of current will be *abcda*.

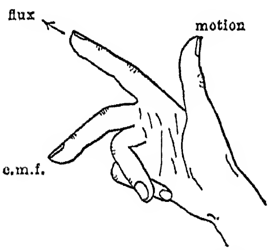


FIG. 6.—Fleming's right-hand rule. Generator action.

(*b*) *Lenz's Law.*—It is found by experiment that an induced current always opposes the action which produces it. This fact is known as Lenz's Law. Stated in another way, the induced current always flows in such a direction as to oppose the change in flux which produces the e.m.f. that in turn causes the current to flow. Thus, again referring to Fig. 5, it will be seen that the motion of the wire (to the right) is such as to cause the loop *abcd* to enclose more and more flux. According to Lenz's Law, the induced current will have such a direction as to oppose this increase of flux, consequently the induced current of itself will tend to produce lines of force directed upward through the loop; and in order to do this, as was explained in Art. 5 (p. 7), the current must then flow in the direction *abcda*.

9. Direction of the Force Due to a Current in a Magnetic Field.—Oersted's discovery that a current of electricity exerts forces upon the poles of a neighboring compass needle and so causes a deflection of the needle when these forces form a couple, leads to the corollary that the conducting wire is likewise acted upon by a force when properly placed in the field of the compass or, in general, in any magnetic field. The direction of the force upon the conducting wire is definitely related to the direction of the field, and may always be determined by the following considerations:

(*a*) *Fleming's Left-hand Rule.*—Hold the thumb, forefinger, and middle finger of the *left* hand mutually perpendicular to one another, as shown in Fig. 7. Point the forefinger in the direction of the lines of force and the middle finger in the direction of flow of the current, then the thumb will point in the direction of the force on the wire. It will be noted that this rule covering *motor action* is the same as Fleming's rule for the direction of the induced e.m.f. (generator action) except that the left hand is used instead of the right hand. Each rule is a sort of mirror image of the other, and in fact if Fig. 6 is observed in a mirror, the image will be identical with Fig. 7.

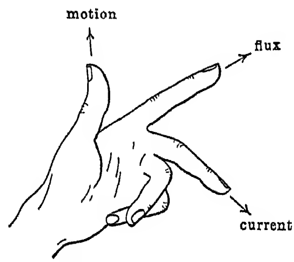
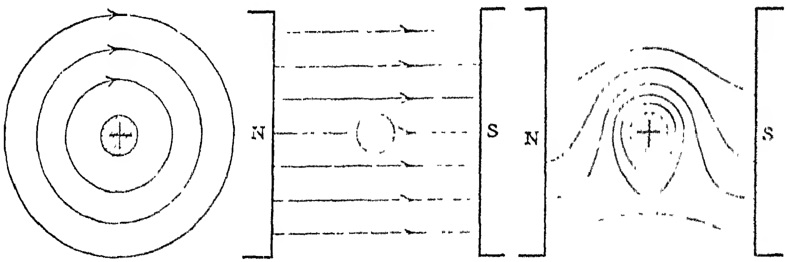


FIG. 7.—Fleming's left-hand rule. Motor action.

(*b*) Referring to Fig. 5, suppose that a current from some external source such as a storage battery is caused to flow through the loop in the direction *abcda*. A force will then be exerted on each of the sides of the loop, and the slider *ab*, if unconstrained, will move in the direction of the force. The left-hand rule given above in paragraph (*a*) indicates that the motion will be to the left. But as soon as motion ensues the wire *ab* will cut across the lines of force and an e.m.f. will be induced in *ab* in a direction opposed to the original direction of flow of current. This is in accordance with Lenz's law, for the original current in the loop may be looked upon as the cause of the motion, and the induced e.m.f. then tends to produce a current in opposition to the original cause of motion. In such a case as this, the induced e.m.f. set up by the motion of the wire *ab*, since it is in opposition to the original current flow, will actually bring about a reduction of the original current unless the e.m.f. of the battery supplying the current is increased sufficiently to balance the counter e.m.f.

(*c*) A long straight conductor in which a current is flowing, and which may be assumed to be in free space remote from any extraneous magnetic field, will be encircled by lines of magnetic force set up by the current in the conductor; it is proved in Art. 10 that these lines are circles whose planes are perpendicular to the axis of the conductor and whose centers lie in that axis.

Physical evidence of this fact may be developed by passing such a conductor through a hole in a sheet of paper or glass on which iron filings are then sprinkled; on tapping the sheet the filings will arrange themselves in concentric circles, in the manner indicated in Fig. 8*a*. If now the current carrying conductor is placed



(*a*) Cross-section of wire carrying current directed away from reader

(*b*) Wire, without current, in transverse uniform field

(*c*) Resultant field due to superposition of (*a*) and (*b*)

FIG. 8.—Distortion of magnetic field produced by current in a conductor.

in a magnetic field which, before the introduction of the conductor, was of uniform intensity and perpendicular to the axis of the wire, as in Fig. 8*b*, the resultant distribution of the lines of force will have the general form of Fig. 8*c*, since above the wire, under the particular conditions assumed in making the drawing, the two component fields cooperate to strengthen the field while below the wire they are in opposition. So far as the current carrying conductor is concerned, the lines of magnetic force act like stretched elastic bands, and in tending to shorten themselves exert a downward force upon the wire.

The facts discussed in Arts. 8 and 9 may be summarized as follows:

1. If a conductor is situated in a magnetic field and is acted upon by a mechanical force in such manner that the conductor is made to cut the lines of force, the induced e.m.f. will have such a direction that the resulting current (assuming the circuit to be closed) will react with the magnetic field in such a way as to set up a mechanical force in opposition to the driving force. Electrical energy is then produced at the expense of mechanical energy, and the entire phenomenon may be classified as *generator* action.

2. If a conductor is situated in a magnetic field and is made to carry a current from some source of electrical energy, the resulting force (if there is any at all) will tend to produce motion in such a direction that the induced e.m.f. will oppose the current. Mechanical energy is then produced at the expense of electrical energy, and the phenomenon is characteristic of *motor* action.

It should be noted that motion of a conductor in a magnetic field will give rise to an induced e.m.f. only when the motion is such as to cause the conductor to cut across the lines of force. Similarly, a wire carrying a current and situated in a magnetic field will experience force action only when the length of the wire has a component across the lines of force.

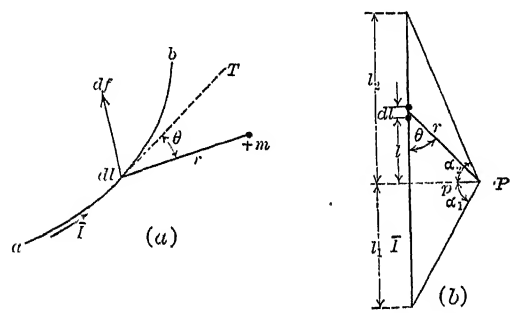


FIG. 9.—Force due to a current in a magnetic field.

10. Force Due to a Current in a Magnetic Field.—While Oersted's discovery established the fact that a conductor carrying a current experiences a force when properly placed in a magnetic field, the numerical relation between the magnitudes of the force, the current, and the field strength were first put into mathematical form by Laplace, as follows:

Let dl, Fig. 9*a*, represent an element of a wire ab, which is carrying a current of I absolute units (abamperes), and let m be a magnet pole of strength m units. The force acting on the element dl is then

$$df = \frac{m}{r^2} I \, dl \sin \theta \text{ dynes}[1] \tag{7}$$

where θ is the angle between the radius vector r and the tangent

[1] This equation is also known as the law of Biot-Savart.

T to the wire at the element. In accordance with the rules given in Art. 9, and under the conditions represented in Fig. 9a, the direction of this force will be upward, at right angles to the plane through r and T. Conversely, the current in the element dl will act upon the pole m with an equal force, directed downward.

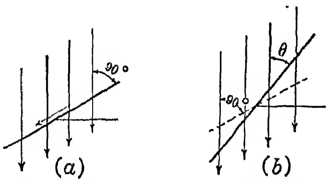


FIG. 10.—Force due to a current in a magnetic field.

Equation (7) cannot be derived. It is empirical, and its validity depends upon the fact that all experimental facts are in agreement with it.

In equation (7) the term $\frac{m}{r^2}$ is the field intensity H at the element dl due to the pole m. If the wire and the pole shown in Fig. 9a are in air, $H = B$, so that the force on the element is

$$df = B\bar{I}\, dl \sin \theta$$

If r is perpendicular to dl, $\sin \theta = 1$, in which case

$$df = B\bar{I}dl$$

It follows, therefore, that if a straight wire l cm. long, carrying a current of $\bar{I}$ abamperes, is placed in a magnetic field having a uniform flux density of B gausses in such a manner that its length is perpendicular to the lines of force (see Fig. 10a), it will be acted upon by a force

$$f = Bl\bar{I} \text{ dynes,} \tag{8}$$

but if the wire makes an angle θ with the direction of the field (see Fig. 10b), the force is

$$f = Bl\bar{I} \sin \theta \tag{9}$$

Equation (8) provides means for establishing a definition of the absolute unit of current (the abampere). For if B, l, and $\bar{I}$ are all made equal to unity, f is likewise unity, from which it follows that *the abampere is a current of such magnitude that if it flows in a straight wire* 1 *cm. long, placed perpendicular to the lines of force of a magnetic field having a flux density of one gauss* (one line per square centimeter), *the wire will experience a side thrust of one dyne.* As will be shown later one abampere is equal to ten amperes; *i.e.,*

$$1 \text{ abampere} = 10 \text{ amperes.}$$

Equation (7) serves to determine the strength of the magnetic field set up in the vicinity of a wire. For if $m = 1$ in equation (7), the value of df becomes equal to the force acting on a unit pole, and this, by definition, is the field intensity at the point occupied by the unit pole. Hence

$$dH = \frac{dB}{\mu} = \frac{\bar{I}dl \sin \theta}{r^2}$$

or

$$dB = \mu \frac{\bar{I}dl \sin \theta}{r^2} \tag{10}$$

where μ is the permeability of the medium surrounding the wire, and which is assumed to be constant.

Let it be required to find the flux density at a point P, Fig. 9b, at a perpendicular distance p cm. from a straight wire $(l_1 + l_2)$ cm. long, l_1 and l_2 being the lengths of the portions on either side of the perpendicular line p. If the current in the wire is $\bar{I}$ abamperes, the flux density at the point P due to an element dl is, by (10)

$$dB = \frac{\mu \bar{I} dl \sin \theta}{l^2 + p^2} = \frac{\mu p \bar{I} dl}{(l^2 + p^2)^{3/2}}$$

and the total flux density is

$$B = \mu p \bar{I} \int_{-l_1}^{+l_2} \frac{dl}{(l^2 + p^2)^{3/2}} = \frac{\mu \bar{I}}{p}(\sin \alpha_2 + \sin \alpha_1) \tag{11}$$

If the wire is infinitely long in both directions, $\sin \alpha_2 = \sin \alpha_1 = 1$, and

$$B = \frac{2\mu \bar{I}}{p} \tag{12}$$

If the wire is in air ($\mu = 1$), equation (12) becomes

$$B = H = \frac{2\bar{I}}{p}$$

which means that at all points equally distant from the axis of the wire the field intensity is the same as to magnitude, while the direction of the field at all such points is tangent to circles drawn through those points in such manner that the circles have their centers on the axis of the wire and their planes perpendicular thereto. It follows, therefore, that these circles are lines of magnetic force.

Example.—A storage battery having a discharge rating of 10,000 amperes is connected to the switchboard by copper bus-bars which have a cross-section of 1 in. by 10 in., and which are spaced 6 in. center to center, the 10-in. faces being in parallel vertical planes. Assuming that the current may be considered to be concentrated at the center of cross-section, what must be the distance between supporting brackets in order that the bus-bars may not deflect more than ¼ in.?

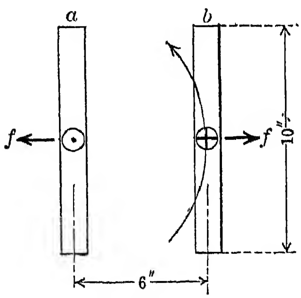


FIG. 11.—Section of bus-bars.

Let Fig. 11 be a cross-section of the bus-bars. Assume that the bus-bars are sufficiently long so that for all practical purposes they may be considered as infinite in length. The current flows in the direction indicated. The flux density at the center of bus-bar b due to the current in a is, by equation (12), since $\mu = 1$,

$$B = \frac{2I}{p} = \frac{2 \times 10{,}000}{6 \times 2.54 \times 10} = 131.2 \text{ gausses}$$

and the lines of force have the direction shown by the dotted arrow. Using the left-hand rule, the direction of the force on b will be to the right; similarly, the force on a will act toward the left, the direction of these two forces therefore agreeing with the known fact that parallel conductors carrying currents in opposite directions mutually repel each other.

The force per inch of length on bus-bar b is then, by equation (8),

$$f = BlI = 131.2 \times 2.54 \times \frac{10{,}000}{10} = 333{,}333 \text{ dynes per in.,}$$
$$= 0.749 \text{ lb. per in.}$$

Each bus-bar is then equivalent to a uniformly loaded beam, which, for simplicity, may be considered to be like a beam fixed at the ends, the ends being the points of attachment to the supporting brackets. It is proved in books on Mechanics that the deflection of such a beam is given by the formula

$$\delta = \frac{wl^4}{384EI}$$

where w is the load per unit length, l is the length between supports, E is the modulus of elasticity of the material of the beam, and I is the moment of inertia of the cross-section. The latter is equal to $I = \frac{1}{12}bh^3$, where b is the width of the beam (10 in. in this problem) and h is the depth (1 in.). Substituting $\delta = \frac{1}{4}$ in., $w = 0.749$ lb. per in., $E = 15 \times 10^6$ (the value for hard-drawn copper), and $I = \frac{1}{12}bh^3 = \frac{5}{6}$, l is found to be 200 in. = 16 ft. 8 in.

11. Unit Current. Unit Quantity of Electricity.—The absolute unit of current defined in the preceding article is the abampere, but in practice the unit ordinarily used is the ampere, which is taken one-tenth as large as the abampere. The definition previously given does not readily lend itself to the accurate measurement of current, consequently the *ampere* is defined as that unvarying current which will deposit silver at the rate of 0.001118 gram per second from a solution of pure silver nitrate, under certain prescribed specifications. Under like conditions the abampere will deposit ten times as much silver per second.

Unit quantity of electricity in the absolute system of units may then be defined as that amount of electricity which will pass a given cross-section of a conductor in one second when the current strength is one abampere. This unit of quantity is called the *abcoulomb*. In the ordinary or practical system of units, unit quantity is that amount of electricity which will pass a given cross-section of a conductor in one second when the current is one ampere; the name of this practical unit of quantity is the *coulomb*. It is readily seen that

$$1 \text{ abcoulomb} = 10 \text{ coulombs}.$$

In general, if the rate at which electricity passes a given cross-section of a conductor is varying, the current is

$$I = \frac{dQ}{dt},$$

which becomes $I = \frac{Q}{t}$ if the flow is unvarying.

12. Magnitude of Induced E.M.F.—Unit difference of electrical potential in the absolute electromagnetic system is said to exist between two points in an electrical field or in an electrical circuit when unit work (the erg) is expended in moving unit

quantity of electricity (the abcoulomb) from the one point to the other. This unit is called the *abvolt*. If, then, Q abcoulombs are moved from one point in a circuit to another point whose electrical potential differs from that of the first point by E abvolts, the work done is

$$W = EQ \text{ ergs}$$

If this work is done in a time t seconds, the power, or rate of doing work, is

$$P = \frac{W}{t} = E\frac{Q}{t} = EI \text{ ergs per second} \tag{13}$$

since $Q/t = I$ (Art. 11).

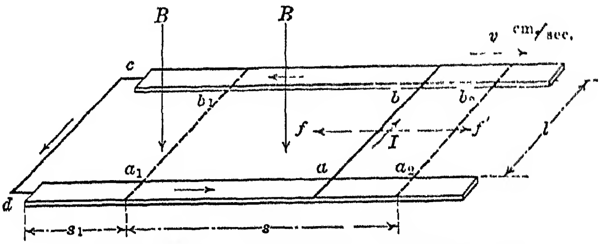


FIG. 12.—Moving conductor in a magnetic field.

Consider now the circuit of Fig. 12, and let the wire ab of length l cm. move to the right with a velocity $v = ds/dt$ cm. per second. There will be generated in the wire an e.m.f. of, say, E abvolts, in accordance with Faraday's law, and a current of I abamperes will be set up. The power developed will be EI ergs per second and the energy developed during a time dt will be

$$dW = EI\,dt \text{ ergs}$$

The current I in the wire ab will produce a thrust of

$$f = BlI \text{ dynes}$$

acting toward the left, hence the work done in overcoming this force through a distance ds will be

$$dW = fds = BlIds$$

and by the principle of the conservation of energy

$$EI\,dt = BlIds$$

or

$$E = Bl\,ds/dt = Blv \text{ abvolts} \tag{14}$$

But the expression Blv is numerically equal to the number of lines of force cut per second by the moving wire. Hence *the e.m.f.* (in absolute units) *is equal to the number of lines of force cut per second.*

In the above discussion it was tacitly assumed that the magnetic field swept across by the moving wire was uniform, and by supposition the velocity was constant. But if the field is not uniform, and if the velocity is variable, equation (14) still holds rigidly true; the e m.f. will simply vary from instant to instant in such a manner that the equation is continuously satisfied.

Let the wire start from position a_1b_1 and move to position a_2b_2. The flux originally linked with circuit a_1b_1cd is

$$\phi_1 = Bls_1$$

and in the final position the flux enclosed is

$$\phi_2 = Bl(s_1 + s)$$

The change of flux during the movement is

$$\phi = \phi_2 - \phi_1 = Bls$$

and if this change occurs in t seconds the average rate of change is

$$\frac{\phi}{t} = Bl\frac{s}{t} = Blv_{average} = \overline{E}_{average}$$

which, in words, states that the average induced e.m.f. is equal to the average rate of change of flux linked with the circuit. If the rate of change of flux is not uniform, the resulting variable e.m.f. will at any instant be given by

$$\overline{E} = \frac{d\phi}{dt}$$

which is a general form of the equation expressing Faraday's law.

If the circuit linked with the flux has N turns, the absolute e.m.f. induced by a change in the flux will be at any instant

$$\overline{E} = N\frac{d\phi}{dt} \text{ abvolts}$$

so far as its numerical value is concerned. But since, by Lenz's law, the induced e.m.f. always tends to set up a current in such a direction as to oppose the inducing action, it follows that a

positive increment of flux through the circuit will induce a negative e.m.f., and vice versa. Hence

$$E = -N\frac{d\phi}{dt} \text{ abvolts} \tag{15}$$

The unit of electromotive force in the absolute system of units is determined by either equation (14) or (15). Thus, referring to (14), if a conductor cuts one line of magnetic force per second, the induced e.m.f. will be 1 abvolt; or, by (15) if a closed coil of a single turn links with magnetic flux which changes at the rate of one line per second, the e.m.f. induced in the coil will be 1 abvolt.

The abvolt is inconveniently small for practical purposes, so that the practical unit, the volt, is taken 10^8 times as large as the abvolt; that is,

$$1 \text{ volt} = 10^8 \text{ abvolts},$$

and equation (15) when converted into practical units, is

$$E = -N\frac{d\phi}{dt} \times 10^{-8} \text{ volts} \tag{16}$$

Since the power in a direct-current circuit is

$$P = EI \text{ ergs per second}$$

when E and I are expressed in abvolts and abamperes, respectively, this expression becomes

$$P = (E \times 10^8)\left(\frac{I}{10}\right) = EI \times 10^7 \text{ ergs per second}$$

when E and I are expressed in volts and amperes, respectively. But 10^7 ergs per second are equivalent to one *watt*, so that

$$P = EI \text{ watts} = \frac{EI}{1000} \text{ kilowatts}$$

Definite standards of electromotive force are provided by certain voltaic cells, which are widely used for calibrating voltmeters. The standard Weston cell has a constant electromotive force on open circuit of 1.01830 volts at 20° C., and the Clark cell has an open-circuit e.m.f. of 1.4328 volts at 15° C. Standard specifications for the construction of these cells are given in the Bulletin of the Bureau of Standards, Vol. 4 (1907).

13. Resistance. Ohm's Law. Joule's Law.—When a source of electromotive force such as a primary or secondary battery is connected to a conducting circuit in such a manner as to provide a closed path that includes the battery and the external conducting circuit, the resultant flow of current is accompanied by the evolution of heat in all parts of the circuit and the energy thereby dissipated is supplied at the expense of the chemical energy of the battery. The evolution of heat is caused by the flow of current through the electrical *resistance* of the circuit, the resistance in the electrical circuit being analogous to the frictional resistance to the flow of water in a hydraulic circuit. In a homogeneous conductor of length l and cross-section a, the resistance is directly proportional to the length and inversely proportional to the cross-section, or

$$r = \rho \frac{l}{a}$$

where the proportionality constant ρ is called the *resistivity* of the material; the formula shows that if l and a are both equal to unity, $r = \rho$, so that the resistivity is the resistance of a portion of the material in question which is of unit length and unit cross-section. For any given material ρ is constant at a given temperature, but is in general a function of the temperature. An approximate relation between resistivity and temperature is

$$\rho = \rho_0 (1 + \alpha t)$$

where ρ_0 is the resistivity at 0° C. and t is the temperature in deg. C. corresponding to the value ρ. The factor α is a constant called the temperature coefficient, and is positive in some materials, negative in others; in some alloys α is practically equal to zero, such substances being particularly useful in the construction of instruments or other devices in which it is desired to have the resistance independent of temperature changes.

If a simple closed circuit is adjusted to have a fixed value of resistance, and the e.m.f. acting in the circuit is varied, the current will be directly proportional to the e.m.f. This experimental fact is called *Ohm's law*. It is expressed by the formula

$$I = \frac{E}{R} \tag{17}$$

or

$$E = IR$$

where R is the resistance of the circuit. If E and I are expressed in volts and amperes, respectively, R will be expressed in *ohms;* if E and I are respectively abvolts and abamperes, R will be expressed in *abohms.* The unit of resistance may therefore be defined as a resistance of such magnitude that unit e.m.f. will produce in it a unit current.

It will be observed from equation (17) that the three units of e.m.f., current and resistance are definitely related to one another, and that if any two of them are arbitrarily defined the third is fixed in terms of the two first chosen. By international agreement, the two units chosen as bases are those of current and resistance; the unit of current, the ampere, has already been defined as that which will deposit silver, under definite specifications, at the rate of 0.001118 gram per second; the unit of resistance, the ohm, is defined as the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14.4521 grams in mass, of a constant cross-sectional area and of a length of 106.300 cm.

In a circuit in which an e.m.f. E volts produces a current of I amperes, the power, or rate of doing work, is

$$P = EI \text{ watts} = EI \times 10^7 \text{ ergs per second.}$$

But from Ohm's law $E = IR$, so that

$$P = I^2R \times 10^7 \text{ ergs per second}$$

which represents the power absorbed in the resistor whose resistance is R ohms. In a time t seconds, during which the current remains constant, the energy supplied to the circuit, and all of which appears as heat, is

$$W = Pt = I^2Rt \times 10^7 \text{ ergs.}$$

Since 10^7 ergs are equivalent to 1 joule, and 1 joule per second is equivalent to one watt, the above expressions can be written

$$\left.\begin{aligned} W &= I^2Rt \text{ joules} \\ P &= I^2R \text{ watts} \end{aligned}\right\} \qquad (18)$$

These equations are the mathematical expression of *Joule's Law,* which may be stated by saying that the total heat developed in a resistor in a given time is proportional to the resistance and to the square of the current. This law was first enunciated by

Joule, about 1843, and his experiments also led to the determination of the mechanical equivalent of heat (Joule's equivalent), namely, that 4.19×10^7 ergs of work are required to raise the temperature of 1 gram of water 1° C.

14. Electromotive Force and Potential Difference.—Consider a battery, either primary or secondary, which is on open circuit. The chemical action in the cells produces an electromotive force the result of which is to cause a displacement of electrons, which are ultimate particles of negative electricity, away from the positive terminal and toward the negative terminal. On the negative terminal there is then a surplus of negatively charged electrons, and on the positive terminal a deficit of electrons, in comparison with the number that would normally exist on them when there is no battery electrolyte. The difference of condition between the two electrodes is much the same as that which would exist between two closed flexible chambers, connected by an air pump in such a way that air is pumped out of the one and into the other. Just as the difference of pressure between two chambers tends to equalize itself by a backward flow of air from the region of high pressure to the region of partial vacuum, but is prevented from so doing by the pump action as long as the latter is in operation, so the excess of electrons on the negative terminal tends to flow back to the partial electrical vacuum at the positive terminal but is prevented from doing it by the chemical action of the battery. Each particle of air in the compression chamber is repelled by all the other particles that are present, giving rise to a disruptive tendency that will cause an explosion if the process is carried far enough. Quite similarly, each electron on the negatively charged terminal is repelled by all the other electrons there present, in accordance with the well-known fact that like charges repel one another; here also there exists an electrical disruptive tendency which will cause a breakdown of the insulation between terminals if the process is carried sufficiently far, and the equalization of electrical pressure, when it occurs, is accompanied by the formation of the familiar spark discharge.

In the pneumatic case discussed above, there is no particular name to describe the pump action. The pump simply keeps up the transfer of air from the suction chamber to the compression chamber until the tendency to discharge backward through the

pump is just balanced by the action of the pump itself; thereupon the transfer of air ceases, and the system is in equilibrium. But in the electrical case the battery action which causes the transfer of electrons from the positive terminal to the negative is called the *electromotive force* of the battery, and the resulting difference of electrical pressure between the terminals is called the *difference of electrical potential*, or simply the *potential difference* (abbreviated as p.d.). It is quite clear, therefore, why there is always a tendency to produce a flow of current between points which have between them a difference of potential; it is simply a result of the mutually repelling forces between the electrons on the negatively charged terminal.

When equalization of electrical pressures takes place between two terminals, the flow of electrons takes place through the connecting conducting circuit in the direction from negative to positive terminal. Unfortunately, the accepted terminology refers to this flow of current as taking place from positive to negative, this usage having become firmly established before the development of the modern electron theory.

It is likewise unfortunate that the term electromotive force was adopted to fit the action described above; for electromotive force is *not a force* in the ordinary sense of that word. Force is *involved* in the notion of the action represented by the term electromotive force, for when electrons are transferred from the positive to the negative terminal, as in the case discussed above, force is required to move each electron against the repelling action of all electrons previously transferred, just as the force exerted by the air pump must increase as the pressure in the compression chamber rises. But electromotive force is the *cause* of the transfer of electrons, and difference of potential between the terminals is the *result,* and both cause and effect are measured in terms of the same unit, the abvolt in the absolute system, the volt in the practical system of units. Referring to Art. 12, it will be noted that a difference of potential of 1 abvolt between two terminals means that work to the extent of 1 erg must be done to transfer 1 abcoulomb from the one terminal to the other; and, similarly, it is true that 1 joule must be expended to transfer 1 coulomb of electricity between terminals which have between them a difference of potential of 1 volt. Conse-

quently both e.m.f. and p.d. are of the nature of *work-per-unit-charge*, and this is not the same as force.

All that has been said about the e.m.f. of batteries applies equally well to e.m.fs. induced by electromagnetic action. In the case of the battery, the primary cause of the action is the chemical reaction between the materials of the cells; in the case of electromagnetic action the transfer of electrons is caused by some unknown reaction between the electrons in the conducting wire and the ether stresses that constitute the magnetic field.

The words "positive" and "negative" as applied to electrical circuits are carried over from the two-fluid theory of electricity. The electron theory is in many respects the same as the one-fluid theory of Franklin, except that what he called positive electricity is now known to be negative, and vice versa. According to the electron theory, the negative terminal is at a higher potential than the positive, and the flow of electrons in a circuit connecting them therefore takes place from negative to positive. But in the usual terminology, the flow of current is said to take place from positive to negative, and the positive terminal is, accordingly, arbitrarily assumed to be of higher potential than the negative. Consequently, when current flows out of a battery through a closed conducting circuit, and back again through the battery, there is a *fall* of potential along the external circuit from the positive to the negative terminal, while within the battery there is a *rise* of potential from the negative to the positive terminal, this rise being due to the electromotive force.

15. Generalized Ohm's Law. *Series and Parallel Circuits.*—Let Fig. 13*a* represent a simple circuit consisting of a battery (or other source of e.m.f.) whose e.m.f. is E volts and whose internal resistance is r ohms; and let the external circuit be a conductor of resistance R ohms. The total resistance is $(r + R)$ ohms, and by Ohm's law the current through all parts of the circuit will be

$$I = \frac{E}{r + R} \text{ amperes.}$$

This equation may be written

$$E = Ir + IR$$

and in this form asserts that the entire electromotive force of the source is used in supplying (1) the drop of potential in the source,

represented by Ir, and which is analogous to the drop of pressure that occurs in a pump when it is causing a circulation of water; and (2) the drop of potential in the external circuit, represented by the term IR. The latter term is obviously equal to the difference of potential, V, that exists between the terminals, and which can be measured by a voltmeter. The expression can then be written

$$V = IR = E - Ir \tag{19}$$

which states that the difference of potential between the terminals is equal to the e.m.f. of the battery less the drop of potential in

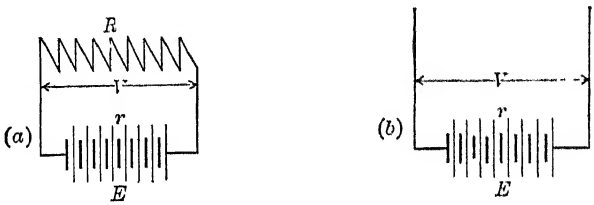


FIG. 13.—Simple series circuit.

the battery; in the particular case where the external circuit is open (R equal to infinity), the current will be zero, whence $V = E$. The equation can also be written

$$E - Ir - IR = 0, \tag{20}$$

which may be interpreted to mean that there is a rise of potential (taken as positive) through the source, caused by the active e.m.f., and a *drop* or *fall* of potential (taken as negative) through the passive resistances r and R.

The case discussed above refers to the conditions in a circuit containing an active source of e.m.f. and a passive receiver circuit. Suppose, however, that we have a storage battery (Fig. 13*b*) whose e.m.f. is E volts and which is being charged from some other source through a circuit between whose terminals there is maintained a constant difference of potential of V volts. The active e.m.f. of the battery (E) opposes V, hence V must be great enough to overcome E and at the same time supply the drop of potential through the battery resistance r; that is

$$V = E + Ir \tag{21}$$

Equations (19), (20), and (21) are more general forms of Ohm's

law than equation (17), and are therefore occasionally referred to as the generalized Ohm's law.

While simple series circuits of the type discussed above are often encountered in engineering practice, most circuits are somewhat more complex, and consist of series circuits, parallel circuits, and combination or series-parallel circuits.

When a number of resistors are connected in series relation, the total resistance of the entire circuit is the sum of the individual resistances, or

$$R = r_1 + r_2 + r_3 + \ldots \tag{22}$$

where R is the total resistance and r_1, r_2, r_3, etc., are the values of the individual resistances. If a direct current of I amperes flows through such a circuit, the total drop of potential from the positive to the negative terminal is

$$\begin{aligned} V = IR &= Ir_1 + Ir_2 + Ir_3 + \ldots \\ &= v_1 + v_2 + v_3 + \ldots \end{aligned} \tag{23}$$

where v_1, v_2, v_3, etc., are the drops across the individual resistors.

When a number of resistors are connected in parallel, the conductance of the entire circuit is equal to the sum of the conductances of the individual paths, the conductance of a circuit being the reciprocal of its resistance.

$$\therefore \frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \ldots \tag{24}$$

Conductance is represented by the letter G or g, hence

$$G = g_1 + g_2 + g_3 + \ldots$$

If the difference of potential between the terminals is V volts,

$$\begin{aligned} I = \frac{V}{R} = GV &= V(g_1 + g_2 + g_3 + \ldots) \\ &= \frac{V}{r_1} + \frac{V}{r_2} + \frac{V}{r_3} + \ldots \\ &= i_1 + i_2 + i_3 + \ldots \end{aligned} \tag{25}$$

where i_1, i_2, i_3, etc., are the currents in the several paths. In applying these formulas to actual circuits, it must be carefully remembered that they are derived on the assumption that the individual portions of the circuit *do not contain active e.m.fs.*, and that if active e.m.fs. are present, the above relations do not in general hold true.

16. Kirchhoff's Laws.—Circuits of the kind used in electrical distribution and in the internal wiring of motors and generators

are in many cases best described as *networks*. A network is characterized by a plurality of closed loops or meshes and a plurality of junction points between the loops, in the manner illustrated by the simple three-wire system of Fig. 14. The solution of problems involving the flow of current in networks of conductors depends upon two experimental facts, known as *Kirchhoff's Laws:*

1. The algebraic sum of the currents at any junction of the conductors in the network is zero.

2. The algebraic sum of the potential drops around any closed loop in the network is zero.

The first of these two laws is a statement of the fact that the sum of all the currents flowing *toward* a junction point is equal to the sum of all the currents flowing *away* from that point. If this were not true, the electrical charge at the junction point would steadily change and the potential of the point would change correspondingly; no effect of this kind has ever been observed under steady conditions of operation.

In applying the second law it is convenient to make a diagram of the network and to give to each active e.m.f. (such as that from a battery or dynamo) an appropriate symbol to indicate its magnitude and an arrow to indicate the direction in which it acts; and each conductor is to be given a symbol to indicate the magnitude of the current flowing in it, and an arrow to indicate the *assumed* direction of the current flow. Let it be agreed that the clockwise direction around any closed loop of the network shall be considered to be the positive direction through the circuit (though cases may arise when it might be more convenient to select the counter-clockwise direction as the positive one). Then any active e.m.f. that is directed positively around the loop is to be given the positive sign; it may in that case be considered to produce a *rise* of potential in the positive direction. A current of i amperes flowing in the positive direction through a resistor of r ohms will produce a *fall* or drop of potential of ir volts, which must, accordingly, be treated as negative.

Example.—Fig. 14 represents a storage battery connected to a three-wire system, the "neutral" wire c being connected to the middle point of the battery. Each half of the battery has an e.m.f. of 115 volts, directed as shown. A and B are the loads which are supposed to consist of lamps, heaters, or other devices which do not contain an active e.m.f., and have resistances of 4

and 8 ohms, respectively. The outer wires, a and b, have each a resistance of 0.1 ohm, and the neutral wire c has a resistance of 0.2 ohm. Each half of the battery has a virtual internal resistance of 0.5 ohm. It is required to find the current in each of the supply lines.

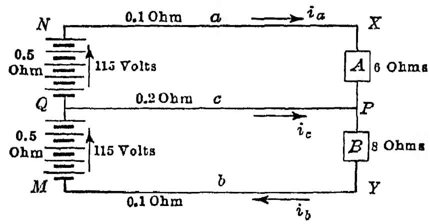


FIG. 14.—Net-work of conductors.

From the first law, the sum of the currents entering point P must equal the sum of the currents leaving it; hence, from the assumed directions of current flow

$$i_a + i_c = i_b$$

From the second law, referring to the upper loop of the diagram,

$$+ 115 - i_a(0.5 + 0.1 + 6) + 0.2i_c = 0$$

And referring to the lower loop

$$+ 115 - 0.2i_c - i_b(8 + 0.1 + 0.5) = 0$$

From these three independent equations it is easily found that $i_a = 17.31$, $i_b = 13.47$ and $i_c = -3.84$. The meaning of the negative sign of i_c is that that current actually flows in a direction opposite to the assumed direction.

A method that will be found useful in writing down and checking the equations incident to problems of this general type involves the construction of what may be called a potential diagram of the circuit. Thus, if in Fig. 14 it be assumed quite arbitrarily that the bottom terminal M of the lower battery is at zero potential (as though this point were grounded), then the upper terminal Q of this battery will have a potential +115 if no current is flowing, and something less than this when there is a current, the difference representing the drop of potential in the battery, in this case amounting to $0.5i_b$. In Fig. 15 let the vertical line OV represent a scale of potential (volts), O being the origin, and the positive direction being upward; the potential of point Q will then be as shown. Between Q and P there must be a fall of potential in accordance with the assumed direction of current flow in conductor c, hence the potential of point P must be lower than that of Q by an amount $0.2i_c$ volts; similarly, there must be a fall of potential of $8i_b$ volts from P to Y, and a further fall of $0.1i_b$ volts from Y to O, to conform to the assumed

direction of current i_b; at the point O, at which the construction was started, there must be a final closure in agreement with Kirchhoff's second law. In the same way, there is a rise of potential from Q to N which will be equal to 115 volts less the drop $0.5i_a$ in battery QN; and finally there is a drop of $0.1i_a$ volts from N

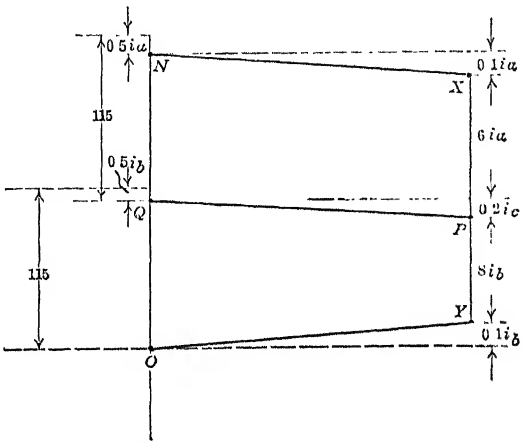


FIG. 15.—Potential diagram of circuit of Fig. 14.

to X, and of $6i_a$ volts from X to P, at which point the upper polygon must close, since the point P has already been fixed. The diagram of Fig. 15 is helpful not only in setting up the equations necessary to a solution of the problem, but also in visualizing their physical meaning; and it shows in a convincing manner how changes in the circuit constants affect the relative values of the potential differences PX and PY, and how under conditions that may easily arise in practice, one or the other

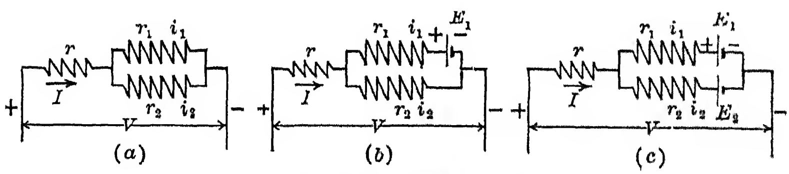


FIG. 16.—Series-parallel circuit.

of these potential differences may exceed the e.m.f. of the generator on its side of the circuit.

Example.—To illustrate the effect of active e.m.fs. in one or more of the branches of a divided circuit, consider the circuits represented in parts (*a*), (*b*), and (*c*) of Fig. 16.

(*a*) The total resistance of the circuit of Fig. 16*a* is

$$R = r + \frac{r_1 r_2}{r_1 + r_2}$$

and the total current is

$$I = \frac{V}{R} = \frac{V}{r + \dfrac{r_1 r_2}{r_1 + r_2}}$$

Further,

$$i_1 = \frac{V - Ir}{r_1} = \frac{V}{R}\frac{r_2}{r_1 + r_2}$$

$$i_2 = \frac{V - Ir}{r_2} = \frac{V}{R}\frac{r_1}{r_1 + r_2}$$

$$\frac{i_1}{i_2} = \frac{r_2}{r_1}$$

(*b*) Applying Kirchhoff's laws to Fig. 16*b*,

$$I = i_1 + i_2$$
$$V = Ir + i_1 r_1 + E_1$$
$$V = Ir + i_2 r_2$$

Whence

$$i_1 = \frac{Vr_2 - E_1(r + r_2)}{r(r_1 + r_2) + r_1 r_2}$$

$$i_2 = \frac{Vr_1 + E_1 r}{r(r_1 + r_2) + r_1 r_2}$$

$$I = \frac{V - E_1 \dfrac{r_2}{r_1 + r_2}}{r + \dfrac{r_1 r_2}{r_1 + r_2}}$$

From these equations it is clear that the branch currents i_1 and i_2 are no longer inversely proportional to the resistances of the branch paths, and also that the total current is not equal to the impressed voltage V divided by the resistance as computed in (*a*).

(*c*) Applying Kirchhoff's laws to Fig. 16*c*,

$$I = i_1 + i_2$$
$$V = Ir + i_1 r_1 + E_1$$
$$V = Ir + i_2 r_2 + E_2$$

whence

$$i_1 = \frac{Vr_2 - E_1(r + r_2) + E_2 r}{r(r_1 + r_2) + r_1 r_2}$$

$$i_2 = \frac{Vr_1 - E_2(r + r_1) + E_1 r}{r(r_1 + r_2) + r_1 r_2}$$

$$I = \frac{V(r_1 + r_2) - E_1 r_2 - E_2 r_1}{r(r_1 + r_2) + r_1 r_2}$$

Here also it is plain that the division of current between the parallel branches does not follow the inverse law when E_1 and E_2 have different values. But if E_1 and E_2 are *numerically equal* and are of the *same sign*, that is, if

$$E_1 = E_2 = E$$

it follows that

$$\frac{i_1}{i_2} = \frac{Vr_2 - E(r + r_2) + Er}{Vr_1 - E(r + r_1) + Er} = \frac{r_2}{r_1}$$

and

$$I = \frac{V - E}{r + \dfrac{r_1 r_2}{r_1 + r_2}}$$

in which case it is seen that it becomes permissible to combine the resistances in accordance with equations like (22) and (24).

17. Magnetic Potential.—Let Fig. 17 represent two magnet poles of strengths m and m' units, respectively, separated by a distance x cm. Each will repel the other with a force of $\dfrac{mm'}{x^2}$ dynes. Let one of the poles, as m', move a distance dx under the influence of this force, then work will be done to the extent of

$$dU = \frac{mm'}{x^2} \cdot dx \text{ ergs}$$

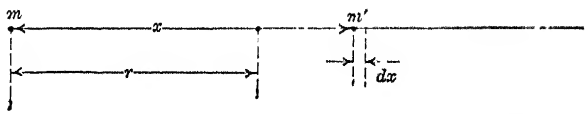


Fig. 17.—Determination of potential energy of two magnet poles.

The entire amount of the work done in separating the two poles to an infinite distance, from an initial separation of r cm., is

$$U = \int_r^\infty \frac{mm'}{x^2} dx = \frac{mm'}{r} \text{ ergs} \tag{26}$$

Since no work has been done upon the system by any outside agency during this process, the work represented by the expression $\dfrac{mm'}{r}$ must have come from the system itself, and therefore represents the *stored* or *potential energy* of the two poles in each other's presence. It represents also the amount of work or energy required to bring one pole from an infinite distance into

the presence of the other, with a separation of r cm.; for this is given by

$$U = \int_{\infty}^{r} \frac{mm'}{x^2}(-dx) = \frac{mm'}{r}$$

or the same as before. If $m' = 1$, the expression becomes $\frac{m}{r}$, which represents the potential energy of a unit magnet pole placed r cm. from a pole m; or it is the *magnetic potential* due to a pole m at a distance of r cm. from the pole.

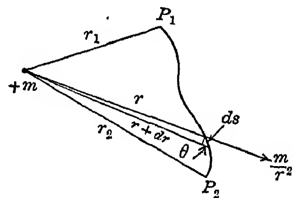


FIG. 18.—Determination of difference of magnetic potential between two points in a magnetic field.

The work required to move a unit magnet pole from one point to another in a magnetic field can be calculated as follows:

Let P_1, Fig. 18, be the initial, and P_2 the final position of the unit pole, and let the path between them be any curve whatsoever. At any general point on the curve, distant r cm. from m, the force will be $\frac{m}{r^2}$, and the direction of this force will be displaced from the elementary path ds by an angle θ. The work done over the distance ds will be

$$dU = \frac{m}{r^2}\,ds \cos\theta = \frac{m}{r^2}\,dr$$

and the total work in going from P_1 to P_2 will be

$$U_{1-2} = \int_{r_1}^{r_2} \frac{m}{r^2}\,dr = \frac{m}{r_1} - \frac{m}{r_2} \tag{27}$$

But $\frac{m}{r_1}$ is the magnetic potential at P_1, and $\frac{m}{r_2}$ is the magnetic potential at P_2. Hence the work done by the agency producing the magnetic field upon a unit magnet pole which moves from one point to another in the field is simply the *difference of magnetic potential* between the points, and is independent of the path followed in the travel from one point to the other.

If the magnetic potential of the terminal point of the travel is higher than that of the starting point, work must be done by an external agency to produce the motion of the unit testing pole, and the work so performed reappears as increased potential

energy of the system. If such a system is left to itself, the stored energy will be dissipated by the separation of the poles under the influence of their mutually repelling forces, provided the poles are free to move.

If, in the above discussion, the pole $+ m$ is replaced by an equal pole of opposite sign, or $- m$, all of the expressions for the forces and potentials will be reversed in sign. Repulsions become attractions and work done *by* the system becomes work done *upon* the system. This case is entirely analogous to that of two heavy particles of ordinary matter which attract each other with a force proportional to the product of the masses of the particles and inversely proportional to the square of the distance between them. In the case of attracting *magnet* poles the *magnetic* potential energy is the work required to move one pole to an infinite distance from the other; in the case of attracting *gravitational* masses the *gravitational* potential energy is also the work required to move one of them to an infinite distance from the other. On the one hand the work is done by moving *magnet* poles in a *magnetic* field; on the other hand by moving *gravitating* matter in a *gravitational* field.

It appears from the above derivation of the expression for magnetic potential at a point that its value is independent of direction. Magnetic potential is a scalar quantity as distinguished from vector quantities like field intensity, forces, etc. But there is an interesting relation between magnetic potential and field intensity, as may be seen from the following considerations:

The magnetic potential at a point distant r cm. from a point pole of strength m is

$$U = \frac{m}{r}$$

Differentiating U with respect to r,

$$\frac{dU}{dr} = -\frac{m}{r^2}$$

But $\frac{m}{r^2}$ is the field intensity H at the point due to the pole m, hence

$$H = -\frac{dU}{dr} \tag{28}$$

In general, if U is the magnetic potential at a point, and U is expressed as a function of the coördinates of the point, the first derivative of the function, taken with respect to any general direction ρ, will be equal, with a change of sign, to the component of field intensity in the direction ρ; that is

$$H_\rho = -\frac{\partial U}{\partial \rho} \tag{29}$$

18. Equipotential Lines and Surfaces.—The locus of all points in a magnetic field which have the same magnetic potential is called an *equipotential surface.* Linear (or curvilinear) elements of such a surface, connecting points of equal potential, are called *equipotential lines.* No work is required to carry a magnet pole from point to point in an equipotential surface or line. It follows, therefore, that the lines of force must intersect the equipotential surfaces at right angles, for if they did not there would be a component of force acting along the tangent to the equipotential surface at the point of intersection, consequently work would be required to move the pole along the surface, which is contrary to the assumption.

19. Field Intensity Due to a Circular Coil.—Let P, Fig. 19, represent a unit magnet pole on the axis of a plane circular coil of N turns and radius r cm. Let P be x cm. from the plane of the coil, and let the current in the latter be $\bar{I}$ abamperes. Then the force acting on an element dl of the coil is, by equation (7),

$$df = \frac{1}{r^2 + x^2} N\bar{I}dl$$

acting in the direction indicated in the figure; and the pole is acted upon by an equal force in the opposite direction. Resolving this force into components respectively parallel to, and perpendicular to, the axis of the coil, it will be clear that the sum of all the perpendicular components is zero since for each element of the coil which gives rise to a perpendicular component of force in one direction, there is a diametrically opposite element which produces an equal and opposite component.

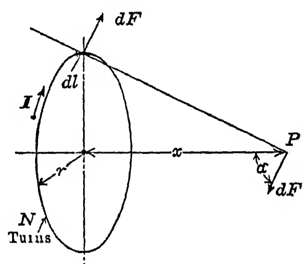


Fig. 19.—Field intensity on axis of coil.

The axial component of df is

$$dH = \frac{N\bar{I}dl}{r^2 + x^2}\cos\alpha = \frac{Nr\bar{I}dl}{(r^2 + x^2)^{3/2}}$$

Hence

$$H = \frac{N\bar{I}r}{(r^2 + x^2)^{3/2}}\int_0^{2\pi r} dl = \frac{2\pi N\bar{I}r^2}{(r^2 + x^2)^{3/2}} = \frac{2\pi NIr^2}{10(r^2 + x^2)^{3/2}} \qquad (30)$$

At the center of the coil, where $x = 0$,

$$H = H_0 = \frac{2\pi N\bar{I}}{r} = \frac{2\pi}{10}\frac{NI}{r} \qquad (31)$$

from which it follows that the absolute unit of current (abampere) may be defined as that current which, when flowing in a circular coil of one turn and 1 cm. radius, will act upon a unit magnet pole at the center with a force of 2π dynes.

20. Field Intensity on the Axis of a Solenoid.—Let Fig. 20 represent a solenoid of N turns uniformly distributed over the length l cm. It is desired to find the field intensity at a point P on the axis, distant D cm. from the center of the solenoid.

Consider an elementary section of the solenoid dx, distant x cm. from P. The element may be considered as a plane circular coil of $\frac{N}{l}dx$ turns; the field intensity due to this elementary ring at the point P is, by equation (30),

$$dH = \frac{2\pi\left(\frac{N}{l}dx\right)Ir^2}{10(r^2 + x^2)^{3/2}}$$

and the total field intensity is then

$$H = \frac{2\pi}{10}\frac{NIr^2}{l}\int_{-\left(\frac{l}{2}-D\right)}^{\frac{l}{2}+D}\frac{dx}{(r^2 + x^2)^{3/2}}$$

$$= \frac{2\pi}{10}\frac{NI}{l}\left[\frac{\frac{l}{2} + D}{\sqrt{r^2 + \left(\frac{l}{2} + D\right)^2}} + \frac{\frac{l}{2} - D}{\sqrt{r^2 + \left(\frac{l}{2} - D\right)^2}}\right] \qquad (32)$$

At the center of the solenoid, where $D = 0$, this becomes

$$H_0 = \frac{2\pi NI}{10\sqrt{r^2 + \frac{l^2}{4}}}$$

which reduces to

$$H_0 = \frac{4\pi NI}{10l} \tag{33}$$

if l is large compared with r.

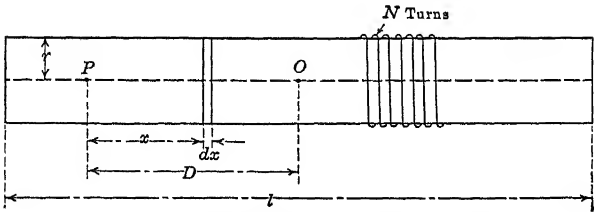


FIG. 20.—Field intensity on axis of solenoid.

At the ends of the solenoid, where $D = \frac{l}{2}$,

$$H = H_e = \frac{2\pi NI}{10l}$$

or half as great as at the center, provided l is large compared with r.

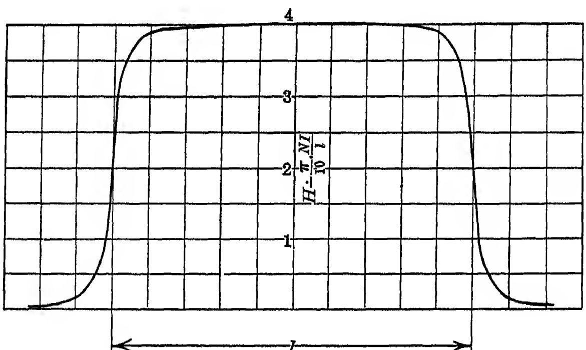


FIG. 21.—Variation of field intensity along axis of solenoid.

Fig. 21 shows the variation of H along the axis of a solenoid whose length is twenty-five times its radius, *i.e.*, $\frac{r}{l} = 0.04$. The value of H_0 is a trifle less than $\frac{4\pi NI}{10l}$, namely, $\frac{3.9872\pi NI}{10l}$; and $H_e = \frac{1.9984\pi NI}{10l}$ instead of $\frac{2\pi NI}{10l}$. It will be observed that H is

very nearly constant over the greater part of the axis, and that it falls off abruptly near the ends.

The physical interpretation of these facts concerning the variation of H along the axis is as follows: For some distance on either side of the middle section of the solenoid the lines of force inside the winding are nearly parallel, hence the field is nearly uniform and H will be practically constant; near the ends of the solenoid the lines diverge in the manner indicated in Fig. 1, and the greater the divergence the more rapidly will H decrease.

21. Magnetic Potential on the Axis of a Circular Coil.—It has been shown in Art. 19 that the field intensity on the axis of a circular coil, at any distance x from the plane of the coil, is given by

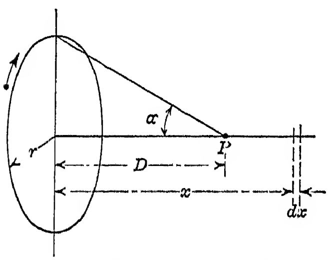


FIG. 22.—Magnetic potential on axis of circular coil.

$$H = \frac{2\pi NIr^2}{(r^2 + x^2)^{3/2}}$$

this being the force in dynes that would act upon a unit magnet pole placed at the point. With the current flowing as indicated in Fig. 22, the unit (positive) pole would be urged to the left, or toward the coil. To move the pole to the right over a distance dx there must be expended

$$dU = Hdx = \frac{2\pi NIr^2dx}{(r^2 + x^2)^{3/2}} \text{ ergs}$$

of work, and the total work required to move the unit pole out to infinity from a point distant D cm. from the coil is

$$U = 2\pi NIr^2 \int_D^\infty \frac{dx}{(r^2 + x^2)^{3/2}} = 2\pi NI \left(1 - \frac{D}{\sqrt{r^2 + D^2}}\right)$$
$$= 2\pi NI\,(1 - \cos\alpha) \qquad (34)$$

where α is the semi-angle of the right cone subtended at the point P by the coil. But $2\pi(1 - \cos\alpha) = \omega$ is the solid angle at the vertex of the cone, hence

$$U = \omega N\bar{I} \qquad (35)$$

If the test pole had been of strength m units, the work done would have been m times as great as the above amount, or

$$U_m = \omega mN\bar{I} \qquad (36)$$

The expression U, equation (35), is the magnetic potential at a general point on the axis of the coil; it represents the work required to move a unit pole from the point out to an infinite distance, when the current flow is as indicated. If the current is reversed, U becomes the work required to bring the unit pole from an infinite distance up to the point in question.

22. General Expression for the Magnetic Potential Due to a Coil of any Shape at any Point.—Equation (36) can be put into a more convenient form, as follows: The total flux emanating from a pole of strength m units is

$$\Phi = 4\pi m \text{ maxwells},$$

for, if the pole is in air, the field intensity at a distance r cm. is $H = \frac{m}{r^2}$, and the resultant flux density is $B = H = \frac{m}{r^2}$ gausses. The locus of all points at a distance of r cm. from m is a sphere whose total area is $4\pi r^2$ sq. cm., so that the total flux across the surface of the sphere is

$$\Phi = B \times \text{area} = \frac{m}{r^2} \times 4\pi r^2 = 4\pi m$$

It follows, therefore, since the flux Φ is uniformly distributed over the entire surface, that the part of the flux, φ, within a solid angle ω, is to the entire flux as ω is to the entire solid angle 4π, hence

$$\frac{\varphi}{\Phi} = \frac{\omega}{4\pi}$$

or

$$\varphi = \omega m$$

Therefore,

$$U_m = \varphi N\bar{I} = \lambda\bar{I} \text{ ergs} \tag{37}$$

where $\lambda = \varphi N$, the product of the flux and the number of turns with which it links, is called the number of *flux linkages*. Or, in other words, the potential energy of a current in a magnetic field produced by some other agency is the product of the current (in abamperes) and the number of flux linkages.

The above expression for the potential energy of a magnet pole in the presence of a current was derived by assuming a circular coil and allowing the magnet pole to move along the axis of the coil. It can be shown, however, that the resulting equations (35)

and (36) are general. For let Fig. 23 represent a coil of any shape which may or may not be plane, and let ω represent the solid angle subtended by it at a point pole m placed in any general position. Then the flux emanating from m and linking the coil is

$$\varphi = \omega m \text{ maxwells}$$

Now allow the pole to move in such a manner that the solid angle changes in magnitude by $d\omega$ in a time dt. The flux linked with the coil will change by

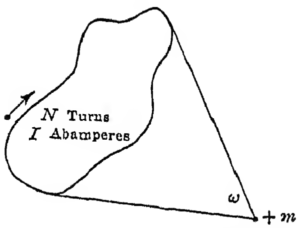


FIG. 23.—Magnetic potential due to coil of any shape.

$$d\varphi = m \; d\omega$$

and there will be induced in the coil an e.m.f.

$$E = -N\frac{d\varphi}{dt} \text{ abvolts}$$

If this change of flux occurs while a current of $\bar{I}$ abamperes is flowing in the coil, the work done is given by

$$dW = -\bar{E}\bar{I}dt = NId\varphi = mNId\omega \text{ ergs}$$

and the total work required to bring the pole from an infinite distance ($\omega = 0$) to a point near the coil ($\omega = \omega'$) is

$$W = mN\bar{I}\int_0^{\omega'} d\omega = m\omega' NI = \varphi NI = \lambda I$$

or the same as equations (36) and (37).

23. Magnetomotive Force.—Let Fig. 24 represent the side and front elevations of a plane coil of any configuration, carrying a current of $\bar{I}$ abamperes, and let a unit magnet pole be placed at a point P in the plane of the coil but outside its boundary. None of the flux emanating from the pole will pass through the coil, and therefore the magnetic potential at P is zero. Now let the unit pole be carried along any path to a point Q infinitely close to the plane of the coil. The solid angle subtended at Q by the coil

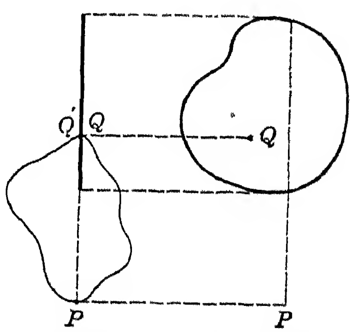


FIG. 24.—Closed path linking with coil.

is 2π, the magnetic potential at Q is then $2\pi N\bar{I}$, and the difference of magnetic potential between P and Q, that is, the amount of work required to carry the pole from P to Q, is $2\pi N\bar{I}$ ergs. If the pole is now carried back to P along any path which requires the pole to be threaded completely through the coil, as $Q'P$, a further amount of work equal to $2\pi N\bar{I}$ ergs must be expended, making a *total of* $4\pi N\bar{I}$ *ergs to carry the pole once around a closed path linking with the coil.* This is called the *magnetomotive force* (m.m.f.) of the coil. The unit of m.m.f. is called the *gilbert.*

It should be particularly noted that m.m.f. is not a *force;* it is of the nature of *work per unit magnet pole.* It is exactly analogous to e.m.f., which is likewise not a force, but work per unit of electrical quantity (see Art. 14).

Although the expression $4\pi N\bar{I}$ has been derived on the assumption that the N turns are concentrated in a plane coil, the entire argument applies equally well to a coil of any shape whatever, including solenoids like Fig. 1. Thus, in Fig, 1, the work required to carry a unit magnet pole once around a closed path like P_1, linking all of the N turns, is

$$4\pi N\bar{I} = \frac{4\pi}{10} NI$$

Similarly the work required to carry a unit magnet pole once around a closed path P_2 is $\frac{4\pi}{10} N'I$, where N' is the number of turns included by the curve P_2.

In Art 20 it was shown that the field intensity at the middle point of the axis of a solenoid is

$$H_0 = \frac{4\pi}{10}\frac{NI}{l} = \frac{\text{m.m.f.}}{\text{length}} = \frac{\text{gilberts}}{\text{centimeters}}$$

which explains the statement in Art. 4, that the unit in which H is measured is the gilbert-per-centimeter.

From the last equation it follows that

$$H_0 l = \frac{4\pi}{10} NI = \text{m.m.f.}$$

which means that if the field intensity were constant all along the axis and equal to H_0, the work required to carry the unit pole from end to end of the solenoid would be

$$\text{force} \times \text{distance} = H_0 l = \frac{4\pi}{10} NI$$

However, the field intensity is not constant along the axis, so that the total amount of work to describe the complete path P_1 of Fig. 1 is

$$\frac{4\pi}{10} NI = \int H dl \tag{38}$$

which states that the *m.m.f. is the line integral of the magnetic force.*

It is interesting to note that the area under the curve of Fig. 21, within the limits of l, represents the work required to carry a unit magnet pole through the solenoid from one end to the other, while the area of the rectangle enclosing the curve, within the same limits, represents the work required to carry the unit pole once around a closed path linking all the turns of the solenoid. It follows, therefore, that the area above the curve, but inside the rectangle, represents to the same scale the work required to carry the magnet pole along the path from end to end of the solenoid, but outside of its windings.

24. B-H Curves.—Hysteresis.—The field intensity at the center of the solenoid of Fig. 20 is

$$H = \frac{4\pi}{10} \frac{NI}{l} \text{ gausses}$$

and if the lines of force passed straight through the solenoid parallel to the axis, and were uniformly distributed over the area of cross-section A, the flux across any section would be, assuming an air core,

$$\phi = AH = \frac{4\pi}{10} \frac{NI}{l} A \text{ maxwells}$$

Now let the solenoid be provided with an iron core of length l cm. and cross-section A sq. cm., as in Fig. 3; induced poles of strengths $+ m$ and $- m$ will then be developed at the ends of the bar, and each of these poles will give rise to a field intensity at the center of the solenoid equal to $\frac{m}{(l/2)^2}$ and opposite in direction to H. The resultant field intensity at the center is then

$$H = \frac{4\pi}{10} \frac{NI}{l} - \frac{2m}{(l/2)^2} = \frac{4\pi}{10} \frac{NI}{l} - \frac{8m}{l^2} \tag{39}$$

In other words, the induced poles exert a demagnetizing effect upon the field which produces them. This demagnetizing or "end effect" becomes negligibly small if the solenoid and core are long, and will be neglected in the remainder of this discussion.

From the pole $+ m$ there will emanate $4\pi m$ lines of force, all of which find their way back to the pole $- m$ through the surrounding air. These lines of force may be assumed to be continued through the iron core back to the starting point. Inside the iron, therefore, the total flux consists of the original HA lines of force and the $4\pi m$ lines of induction, or

$$\Phi = AH + 4\pi m$$

Assuming the flux Φ to be uniformly distributed over the cross-section A, the flux density is

$$B = \frac{\Phi}{A} = H + 4\pi \frac{m}{A} = H \left(1 + 4\pi \frac{m}{AH}\right) = \mu H \qquad (40)$$

where

$$\mu = \frac{B}{H} = 1 + 4\pi \frac{m}{AH} \qquad (41)$$

is the permeability of the material of the core. It is the ratio of the flux density in the material to the intensity of the inducing field, and is therefore a measure of the specific magnetic conductance of the material. Its magnitude is dependent upon the ratio $\frac{m}{AH}$ or $\frac{m}{H}$, that is, the ratio of the strength of the induced pole to the intensity of the inducing field. The better the material from a magnetic standpoint, or the more it is susceptible to magnetization, the greater will be the strength of the induced pole for a given inducing field, hence the ratio $\frac{m}{AH}$ is called the *susceptibility*.

There is no known relation between m and H, so that it is impossible to express either μ or B in terms of H other than empirically. The relation must be found experimentally for each material. Curves showing the relation between B and H are called normal B-H curves, or magnetization curves. Fig. 25 shows a number of such curves for several different kinds of commercial iron and steel, and Fig. 26 shows the relation between *reluctivity* $(1/\mu)$, and the excitation in ampere-turns per inch, for

the same materials whose *B-H* curves are shown in Fig. 25. In the equation $H = \frac{4\pi}{10}\frac{NI}{l}$, the term NI represents the number

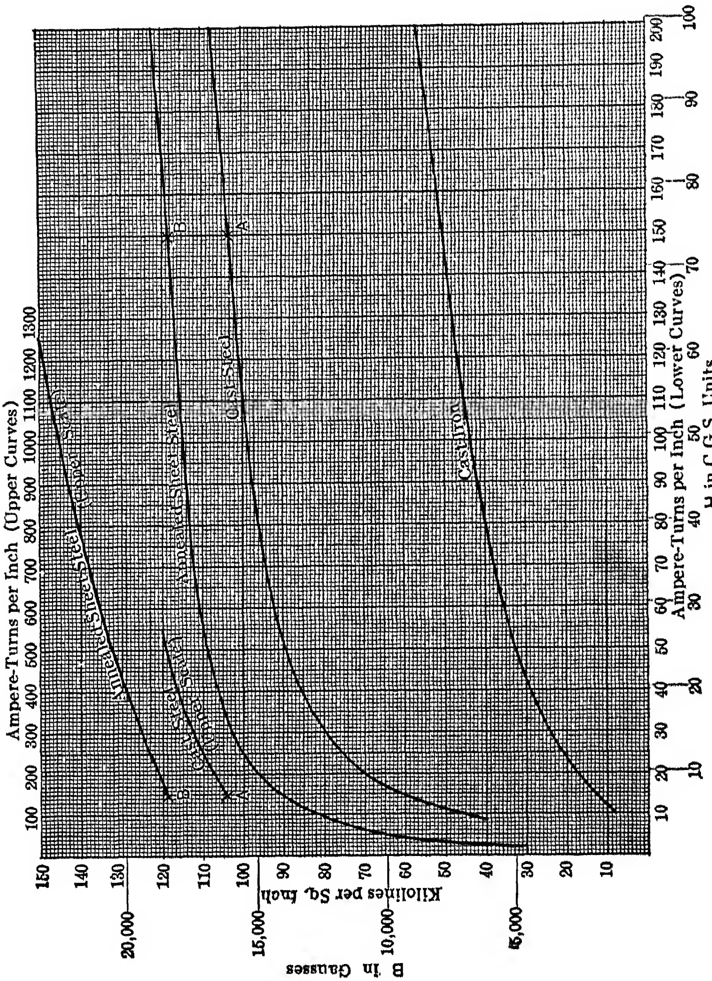


FIG. 25.—B-H curves.

of *ampere-turns* of the exciting winding, and NI/l expresses the number of ampere-turns per cm. It follows that

$$\text{ampere-turns per cm.} = \frac{NI}{l} = \frac{10}{4\pi}H = 0.8H$$

and

$$\text{ampere-turns per inch} = \frac{10}{4\pi} H \times 2.54 = 2.02\ H$$

In practical calculations it is more convenient to deal with ampere-turns per cm. (or with ampere-turns per inch) than with H,

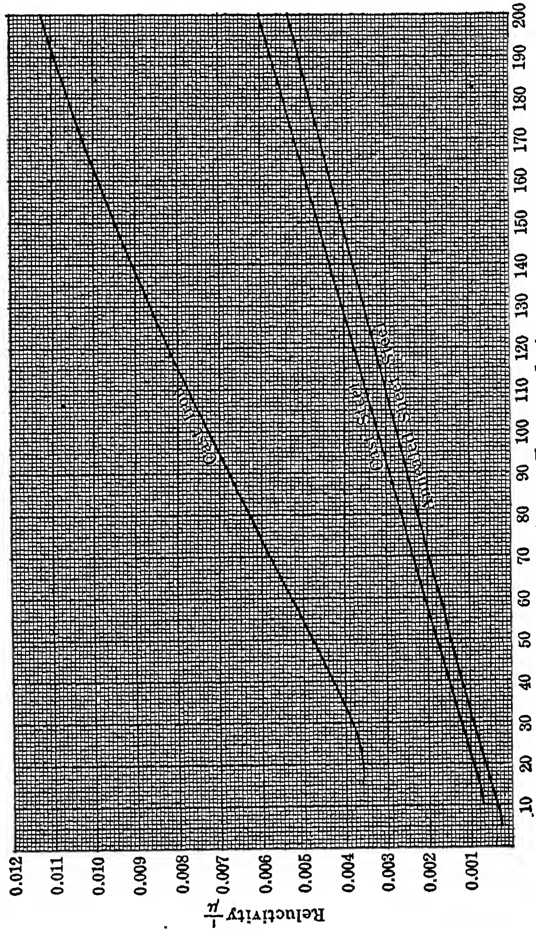


Fig. 26.—Reluctivity curves.

for if the value of the former quantity, corresponding to a given value of B, can be found, the total excitation (in ampere-turns) is simply the product of ampere-turns per cm. (or per inch) and the length of the circuit, expressed in cm. (or in inches). Con-

sequently, magnetization curves are commonly drawn with B (lines per sq. cm.) as ordinates and with $0.8H$ (= ampere-turns per cm.) as abscissas; or, if English units are employed, the curves are drawn with ordinates $B \times (2.54)^2$ (= lines per sq. in.), and with abscissas 2.02 H (= ampere-turns per inch).

Hysteresis.—Let Fig. 28 represent a sample of iron or steel in the form of a ring, upon which is wound a uniformly distributed magnetizing coil of N turns and a "search coil" consisting of n turns of fine wire. Let the magnetizing coil N be connected through a switch and an adjustable resistance to a suitable source of current, and let the search coil be connected through a resistance to a ballistic galvanometer. Starting with the test sample of iron entirely demagnetized, and with the exciting circuit open, let the adjustable resistance be set to such a value that on closing the circuit a small current will flow. The current sets up a magnetic flux within the core, the flux cuts the turns of the search coil and induces in it a transient e.m.f., with the result that a single pulse of current is produced in the ballistic galvanometer, and the latter deflects. The first swing of the galvanometer is then proportional to the change of flux through the sample, and by properly calibrating the galvanometer the flux can be determined; since the cross-section of the sample is known, the flux density B corresponding to the magnetizing force H $\left(= \frac{4\pi}{10} \frac{NI}{l}\right)$ is also determined.

If the magnetizing current is now suddenly increased to a larger value by short circuiting a portion of the adjustable resistance in series with coil N, the increment of current will produce an increment of flux which can be computed from the new swing of the galvanometer. In this way, increasing the current step by step, there will be obtained a series of pairs of values of B and H, which, when plotted, will be the normal B-H curve.

Suppose that this process has been carried out, without at any time decreasing the current, until some particular value of H, say $H_{max.}$, Fig. 27, has been reached, and that the magnetizing current is then decreased step by step until it is again zero. It will be found that the curve of descending values of B and H does not coincide with the original curve, but lies above it, as shown in the figure. When the magnetizing current has been

reduced to zero, reverse the connections between the coil N and the source of current, and again increase the current (which must now be considered to be negative since its direction is opposite to that of the original flow) until a value of $H_{max.}$ has again been reached, whereupon the current is again decreased in magnitude, step by step, until it is zero; then again reverse the connections between coil N and the source of current, and increase the current step by step until the magnetizing force is again equivalent to $H_{max.}$. The relation between B and H during this complete cycle of changes will have the form of the loop of Fig. 27.

The form of the loop indicates that the changes in the magnetization of the core lag behind the changes in the magnetizing force. This phenomenon is called *hysteresis*, the term being derived from a Greek word meaning "to lag behind." Hysteresis is caused by a sort of molecular friction, hence represents dissipation of energy, and if the successive changes in magnetizing force

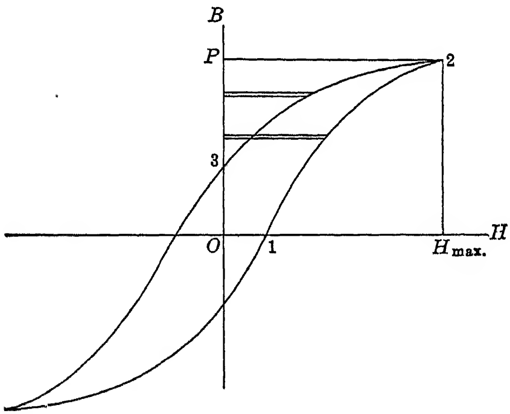


FIG. 27.—Hysteresis loop.

occur with sufficient rapidity, as when the magnetizing current is an alternating one, the molecular friction will manifest itself by a rise of temperature of the core.

The amount of energy dissipated during one complete cycle can be shown to be proportional to the area of the hysteresis loop. Thus, in Fig. 27, let the magnetizing force increase from H to $(H + dH)$, and let the corresponding change in flux density be from B to $(B + dB)$. The flux linked with the magnetizing

coil will change by the amount $d\varphi = AdB$, where A is the cross-section of the core, and the e.m.f. induced in the coil will be

$$E = -N\frac{d\varphi}{dt} \times 10^{-8} = -AN\frac{dB}{dt} \times 10^{-8} \text{ volts}$$

Since the current in the coil is

$$I = \frac{10}{4\pi}\frac{Hl}{N}$$

the energy consumed in the circuit in the time dt is

$$dW = -EI\,dt \times 10^7 = \frac{Al}{4\pi} HdB \text{ ergs}$$

But HdB represents the area of the element indicated in Fig. 27, hence the total energy dissipated in the interval corresponding to the portion of the loop between points 1 and 2 is

$$W = \frac{Al}{4\pi}\int HdB$$

where $\int HdB$ represents the area $012P$.

During the interval between points 2 and 3 the flux is decreasing instead of increasing, while the current is in the same direction as before, so that energy is returned to the circuit instead of being absorbed. This returned energy is

$$W' = \frac{Al}{4\pi}\int_{B=B_{max.}}^{B=03} HdB$$

where the integral represents the area $23P$. The net energy loss during this part of the cycle is therefore proportional to the part of the loop 0123. Treating the remainder of the cycle in a similar manner, the total dissipation of energy (in ergs) is

$$W = \frac{Al}{4\pi}\int_{-B_{max.}}^{B_{max.}} HdB$$

where Al is the volume of the core, and the integral is proportional to the area of the loop.

The above expression cannot be integrated for the reason that there is no known relation between B and H; but it has been found by Steinmetz that the relation between the hysteresis loss per cubic centimeter per cycle and the maximum value of the flux density is

$$w = \text{constant} \times (B_{max.})^{1.6} \qquad (42)$$

where the constant term depends upon the material of the core (see Art. 172, Chap. X).

25. The Law of the Magnetic Circuit.—*Magnetic Reluctance.*—The demagnetizing effect of the ends of the core of Fig. 3, referred to in the preceding section (equation (39)) can be eliminated by bending the core into a closed ring form, as in Fig. 28. The value of H will then be uniform around the entire circular axis of the coil, and will be

$$H = \frac{4\pi}{10}\frac{NI}{l}$$

where l is the mean length of the core in centimeters. The total flux through the core will then be

$$\Phi = AB = A\mu H = \frac{4\pi}{10}\frac{NI}{l}\mu A$$

or

$$\Phi = \frac{\frac{4\pi}{10}NI}{\frac{l}{\mu A}} \qquad (43)$$

The numerator of equation (43) is the m.m.f. of the solenoid and the denominator is the *reluctance* of the magnetic circuit. It will be noted that the expression for reluctance is of the same form as that for the resistance of an electrical circuit, for it is proportional to the length and inversely proportional to the cross-section; moreover, the permeability μ appears in the expression for the reluctance in exactly the same manner as does the specific conductance in the expression for electrical resistance, hence the reference to permeability (Art. 24) as "specific magnetic conductance." The unit in which reluctance is measured is called the *oersted.* The reciprocal of reluctance, or $\frac{\mu A}{l}$, is called *permeance.*

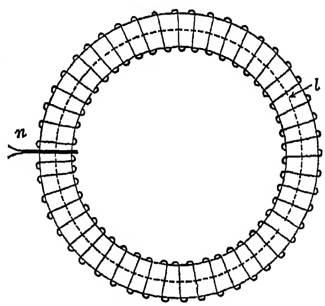


Fig. 28.—Ring core.

Equation (43) is of the form

$$\text{flux} = \frac{\text{m.m.f.}}{\text{reluctance}}$$

or, in terms of the units themselves,

$$\text{maxwells} = \frac{\text{gilberts}}{\text{oersteds}},$$

which corresponds term by term with Ohm's law of the electric circuit, namely,

$$\text{current} = \frac{\text{e.m.f.}}{\text{resistance}}$$

or

$$\text{amperes} = \frac{\text{volts}}{\text{ohms}}$$

The relations represented by equation (43) constitute the so-called law of the magnetic circuit.

26. Applications of Law of Magnetic Circuit.—Magnetic circuits, like electric circuits, may be joined in series, in parallel, or in series-parallel, and the solution of problems involving any of these combinations is in every case carried out by methods that are the exact analogues of those used in the corresponding electrical circuits. In the form in which such problems arise in practice, it is either necessary to compute the number of ampere-turns required to maintain a given flux through the circuit, or conversely, to compute the flux that will be produced by a given number of ampere-turns.

Typical forms of magnetic circuits are represented in Fig. 29, part *a* representing a simple series circuit, part *b* two circuits in parallel; in each case the corresponding type of electrical circuit is indicated.

The following examples will serve to illustrate the methods to be employed in the solution of ordinary problems; more detailed treatment of the problems encountered in the calculations of performance characteristics of generators and motors is given in Chap. IV.

1. Series Circuits.—A circuit consisting of a number of reluctances in series will have a total reluctance given by

$$\frac{l_1}{\mu_1 A_1} + \frac{l_2}{\mu_2 A_2} + \cdot \cdot \cdot \cdot + \frac{l_n}{\mu_n A_n}.$$

and the resultant flux through the circuit will be

$$\Phi = \frac{\frac{4\pi}{10} NI}{\frac{l_1}{\mu_1 A_1} + \frac{l_2}{\mu_2 A_2} + \cdot \cdot \cdot \cdot + \frac{l_n}{\mu_n A_n}}$$

where NI is the total number of ampere-turns acting upon the circuit as a whole. This equation can be written

$$\frac{4\pi}{10}NI = \Phi\frac{l_1}{\mu_1 A_1} + \Phi\frac{l_2}{\mu_2 A_2} + \ . \ . \ . \ . \times \Phi\frac{l_n}{\mu_n A_n} \qquad (44)$$

and in this form each term of the right-hand member represents the m.m.f. (in gilberts) required to maintain the flux through the corresponding portion of the circuit. The total m.m.f. is then merely the sum of the m.m.fs. required by the individual parts. In the analogous electrical circuit

$$I = \frac{E}{R_1 + R_2 + \ . \ . \ . + R_n}$$

and

$$E = IR_1 + IR_2 + \ . \ . \ . \ . + IR_n$$

or the total e.m.f. required to maintain the current through the circuit is the sum of the potential drops in each part of the circuit.

While the foregoing relations between ampere-turns, flux and reluctance are strictly correct, equations like (44) which embody these relations are not convenient for practical calculations for the reason that they involve the permeability of each portion of the circuit, and the permeability is in turn a function of the flux density and the kind of material in each part of the circuit, and must be computed from the relation $\mu = \frac{B}{H}$ before its value can be substituted. The practical, because simpler and more direct, method of computing the ampere-turns follows directly from equation (44), as follows:

Each term on the right-hand side of (44) is of the form

$$\Phi\frac{l_x}{\mu_x A_x} = \frac{\Phi}{\mu_x A_x} l_x = H_x l_x$$

where H_x is the number of gilberts per centimeter in the part l_x of the circuit. But $H_x = \frac{4\pi}{10}\frac{(NI)_x}{l_x}$, where $(NI)_x$ is the number of ampere-turns required to maintain the flux Φ in the path l_x, and $\frac{(NI)_x}{l_x}$ is evidently the corresponding number of *ampere-turns per centimeter* (a.t./cm.) Hence, substituting in (44), and canceling the common factor $\frac{4\pi}{10}$.

$$NI = \left(\frac{\text{a.t.}}{\text{cm.}}\right)_1 l_1 + \left(\frac{\text{a.t.}}{\text{cm.}}\right)_2 l_2 + \ . \ . \ . \ . + \left(\frac{\text{a.t.}}{\text{cm.}}\right)_n l_n \qquad (45)$$

If inch units are used,

$$NI = \left(\frac{\text{a.t.}}{\text{in.}}\right)_1 l_1'' + \left(\frac{\text{a.t.}}{\text{in.}}\right)_2 l_2'' + \ . \ . \ . \ . \ \left(\frac{\text{a.t.}}{\text{in.}}\right)_n l_n'' \qquad (46)$$

The numerical value of the quantity $\left(\frac{\text{a.t.}}{\text{in.}}\right)$ for a given material can be obtained directly from the curves of Fig. 25 when the flux density is known. Thus, in Fig. 29*a*, let it be required to find the number of ampere-turns necessary to produce a total flux of 160,000 maxwells on the assumption that the core is made of cast iron. Assume that the mean path of the lines of force follows the center of gravity of the cross-section and that at the corners the mean path follows quadrants of circles.

$\therefore l_1$ = length of path in cast iron $= 2(6 + 4) + 2\pi - 0.125 =$ 26.15 in.

l_a = length of path in air = 0.125 in.

B = flux density in iron and in air-gap $= \frac{160,000}{4} = 40,000$ lines per sq. in.

From the curve for cast iron in Fig. 25 it is found that a flux density of 40,000 lines per sq. in. corresponds to an excitation of 79 ampere-turns per inch length of core. Hence the number of ampere-turns required by the core is $79 \times 26.15 = 2060$. The number of ampere-turns required to maintain the flux through the air-gap, where $\mu = 1$, may be found from the relation $B = H = \frac{4\pi}{10}\frac{NI}{l}$ (where all quantities are in metric units), from which

$$NI = \frac{10}{4\pi}Bl = 0.8 \times \frac{\text{lines per sq. in.}}{(2.54)^2} \times (\text{air-gap in inches} \times 2.54)$$

$$= 0.3133B''l''$$

where B'' = flux density in lines per sq. in. and l'' is the length of the air-gap in inches. Hence the ampere-turns for the air-gap are

$$NI = 0.3133 \times 40,000 \times \tfrac{1}{8} = 1567$$

and the total excitation for the entire circuit is $2060 + 1567 =$ 3627 ampere-turns.

2. Parallel and Series-parallel Circuits.—In Fig. 29*b* two magnetic circuits, each of the type illustrated in Fig. 29*a*,

are connected in parallel. Just as in the corresponding electrical circuit the entire battery e.m.f. acts equally on each of the parallel electric circuits, so does the entire m.m.f. of the exciting circuit act on each of the parallel magnetic circuits. In the case of the magnetic circuit of Fig. 29*b*, the flux in each part is to be computed as though the other part were not present; if the parts are exactly alike the flux will be the same in each.

Suppose, for example, that the left-hand circuit of Fig. 29*b* is exactly the same as that of Fig. 29*a*, but that the right-hand circuit, though having identical dimensions, is made of cast steel instead of cast iron. Assuming that the flux through the left-hand branch is again 160,000 maxwells, the coil must supply an excitation of 3627 ampere-turns. It does not follow that the flux through the cast-steel circuit will be 160,000 maxwells, and indeed it does not have that value because the reluctance of the cast steel is less than that of the cast iron. The problem is then to find that value of flux through the cast-steel circuit which will require 3627 ampere-turns for its maintenance. This can be done by trial, as follows:

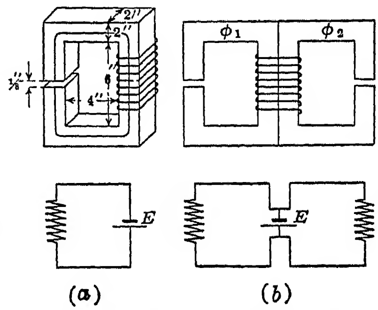


FIG. 29.—Typical magnetic and electric circuits.

Assume a series of values of the total flux, and for each value compute the corresponding total number of ampere-turns. Plot flux and ampere-turns, extending the computations far enough so that a curve may be drawn that will include within its range the given number of ampere-turns; the actual flux corresponding to the latter can then be read from the curve. In the case under discussion the total flux through the cast-steel circuit is in this way found to be approximately 303,200 maxwells.

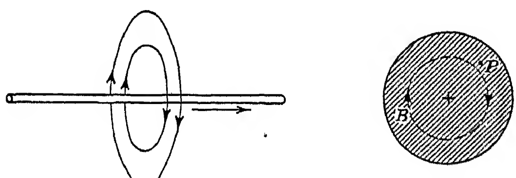
In such a case as that illustrated by this particular problem, the reluctance of the cast-steel part of the circuit is so small compared with that of the air-gap that a first approximation to the final result may be found by assuming that the reluctance of the path through the steel is negligible, and that the entire excitation is consumed in maintaining the flux through the air-gap. On

this basis the flux density in lines per sq. in. would be given by

$$3627 = 0.3133 \times B'' \times \tfrac{1}{8}$$

or $B'' = 92{,}800$ lines per sq. in. and $\Phi = 371{,}200$. Reference to the curves of Fig. 25 shows that at this flux density the steel would require 1635 ampere-turns, so that the entire circuit would require $3627 + 1635 = 5262$ ampere-turns, or considerably more than the available number. It is then necessary to select a smaller value of B'', repeating the calculations until the given number of ampere-turns is included in the range of trial values.

Kirchhoff's laws are applicable to the magnetic circuit as well as the electric circuit. Thus at any junction in a magnetic circuit, the number of lines of induction coming up to the junction must be equal to the number leaving it, for the reason that lines of induction are always closed loops. This is equivalent to Kirchhoff's first law of the electric circuit. Again, in any closed magnetic circuit, the algebraic sum of the drops of magnetic potential must be zero. If in any part of the closed magnetic circuit the flux is Φ and the reluctance R, the drop of magnetic potential is ΦR, and the summation of all such drops must then be equal to the summation of all the active m.m.fs., with due attention to the sign of each term, following the rules described in Art. 16.



(a) (b)

FIG. 30.—Lines of force surrounding a conductor.

27. Self-induction.—When a straight wire, Fig. 30a, carries a current in the direction indicated, the wire will be surrounded by magnetic lines of force as shown. As the current increases from zero to any arbitrary value, the flux will increase proportionally from zero, and may be thought of as issuing from the center of the wire and expanding outward, like spreading ripples on a pond. The lines of force thus expanding cut across the wire in the manner indicated in Fig. 30b, which represents a cross-

section of the wire in (a) when viewed from the left. The expanding line of force B is about to cut across the longitudinal filament of the wire shown at P, the motion of the line of force at this point being radially outward. Relatively, the effect is the same as though the filament were moving radially inward; so that if Fleming's (right-hand) rule is applied, it is found that the induced e.m.f. is directed outward from the plane of the paper, or in opposition to the direction of the current flow. The whole effect is in accord with Lenz's law; the original change in the current strength which produced the change in the flux immediately calls into existence an opposing e.m.f. which tends to retard the change in current. Conversely, the same line of reasoning will show that an initial decrease of current induces an e.m.f. of reversed direction, which tends to maintain the current at its original strength. This e.m.f. being self-induced, is called the *e.m.f. of self-induction.*

Let Fig. 31 represent a coil of wire wound on a core having constant permeability μ, a cross-section of A sq. cm., and a mean length of magnetic path of l cm. The mean path is to be taken as passing through the center of gravity of the cross-section of the core. On passing a current of i amperes through the coil there will be produced a flux

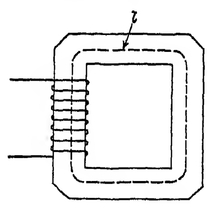


FIG. 31.—Inductive circuit.

$$\phi = \frac{\frac{4\pi}{10} Ni}{\frac{l}{\mu A}} = \frac{4\pi}{10} \frac{Ni}{l} \mu A$$

and a change of current di will produce a change of flux of

$$d\phi = \frac{4\pi}{10} \frac{N di}{l} \mu A$$

This change of flux will then induce an e.m.f.

$$E = -N \frac{d\phi}{dt} \times 10^{-8} = -\frac{4\pi}{10} \frac{N^2 \mu A}{l} \frac{di}{dt} \times 10^{-8} = -L \frac{di}{dt} \qquad (47)$$

where

$$L = \frac{4\pi}{10} \frac{N^2 \mu A}{l} \times 10^{-8} \qquad (48)$$

The quantity L is called the *coefficient of self-induction* or the *self-inductance* of the circuit, and in the practical system of units

is measured in terms of a unit called the *henry*. It is evident from equation (48) that the self-inductance is proportional to the square of the number of turns linked with the flux, and is dependent upon the shape, size and material of the magnetic circuit. Its magnitude is of very great importance in all electrical circuits in which the current is changing in strength, as for instance, in those coils of a direct-current generator or motor which are undergoing commutation (Chap. VIII).

From equation (47) it is seen that the self-inductance L of a circuit is numerically equal to the e.m.f. induced in it by a current which is changing at the rate of 1 ampere per second $\left(\frac{di}{dt} = 1\right)$; that is, a circuit has a self-inductance of 1 henry if a change of current of 1 ampere per second induces an e.m.f. of 1 volt. The self-inductance may also be defined in another way. Thus from equation (47)

$$L \frac{di}{dt} = N \frac{d\phi}{dt} \times 10^{-8}$$

or

$$L = N \frac{d\phi}{di} \times 10^{-8} \tag{49}$$

in this equation $\frac{d\phi}{di}$ is numerically equal to the rate of change of flux with current, or it is the number of lines of force produced by 1 ampere. Equation (49) says that the product of the number of lines produced by 1 ampere, multiplied by the number of turns which this flux links, and divided by 10^8, is equal to the coefficient of self-induction. The product of flux per ampere by the number of turns with which this flux links is called the number of flux linkages per ampere, so that, briefly, *the self-inductance is equal to the number of flux linkages per ampere, divided by* 10^8.

28. Mutual Induction.—If two circuits of N_1 and N_2 turns, respectively, are so placed with respect to each other that the magnetic field due to a current in one of these links in whole or in part with the other, a change in the current strength in the first circuit will induce an *e.m.f. of mutual induction* in the second circuit. It is clear that the magnitude of this will depend upon the geometrical shapes and relative positions of the two circuits, as well as upon the rate of change of current in the inducing circuit.

Let a current of i_1 amperes in the first circuit produce a flux Φ_1 such that

$$\Phi_1 = \frac{\frac{4\pi}{10} N_1 i_1}{\frac{l_1}{\mu_1 A_1}} = C_1 N_1 i_1 \tag{50}$$

A part of this flux, or

$$\varphi_1 = K_1 \Phi_1 = K_1 C_1 N_1 i_1 \tag{51}$$

(where $K_1 \leqq 1$) will link with the second circuit of N_2 turns, so that the total number of linkages with the second circuit is

$$\lambda_{21} = N_2 \varphi_1 = K_1 C_1 N_1 N_2 i_1 \tag{52}$$

and if the second circuit is traversed by a circuit of i_2 amperes its potential energy in the presence of the first circuit is, by Art. 22,

$$U_{21} = \lambda_{21} \frac{i_2}{10} = \tfrac{1}{10} K_1 C_1 N_1 N_2 i_1 i_2 \qquad \text{ergs} \tag{53}$$

Similarly, the current i_2 in the second circuit will produce a total flux

$$\Phi_2 = \frac{\frac{4\pi}{10} N_2 i_2}{\frac{l_2}{\mu_2 A_2}} = C_2 N_2 i_2 \tag{54}$$

of which a part

$$\varphi_2 = K_2 \Phi_2 = K_2 C_2 N_2 i_2 \tag{55}$$

(where $K_2 \leqq 1$) will link with the first circuit of N_1 turns, so that the total number of linkages with the first circuit is

$$\lambda_{12} = N_1 \varphi_2 = K_2 C_2 N_1 N_2 i_2 \tag{56}$$

The potential energy of the first circuit in the presence of the second is

$$U_{12} = \lambda_{12} \frac{i_1}{10} = \tfrac{1}{10} K_2 C_2 N_1 N_2 i_1 i_2 \qquad \text{ergs} \tag{57}$$

But U_{21} must be equal to U_{12}, since the potential energy of the system can have but one value;

$$\therefore K_1 C_1 N_1 N_2 = K_2 C_2 N_1 N_2 \tag{58}$$

From (52)

$$K_1 C_1 N_1 N_2 = \frac{N_2 \varphi_1}{i_1}$$

or it is the number of flux linkages with the second circuit due to unit current in the first circuit; and from (56)

$$K_2 C_2 N_1 N_2 = \frac{N_1 \varphi_2}{i_2}$$

which represents the number of flux linkages with the first circuit due to unit current in the second. Hence, from (58), it follows that unit current in one circuit will produce the same number of linkages in the other, as unit current in the latter will produce in the former.

When the current in circuit No. 1 changes, the e.m.f. induced in circuit No. 2 is

$$e_2 = -N_2 \frac{d\varphi_1}{dt} \times 10^{-8} = -K_1C_1N_1N_2 \frac{di_1}{dt} \times 10^{-8}$$

and when the current in circuit No. 2 changes, there will be induced in circuit No. 1 an e.m.f.

$$e_1 = -N_1 \frac{d\varphi_2}{dt} \times 10^{-8} = -K_2C_2N_1N_2 \frac{di_2}{dt} \times 10^{-8}$$

From (58), these equations may be written

$$\left.\begin{aligned} e_2 &= -M\frac{di_1}{dt} \\ \text{and} \qquad e_1 &= -M\frac{di_2}{dt} \end{aligned}\right\} \qquad (59)$$

where

$$M = K_1C_1N_1N_2 \times 10^{-8} = K_2C_2N_1N_2 \times 10^{-8} \qquad (60)$$

is the *number of flux linkages with one circuit due to unit current* (the ampere) *in the other, divided by* 10^8. This is called the *coefficient of mutual induction*, or the *mutual inductance*, of the two circuits. It is obviously of the same nature as self-inductance, and is measured in henrys. From (59) it follows also that the *mutual inductance of two circuits is numerically equal to the e.m.f. induced in one of them when the current in the other changes at the rate of* 1 *ampere per second.*

It is clear from equation (50) that the self-inductance of circuit No. 1 is

$$L_1 = \frac{N_1\Phi_1}{i_1} \times 10^{-8} = C_1N_1^2 \times 10^{-8} \qquad (61)$$

and from equation (54) that

$$L_2 = \frac{N_2\Phi_2}{i_2} \times 10^{-8} = C_2N_2^2 \times 10^{-8} \qquad (62)$$

Hence, from (60), (61) and (62)

$$M^2 = K_1K_2L_1L_2 \qquad (63)$$

If the circuits are so related that there is no leakage of flux between them, that is, if all of the flux produced by one circuit links with all of the turns of the other,

$$K_1 = K_2 = 1$$

and

$$M = \sqrt{L_1 L_2}$$

or the mutual inductance of two perfectly coupled circuits is a mean proportional between their respective self-inductances. The factor $\sqrt{K_1 K_2}$ is sometimes called the coefficient of coupling.

The phenomenon of mutual induction is utilized in the induction coil and in the alternating-current transformer, both of which consist of an iron core upon which are wound two coils, the primary and the secondary, insulated from the core and from each other. An interrupted or alternating current in one winding sets up a periodically varying flux which in turn induces an alternating e.m.f. in the other winding. Mutual induction is also of importance as a factor in the commutation process in direct-current machines.

29. Energy Stored in a Magnetic Field.—A coil or circuit of self-inductance L henrys carrying a variable current will have induced in it an e.m.f.

$$e = -L\frac{di}{dt} \text{ volts}$$

If the current is i amperes at the moment when the rate of change of current is $\frac{di}{dt}$ amperes per second, the power required to effect the change of current is

$$(-e)i = Li\frac{di}{dt} \text{ watts}$$

and the work done in the time dt is

$$dW = (-e)idt = Lidi \text{ joules}$$

The total amount of work required to raise the current from zero to a value i is, therefore,

$$W = \int_0^i Lidi = \tfrac{1}{2}Li^2 \text{ joules} \qquad (64)$$

This energy is not lost, but is stored in the magnetic field, and may be recovered by allowing the magnetic field to collapse to

zero value. It is this energy which appears in the spark or arc formed on opening an inductive circuit.

It is instructive to compare equation (64) with the equation for the kinetic energy of a moving body. This is of the form

$$W = \tfrac{1}{2}mv^2$$

where m is the mass of the body and v its velocity. In the case of the electric circuit the current i is the quantity of electricity that passes a given point in a second, and is analogous to velocity. The self-inductance L represents a sort of electrical inertia, since it operates to resist any change in the current flow, or electrical velocity; it is therefore analogous to the mass of a mechanical system. The energy $\frac{1}{2}Li^2$ may, therefore, be considered as the kinetic energy of electricity in motion.

When two circuits of self-inductances L_1 and L_2 have a mutual inductance M, there is stored in the system an amount of energy equal to

$$W = \tfrac{1}{2}L_1i_1^2 + \tfrac{1}{2}L_2i_2^2 + Mi_1i_2 \text{ joules} \qquad (65)$$

provided the two currents magnetize in the same direction. The derivation of the first two terms of equation (65) is obvious from (64); the last term, Mi_1i_2, can be derived as follows:

The potential energy of one circuit in the presence of the other is from (53) and (57)

$$W = \tfrac{1}{10}K_1C_1N_1N_2i_1i_2 = \tfrac{1}{10}K_2C_2N_1N_2i_1i_2 \text{ ergs}$$

and by (60) this becomes

$$\begin{aligned} W = \tfrac{1}{10}(M \times 10^8)i_1i_2 &= Mi_1i_2 \times 10^7 \text{ ergs} \\ &= Mi_1i_2 \text{ joules} \end{aligned}$$

If the two circuits magnetize in opposite directions, their mutual potential energy is evidently reversed in sign, so that the stored energy of the system is

$$W = \tfrac{1}{2}L_1i_1^2 + \tfrac{1}{2}L_2i_2^2 - Mi_1i_2 \qquad (66)$$

30. Tractive Effort of Electromagnets.—Let a unit magnet pole be placed at the point P, Fig. 32, on the axis of a cylindrical bar magnet of radius r cm., and distant a cm. from the end of the bar magnet; let the strength of the pole of the magnet be m units, assumed to be uniformly distributed over the end surface of the

cylinder. The pole strength per unit area, or the *intensity of magnetization*, is then

$$\sigma = \frac{m}{\pi r^2} = \frac{m}{A} \tag{67}$$

Considering an annular element of radius x and width dx on the end surface of the magnet, the force which it will exert upon the unit pole at P is

$$dF = \frac{2\pi\sigma x dx}{a^2 + x^2} \cdot \frac{a}{\sqrt{a^2 + x^2}}$$

and the total force due to the entire pole of the magnet is

$$F = 2\pi\sigma a \int_0^r \frac{x dx}{(a^2 + x^2)^{3/2}} = 2\pi\sigma(1 - \cos\theta) \tag{68}$$

where θ is the semi-angle of the right cone subtended at the point P by the end of the magnet. If the distance a is made very small,

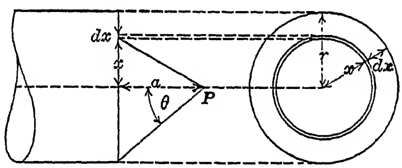


FIG. 32.—Field intensity on axis of bar magnet.

or if a is small relatively to the dimensions of the end area of the magnet, $\cos\theta$ approaches zero as a limit, in which case

$$F = 2\pi\sigma \tag{69}$$

If two bar magnets are placed end to end with a very small separation, and if the intensities of magnetization of the adjacent surfaces are $+\sigma$ and $-\sigma$, respectively, the attraction of one of the magnets upon an elementary magnet pole of area dA on the other will be $dF = 2\pi\sigma \times \sigma dA$; and the total attraction between adjacent poles will be

$$F = 2\pi\sigma^2 A \text{ dynes} \tag{70}$$

From equation (67), $\sigma = \frac{m}{A}$; and since the flux issuing from a pole of m units is $\Phi = 4\pi m$, it follows that $\sigma = \frac{\Phi}{4\pi A}$; whence, from (70),

$$F = \frac{\Phi^2}{8\pi A} = \frac{B^2 A}{8\pi} \text{ dynes} \tag{71}$$

This is the fundamental equation underlying the design of tractive or lifting electromagnets.

PROBLEMS

1. A square coil 30 cm. on each side is placed in a vertical plane that makes an angle of 30 deg. with the plane of the magnetic meridian. If the total intensity of the earth's magnetic field is 0.41 gauss and the angle of dip is 60 deg., what is the total flux that passes through the coil?

2. Two identical slender bar magnets, *A* and *B*, each 20 cm. long and having concentrated poles of 150 c.g.s. units at their ends, are placed with their axes in parallel lines that are 3 cm. apart, and so that the north pole of *A* is 5 cm. from the south pole of *B*. What is the moment of the couple tending to rotate *A* about its middle point, and what is its direction?

3. A rectangular coil 30 cm. by 60 cm. having one turn is placed in the magnetic meridian with its 30 cm. sides vertical. A small compass needle is placed 20 cm. from the plane of the coil and on a line passing through the center of the coil and perpendicular to its plane. If the intensity and direction of the earth's magnetic field are as given in Problem 1, what will be the angular deflection of the needle if the current in the coil is 10 amperes?

4. Solve the problem worked out in Art. 10, Chap. I, on the assumption that the current in each bus-bar flows uniformly along the vertical plane through the central axis.

5. A fly-wheel having a diameter of 15 ft. is mounted with its plane in an east and west direction, and rotates at 100 r.p.m. If the intensity and direction of the earth's magnetic field are as given in Problem 1, what is the difference of potential between the rim and axle of the wheel, the diameter of the shaft being one foot?

6. A concentrated circular coil of 15 turns and radius 15 cm. is revolved on a horizontal axis pointing northeast and southwest at the rate of 10 rev. per sec. What is the average e.m.f. generated in the coil if the intensity and direction of the earth's magnetic field are as given in Problem 1? In what position of the coil will the e.m.f. have maximum value, and what is the maximum value of the e.m.f.?

7. A concentrated circular coil of 100 turns and 15 cm. radius is mounted on trunnion bearings whose axis coincides with a diameter of the coil and which is perpendicular to the magnetic meridian. The plane of the coil is perpendicular to the lines of force of the earth's magnetic field, the intensity and direction of which are specified in Problem 1. If the total resistance of the coil and the circuit to which it is connected is 200 ohms, what is the total quantity of electricity discharged through the circuit on quickly rotating the coil through half a revolution? 2.9×10^{-6} coul.

8. A coil of insulated wire having a resistance of 250 ohms is mounted in a tube through which is passed a stream of water at the rate of 200 cu. cm. per min. The temperature of the surrounding air is 25° C., and the initial temperature of the water is 15° C. The current is adjusted until the tem-

perature of the outflowing water is constant at 35° C. What is the strength of the current? Would the solution be the same if the air temperature were other than 25° C?

9. A storage battery of 12 cells has an e.m.f., when discharged, of 1.8 volts per cell. Its internal resistance is 0.005 ohm per cell and the normal charging rate is 10 amperes. If the cells are connected in series and are to be charged from a 110-volt circuit, what resistance must be inserted in the line to yield the normal current? Toward the end of the charging period the battery e.m.f. rises to 2.5 volts per cell; what is the charging current?

10. Four binding posts, A, B, C and D, are arranged to form a square, the corners of which are lettered in the sequence given. Between A and B, B and C, and C and D are inserted resistors of 5 ohms each, while a resistor of 15 ohms is inserted between D and A. A coil of 10 ohms resistance is connected between B and D, and a battery having an e.m.f. of 24 volts and an internal resistance of 0.02 ohm is connected between A and C. Find the current in each branch of the circuit.

11. Solve Problem 10 on the assumption that branch BC contains, in addition to the 5-ohm resistor, a battery of negligible resistance and an e.m.f. of 4 volts directed from C to B; the positive pole of the 24-volt battery is connected to terminal A.

12. Referring to Fig. 14, assume that the lower loop of the three-wire circuit supplies a 110-volt motor connected in parallel with resistor B, the resistances of A and B remaining unchanged, and that the load on the motor is such as to require an input current of 10 amperes. (*a*) Construct the potential diagram of the circuit and compute the currents in the three supply lines and the potential difference at the motor terminals. (*b*) The above motor is removed, and a 220-volt motor, requiring an input current of 5 amperes, is connected between points X and Y; determine the same quantities as in (*a*), and construct the potential diagram.

13. How much work is required to turn magnet A of Problem 2 end for end?

14. Parallel to the rectangular coil of Problem 3, and 30 cm. distant from it, there is placed a circular coil of two turns having a radius of 20 cm. The centers of both coils are on a line perpendicular to their planes. Midway between them, and on the line joining their centers, is placed a small compass needle. If the current in the rectangular coil is 5 amp., how much current must flow through the circular coil in order that the needle may not be deflected? What must be the relative directions of the currents in the two coils?

15. A slender bar magnet 25 cm. long having concentrated poles of 200 c.g.s. units at its ends is placed on the axis of a two-turn circular coil of radius 20 cm. The south pole of the magnet lies nearest to the plane of the coil and is originally 20 cm. away from it. The coil carries a current of 15 amp. flowing in a clockwise direction when viewed from the magnet. How much work must be done to move the magnet along the axis until the south pole is in the plane of the coil?

16. A solenoid has a right-handed winding of 100 turns, the mean diameter being 5 cm. and the length 25 cm. The axis of the solenoid is horizontal and lies in the magnetic meridian. If the solenoid carries a current of 15 amp., flowing from the southern to the northern end, how much work is required to rotate the solenoid through an angle of 90 deg. about a vertical axis passing through the middle of the solenoid?

17. A cast-iron ring has a square cross-section of 1 sq. in. and a mean diameter of 12 in. How many ampere-turns are required to produce a flux of 36,000 maxwells? Compute the permeability and reluctance of the ring. What is the permeability when the flux is reduced to 20,000 maxwells?

18. A magnetic circuit made of sheet steel punchings is built up to the dimensions of Fig. 29*b*. The net thickness of the core is 90 per cent. of the gross thickness because of scale and air spaces between the punchings. Find the flux through a coil wound on the central core if the coil produces a m.m.f. equivalent to 3500 ampere-turns. Find the permeability of the sheet steel on each side of the circuit and the reluctance of the gaps.

19. Compute the self-inductance of the cast-iron ring of Problem 17 assuming that the winding has 800 turns. If the current has such a value that the flux is 36,000 maxwells, what e.m.f. will be induced if the current and flux are reduced to zero, at a uniform rate, in 0.001 sec.?

20. Two circular coils, A and B, are mounted concentrically, one inside the other. A current in A produces a flux of which 75 per cent. links with B, and a current in B produces a flux of which 95 per cent. links with A. When the two coils are connected in series so that they magnetize in the same direction, the total self-inductance is found to be 0.5 henry; when they magnetize in opposite directions the total self-inductance is 0.06 henry. Find (*a*) the self-inductance of each coil; (*b*) the coefficient of mutual induction; (*c*) the amount of work required to turn coil B through 90 deg., starting from the position in which the coils magnetize in the same direction, assuming that they are connected in series and are carrying a current of 75 amp.

21. Coil A of Problem 19 is wound in two equal parts A_1 and A_2, in such a way that there is perfect coupling between them. Coil B consists likewise of two equal parts, perfectly coupled. Find all the possible values of self-inductance that can be obtained from combinations of the four windings, assuming that the planes of the coils are coincident.

22. If the cast-iron ring of Problem 16 is split into two semicircular parts, the air-gap at each joint being 0.001 in., what is the pull, in pounds, required to separate the two halves? What is the pull between the two parts of the ring when they have been separated $\frac{1}{16}$ in., the excitation remaining the same as before?

CHAPTER II

THE DYNAMO

31. Dynamo, Generator and Motor.—A dynamo-electric machine, or a *dynamo*, may be defined as a machine for the conversion of mechanical energy into electrical energy, or conversely, for the conversion of electrical energy into mechanical energy. When used for the first purpose it is called a *generator*, and when used for the second it is called a *motor*. In other words, the word dynamo is a generic term which includes the other two; a dynamo is a reversible machine, being capable of operation either as generator or motor.

Every generator consists of a conductor, or set of conductors, subjected to the influence of a varying magnetic field, so that e.m.fs. are induced in them. Current will be produced when the circuit of these active conductors is completed through an external receiver circuit. On the other hand, motor action results when current from some external source is sent through a set of conductors suitably located in a magnetic field.

In the case of generator action, each conductor is the seat of an induced e.m.f. of

$$\bar{E} = Blv \text{ abvolts} \tag{1}$$

where l is the length of the wire in centimeters, B is the density of the field through which it is moving, and v is its velocity in centimeters per second in a direction perpendicular to that of the field and to its own length. On closing the circuit there will flow a current of, say, $\bar{I}$ abamperes, the value of which will depend upon the resistance of the circuit as a whole, in accordance with Ohm's law. The conductor will then be acted upon by a force of

$$F = Bl\bar{I} \text{ dynes} \tag{2}$$

in a direction opposite to its motion; hence, to maintain the action, a driving force must be applied to the conductor and work must be done at the rate of $Fv = Bl\bar{I}v = \bar{E}\bar{I}$ ergs per second.

In the case of motor action, each conductor is caused to carry a current of $\bar{I}$ abamperes, so that it is acted upon by a lateral thrust of

$$F = Bl\bar{I} \text{ dynes}$$

motion of the wire results, and under the influence of the field intensity B and velocity v there is induced in the wire an e.m.f.

$$\bar{E} = Blv \text{ abvolts}$$

in a direction opposite to the current. To maintain the current flow there must be impressed an e.m.f. of sufficient magnitude to balance this counter-generated e.m.f., and work is done by the electrical source of supply at the rate of $\bar{E}\bar{I} = Blv\bar{I} = Fv$ ergs per second.

It will then be clear that in the case of an actual generator consisting of a number of conductors on the periphery of a cylindrical armature and suitably connected, the flow of current caused by the generated e.m.f. acting through the closed external circuit is accompanied by the appearance of a counter torque opposing that of the prime mover; and that in a motor, the rotation of the armature caused by the torque resulting from the reaction of the current on the magnetic field sets up a counter e.m.f. which must be overcome by the voltage impressed on the terminals of the motor.

In this discussion ideal conditions have been tacitly assumed, namely, that all of the energy supplied reappears as useful energy after the conversion process has been completed. As a matter of fact this condition is never realized in practice; the energy supplied must be greater than that usefully converted by an amount equal to the loss of energy inevitable in the conversion.

The *armature* of a dynamo is the part in which the e.m.f. is generated in the case of a generator, or the part which carries the working current in the case of a motor. The *field* member is the part which produces the magnetic field. The relative motion of the one structure with respect to the other is most easily obtained by making one or the other rotate, so that in general the two have concentric cylindrical forms. Either may be the rotating member; if the armature rotates, the machine is called a revolving armature machine, while if the field rotates it is called a revolving field machine.

There are two distinct types of dynamo-electric machines.

according to the nature of the e.m.f. and current produced; they are (1) *alternating-current* machines, and (2) *direct-current* machines. The first type, when used as a generator, is called an *alternator* and produces an e.m.f. which acts alternately in opposite directions, so that when the armature circuit is completed the current in the circuit flows first in one direction and then in the other. The second type produces a current through the external circuit which flows in one direction only. A direct current, though characterized by constancy of direction, may, however, vary in magnitude from instant to instant, that is, it may be pulsating; or it may be constant in magnitude as well as in direction. In the former case, the current is said to be a *direct current;* in the latter case the current is said to be a *continuous current.*

The alternating-current generator or motor is the simplest form of dynamo. Reduced to the most elementary type, it

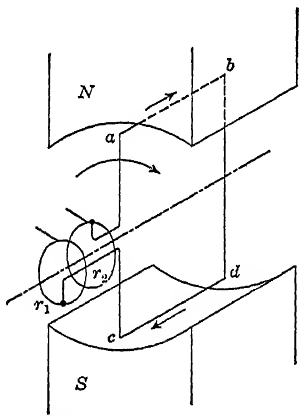


Fig. 33.—Elementary alternator.

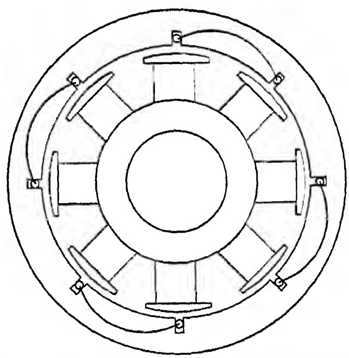


Fig. 34.—Multipolar revolving field alternator.

consists of a loop of wire, *abcd,* Fig. 33, rotating in a magnetic field that passes across from pole N to pole S. It is understood that the pole pieces N and S are the extremities of the field structure, and that the excitation of the magnets is effected by a direct current from some suitable source circulating in coils wound on the field structure. The ends of the armature coil are attached to the insulated slip-rings r_1, r_2. In the position shown in the figure, wire *ab* will have generated in it an e.m.f. directed from front to back, while the e.m.f. in *cd* will be directed from back to

front; the collecting brush touching ring r_1 will therefore be positive, and that touching r_2 will be negative. After half a revolution it will be seen that the polarity of the terminals reverses, so that each terminal is alternately of opposite polarity.

In practice alternating-current machines usually have more than the two poles shown in Fig. 33; in other words, they are *multipolar*. The winding consists of a number of coils connected in series in such manner that the e.m.fs. of the individual coils add together. Fig. 34 represents diagrammatically an 8-pole revolving field machine with the winding of the stationary armature arranged in eight slots. Fig. 35 is a development of this particular type of winding as it would appear if the cylindrical surface of the armature were rolled out into a plane.

With the exception of the homopolar machine described in Art. 50, all standard forms of direct-current generators and motors consist of a wire- or bar-wound armature arranged to rotate between inwardly projecting poles of alternate polarity, in the manner illustrated in Fig. 46. Each of the armature conductors is, therefore, the seat of an alternating e.m.f. which changes its direction each time the conductor moves from the influence of one pole to that of the adjacent pole. It is the func-

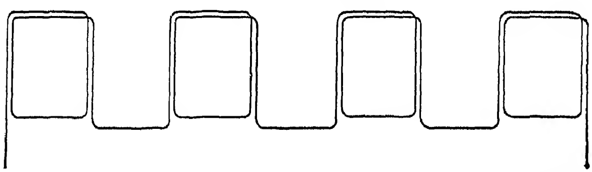


Fig. 35.—Developed armature winding of alternator of Fig. 32.

tion of the commutator to convert this internal alternating e.m.f. into a uni-directional e.m.f. in the external circuit; but so far as the armature winding itself is concerned, every direct-current machine (with the exception of the homopolar machine) is essentially an alternating-current machine, hence it is important to analyze the development of the e.m.f. in an alternator in order to understand thoroughly what is happening in the case of the direct-current machine.

32. E.M.F. of Elementary Alternator.—Consider first the elementary alternator of Fig. 36, whose armature winding consists of a concentrated coil having Z conductors $\left(\text{or } N = \frac{Z}{2} \text{ turns}\right)$ on

the external periphery of the armature core A. If the air-gap between the pole faces N, S and the armature core is uniform, as is usual in direct-current machines (except at the pole tips), the lines of force of the magnetic field will tend to cross the gap on radial lines, and the field strength will have practically uniform strength everywhere under the poles; at the pole tips the lines of force will "fringe," the spreading apart of the lines indicating that the strength of the field tapers off more or less gradually to zero value midway between the poles. These facts are represented in Fig. 36, where the line marked B is so drawn that its ordinates represent the radial component of the flux density in the air-gap all around the periphery of the armature, the latter being developed, *i.e.*, rolled out into a plane. If it were not for the fringing of the flux at the pole tips, the flux distribution would be represented by the rectangular diagram shown in broken lines.

If the diameter of the armature is d cm. and its active length is l cm., and if it is driven at a speed of n revolutions per minute, the tangential or peripheral velocity of the conductors is

$$v = \pi d \frac{n}{60} \text{ cm./sec.}$$

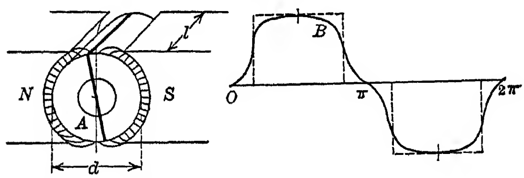


FIG. 36.—Flux distribution around armature.

and at the instant when the conductor is cutting through the magnetic field where the flux density has a radial component of B gausses, the generated e.m.f. per conductor is

$$e = Blv \times 10^{-8}. = \pi dl \frac{n}{60} B \times 10^{-8} \text{ volts} \qquad (3)$$

Because of the assumed concentration of the Z conductors in a diametral plane, and because of the further assumption that the flux distribution is symmetrical around the armature, the e.m.f. is the same in all of the conductors at the same instant, hence the total instantaneous value of the generated e.m.f. is

$$e = \pi dl \frac{n}{60} ZB \times 10^{-8} \text{ volts} \qquad (4)$$

Since all of the terms on the right-hand side of this expression are constant with the exception of B, it follows that the variation of e.m.f. with respect to *time* is identical in form with the curve B of Fig. 36, which shows the *space* distribution of the flux density. The e.m.f. is zero at the instant when the active edges of the coil are passing through the neutral axis midway between the poles, rises sharply as the active coil edges pass under the poles, remains fairly constant as they pass under the pole faces, falls again to zero when the coil is again in the neutral plane, and then goes through these changes with a reversal of direction. Thereafter, the same cycle of changes is repeated indefinitely.

33. General Case of the E.M.F. of an Alternator.—The discussion of Art. 32 was based upon the assumption of a bipolar field structure, and a full-pitch armature coil, that is, a coil spanning the arc from center to center of poles. Generally, however, there is more than a single pair of poles, and the coil spread may be greater or less than the pole pitch.

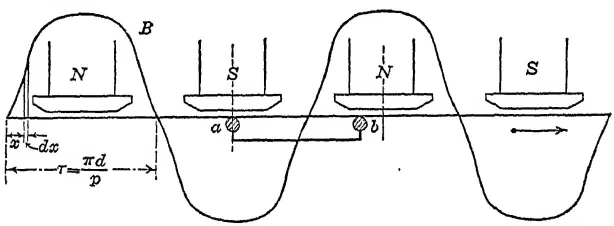


FIG. 37.—Multipolar alternator, non-sinusoidal flux distribution.

Let Fig. 37 represent a partial development of an alternator having p poles (like Fig. 34), and let the distribution of flux at the armature surface be represented by curve B; further, let the armature have a diameter of d cm., the conductors have an active length of l cm. in a direction parallel to the shaft, and let the speed of rotation be n revolutions per minute. The instantaneous e.m.f. generated in each conductor is the same as equation (3)

$$e = Blv \times 10^{-8} = Bl\pi d \frac{n}{60} \times 10^{-8}$$

and the graph of this e.m.f. will then be a curve which is the same as that showing the flux distribution, except for a change in scale. The average e.m.f. per conductor is

$$E_{aver.} = \frac{1}{\tau}\int_0^{\tau = \frac{\pi d}{p}} e\,dx = \frac{1}{\tau}\frac{n}{60}\pi d\int_0^{\tau} Bl\,dx \times 10^{-8} = p\frac{n}{60}\Phi \times 10^{-8}$$

where $\Phi = \int_0^{\tau} Bldx$ is the flux per pole. The last result might have been anticipated from the fact that the average e.m.f is equal to the number of lines of force cut per second, divided by 10^8; thus each conductor in one revolution cuts Φ lines per pole, or $p\Phi$ lines per revolution, hence $p\Phi \frac{n}{60}$ lines per second. It is interesting to note that Φ is the integral of the B function; conversely, B is the first derivative of the flux function.

If the armature is wound with Z conductors, all connected in series as in Fig. 35, in such a manner that the coils are of full pitch, the total average e.m.f. is

$$E_{aver.} = p\Phi Z \frac{n}{60} \times 10^{-8} \qquad (8)$$

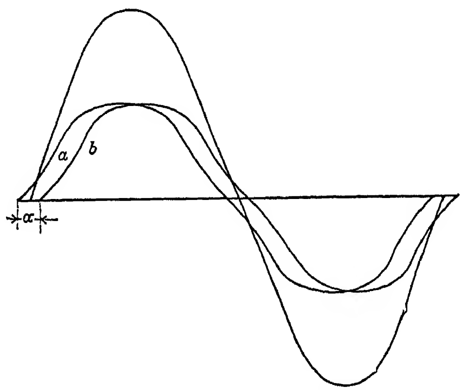


FIG. 38.—E.M.F. in coil of fractional pitch.

As pointed out above, each conductor is the seat of an e.m.f. whose variation from instant to instant is represented graphically by a curve identical (except for a change of scale) with the curve of flux distribution, Fig. 37. If the conductors are arranged as in Fig. 34 so that the coil spread is the same as the pole pitch, the e.m.f. in all conductors will be simultaneously in the same phase of the variation, and the total instantaneous e.m.f. will be simply Z times that of a single conductor. But if the coil spread differs from the pole pitch, as indicated by coil *ab*, Fig. 37, the instantaneous e.m.fs. of the two sides of the coil will differ in phase, that of coil-edge *a* following curve *a*, Fig. 38, and that

of coil edge b following curve b, where the displacement α between the curves corresponds to the amount by which the sides of the coil ab of Fig. 37 fall short of being a pole pitch apart. The total instantaneous e.m.f. of the coil is obtained by adding the ordinates of the individual e.m.f. curves. It is evident from Fig. 38 that the maximum e.m.f. of such a "fractional pitch" winding is less than that of a full-pitch winding of the same number of conductors.

34. Rectification of an Alternating E.M.F.—If the terminals a and c of the elementary alternator of Fig. 33 are connected, respectively, to the two insulated segments of a *commutator* C, as in Fig. 39, and stationary brushes, b_1, b_2, are mounted so as to make sliding contact with the revolving commutator segments, the plane of the brushes being coincident with that through the shaft and the polar axis, the brush b_1 will always be of negative

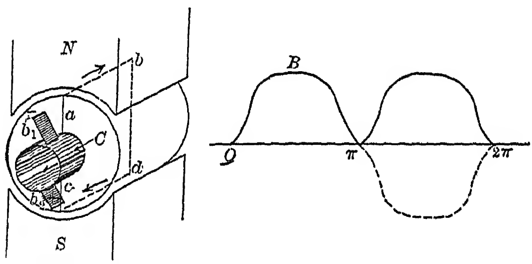


FIG. 39.—Two-part commutator. Rectification of e.m.f.

polarity and brush b_2 will always be of positive polarity. The reversal of the e.m.f. of the coil takes place simultaneously with the passage of the brushes across the gaps between the segments of the commutator. If the flux distribution is like that of curve B, Fig. 36, the brush voltage will vary in the manner shown in Fig. 39; the negative half loop of the original alternating voltage is reversed, so far as the external circuit is concerned, hence the voltage in the external circuit connected to the brushes b_1 and b_2 will be unidirectional, but its magnitude will pulsate between zero and a maximum value.

If the Z peripheral conductors constituting the loop of Fig. 39 are replaced by a winding like that of Fig. 40, the latter likewise having Z peripheral conductors, the generated e.m.f. will remain the same in magnitude and in the manner of its variation. The

former winding is of the *drum* type, the latter of the *ring* type. It will be noted that in the drum winding there is one complete turn for each pair of conductors, while in the ring type there is a turn for each conductor; but in the ring winding the wires inside the core play no part in generating e.m.f. since they do not cut lines of force. Consequently, for equal numbers of peripheral conductors in the two types of windings, the eléctrical characteristics are identical except for minor differences due to unequal length of wire and therefore of resistance.

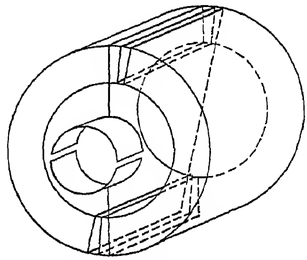


Fig. 40.—Elementary ring-wound armature.

35. Effect of Distributed Winding.—An e.m.f. varying as in Fig. 39 is not desirable, and means must be found to make it more nearly continuous. The large amplitude of the pulsation in that figure is due to the fact that the entire armature is inactive each time the coil edges pass through the neutral zone between the poles, that is, each time commutation takes place; if the winding can be so disposed that small sections, each consisting of relatively few turns, undergo commutation successively, the pulsations will become insignificant when the

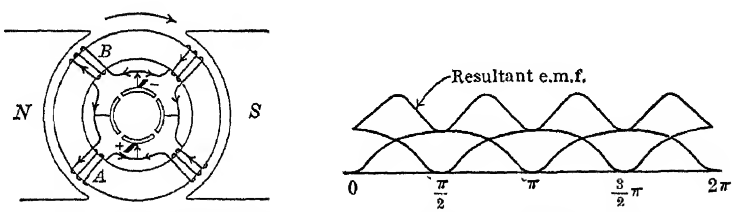


Fig. 41.—Ring winding with four sections. Pulsation of e.m.f.

number of such winding sections is sufficiently great. Thus, Fig. 41 is a diagrammatic sketch of the armature of Fig. 40, but with the original Z conductors arranged in four equidistant groups of concentrated coils; the end of each coil is connected to the beginning of the next, and there is a connection between each of these junction points and a segment of a four-part commutator. A study of the directions of the e.m.fs. generated in

the coils shows that the brushes must now be placed along an axis perpendicular to the polar axis, in order that current may be delivered effectively to the external circuit. It will also be clear that although the four individual coils form a closed ring so far as the internal armature circuit is concerned, they are connected to the external circuit in such a manner that with respect to the external circuit the armature winding consists of two equal halves in parallel with each other. Each half of the armature winding consists of a pair of winding sections connected in series. Since the winding consists of two equal parts in parallel, the voltage at the brushes will be equal to that of either half alone. Considering the particular half of the armature winding made up of sections A and B, it will be observed that section A generates a wave of e.m.f. similar to that of Fig. 39, but of only one-fourth the amplitude since coil A has only one-fourth as many active conductors as the coil of Fig. 40. Similarly, section B generates a wave exactly like that of section A, but the two waves differ in phase by 90 deg., as shown in Fig. 41. The resultant brush voltage will be obtained by adding the ordinates of these two component curves, as shown. There are now four pulsations instead of the original two, but the range from minimum to maximum is much reduced.

Carrying the subdivision of the winding one step further by arranging the Z peripheral conductors in eight equidistant groups of coils as shown in Fig. 42, each half of the resultant ring winding will consist of four sections in series. In each section there will be generated an e.m.f. which will vary from instant to instant in the manner shown by curve B of Fig. 39, but the maximum value will be only one-eighth as great as in that figure; the e.m.fs. in the four coils which at any moment are in series in one-half of the ring differ in phase from each other by 45 degrees, as shown in Fig. 42, so that the resultant e.m.f. between the brushes is at any moment the sum of the ordinates of these four curves. It will be seen that there are now eight pulsations instead of the original two, and the amplitude of the pulsations is decidedly less than before.

Fig. 42 shows the armature in two different positions; in part a, the four coils marked A, B, C, D, are directly in series, each contributing a definite e.m.f. to the total; in part b, rotation of

the armature has carried coil D to the position where commutation occurs, and under this condition coil D does not contribute e.m.f. to the circuit formed by coils A, B, and C, consequently the number of active coils per circuit is reduced from four to three. A moment later, coil D will form a part of the circuit on the right-hand side, and one of the coils from the right-hand side will be transferred to the left-hand side; thereafter the process is repeated indefinitely. In position (*a*) the total e.m.f. developed by the four sections in series is a maximum; in position (*b*) the total e.m.f. is a minimum. Clearly, therefore, the varia-

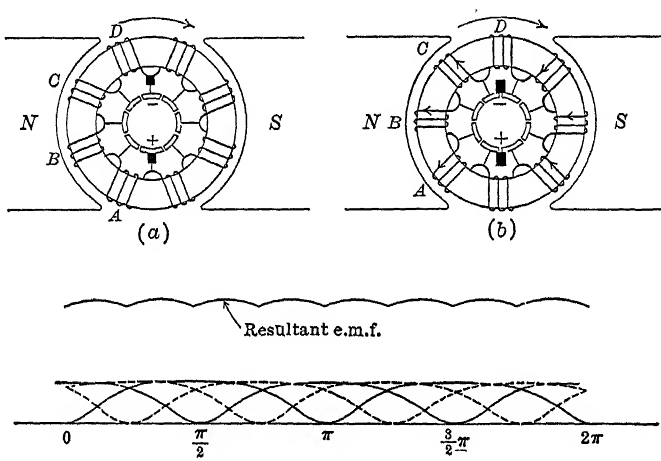


FIG. 42.—Ring winding with eight sections. Pulsation of e.m.f.

tion will become less and less as the angular separation between adjacent coils is reduced, that is, as the number of sections is increased.

The smoothing effect upon the resultant e.m.f. produced by increasing the number of winding sections is entirely analogous to the effect upon the torque of a gas engine produced by increasing the number of cylinders, provided the successive crank pins are uniformly displaced around the crank shaft.

36. Average E.M.F. of a Direct-current Armature.—If a ring winding of the type discussed in the preceding article is rotated in a multipolar field structure, as indicated in Fig. 43, the e.m.fs. generated in the individual winding sections will be directed in the manner shown by the small arrow heads. In

the case represented in the diagram the e.m.fs. are so directed that the entire winding is divided into four belts, one per pole, in each of which the individual e.m.fs. are cumulative. In order to take full advantage of this distribution, brushes must be placed at each neutral point; half of the brushes will be of positive polarity, the other half negative. If all of the positive brushes are connected to one another and to the positive side of the external circuit, and if the negative brushes are likewise connected to one another and to the negative side of the external circuit, then, with respect to the external circuit, the armature winding consists of four paths (in the case illustrated) which are in parallel with one another. The e.m.f. will be the same in each of these paths provided the entire armature winding is symmetrical and provided the flux in each of the poles is the same; and the total e.m.f. of the machine as a whole will be the same as that in any one of the paths.

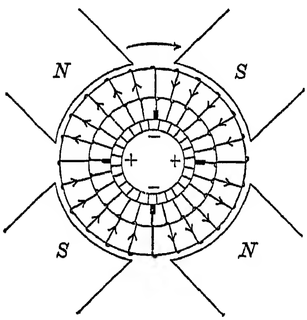


FIG. 43.—Ring-wound armature in multipolar field.

In the particular case illustrated in Fig. 43 the number of paths is equal to the number of poles, this being characteristic of all simple ring windings of the type illustrated. But it will be shown in Chapter III that the number of paths, a, is not necessarily equal to the number of poles, p, in all armature windings, and that by suitably connecting the individual coils of the winding the number of paths may be made any even number, from two up.

It is clearly of fundamental importance to be able to compute the average e.m.f. developed in the armature winding of a direct-current machine. Thus, let it be required to find the average e.m.f. generated in a winding having the following data:

Z = total number of peripheral conductors.
a = number of parallel paths through armature.
p = number of poles.
Φ = flux per pole.
n = revolutions of armature per minute.

Each conductor cuts $p\Phi$ lines per revolution, or $p\Phi\frac{n}{60}$ lines per

second, so that the average e.m.f. per conductor is $p\Phi\frac{n}{60} \times 10^{-8}$ volts; since the entire number of conductors is divided into a paths connected in parallel with one another, the number of *conductors in series per path* is $\frac{Z}{a}$; the average e.m.f. per path, and, therefore, of the armature as a whole, is

$$E = \frac{Z}{a} p\Phi \frac{n}{60} \times 10^{-8} = \frac{p}{a} \frac{\Phi Z n}{60 \times 10^8} \text{ volts} \tag{7}$$

This is the *general equation for the generated e.m.f. of a direct-current machine, provided* the brushes are so placed that the winding sections of any one group are simultaneously under the influence of one pole. Thus, if the brushes of the armature of Fig. 44 are so placed that commutation takes place in coils opposite the middle of the pole shoes, the potential difference between them will be zero; for in that case each path through the armature is made up of conductors half of which are subjected to the inductive action of one pole and the other half to the influence of a pole of opposite polarity, with the result that the e.m.fs. generated in the two halves of each path are equal and opposite.

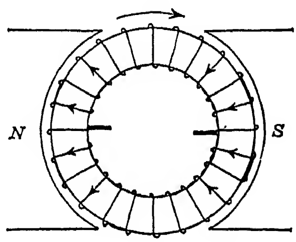


FIG. 44.—Brushes displaced from proper position.

37. Magnitude of E.M.F. Pulsations.—Comparison of the curves of Figs. 39, 41, and 42 with the corresponding windings brings out very clearly that subdividing the winding into relatively few distributed sections causes a marked reduction in the magnitude of the pulsations of e.m.f. above and below the average value. Further subdivision of the winding produces a still further suppression of the pulsations, though beyond a certain point the smoothing out of the wave of e.m.f. proceeds at a greatly reduced rate. In commercial machines the number of winding sections is determined by such considerations as the attainment of sparkless commutation, and the number so fixed is sufficiently great to make the pulsations of e.m.f. of minor importance. Nevertheless it is of interest to investigate the relation between the number of winding sections and the magnitude of the voltage

machine has two poles, like that of Fig. 42, and that the winding consists of Z conductors divided into s sections having $\frac{Z}{s}$ turns each; and further that the flux distribution, instead of having the form shown in Fig. 36, is sinusoidal. This means that the radial component of flux density at any point in the air-gap is proportional to the sine of the angle measured from the neutral axis to the point in question; thus, if B_m is the radial component of flux density under the middle of the pole face, the flux density at any point situated θ degrees from the neutral axis is

$$B = B_m \sin\theta \tag{8}$$

In one of the winding elements which at a given instant is displaced θ degrees from the neutral axis the instantaneous value of the generated e.m.f. is then

$$e_1 = \frac{Z}{s} Blv \times 10^{-8} = \pi \frac{Z}{s} B_m ld \frac{n}{60} \sin\theta \times 10^{-8} = E_m \sin\theta \tag{9}$$

where

$$E_m = \pi \frac{Z}{s} B_m ld \frac{n}{60} \times 10^{-8} \tag{10}$$

is the maximum value of the e.m.f. generated in the winding element of $\frac{Z}{s}$ turns at the particular instant when $\theta = \frac{\pi}{2}$, that is, when the coil is passing under the middle of the pole face. Since there are s winding sections uniformly distributed around the armature, the angle between adjacent sections is $\frac{2\pi}{s}$, and at the instant when the coil above referred to occupies the position determined by the angle θ, the next coil ahead of it occupies the position $\left(\theta + \frac{2\pi}{s}\right)$; the next one beyond occupies the position $\left(\theta + 2.\frac{2\pi}{s}\right)$, and so on. It follows, therefore, that the instantaneous values of e.m.f. in the $\frac{s}{2}$ successive winding sections in series with each other are

$$e_1 = E_m \sin\theta$$

$$e_2 = E_m \sin\left(\theta + \frac{2\pi}{s}\right)$$

$$e_3 = E_m \sin\left(\theta + 2.\frac{2\pi}{s}\right)$$

. . . .

$$e_{s/2} = E_m \sin\left[\theta + \left(\frac{s}{2} - 1\right)\frac{2\pi}{s}\right] = E_m \sin\left[\theta + \left(\pi - \frac{2\pi}{s}\right)\right]$$

Thus, in a winding like Fig. 42 where $s = 8$, the e.m.fs. in the successive winding sections are 45 deg. apart, and their values are indicated in Fig. 45 for three different values of the angle θ, corresponding to three successive positions of the armature as

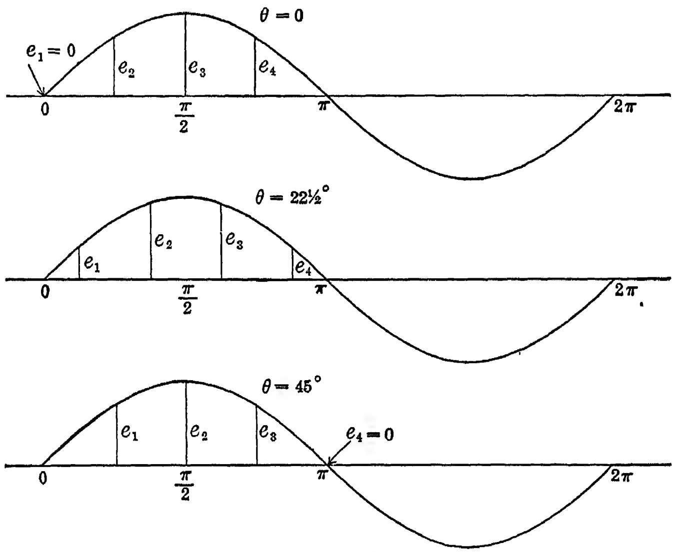


FIG. 45.—Successive phases of e.m.f. in eight-coil ring winding, sinusoidal flux distribution.

the latter rotates. The value of the total e.m.f. contributed by all of the $\frac{s}{2}$ sections is at any instant

$$\Sigma e = e_1 + e_2 + e_3 + \ldots\ldots + e_{s/2}$$

$$= E_m\left\{\sin\theta + \sin\left(\theta + \frac{2\pi}{s}\right) + \ldots\ldots + \sin\left[\theta + \left(\pi - \frac{2\pi}{s}\right)\right]\right\} \quad (11)$$

The minimum value of this expression occurs when $\theta = 0$, as

was pointed out in connection with Fig. 42, and the maximum value occurs when $\theta = \frac{\pi}{s}$; it follows, therefore, that

$$E_{min} = E_m\left[\sin\frac{2\pi}{s} + \sin\frac{4\pi}{s} + \ .\ .\ .\ .\ .\ + \sin\left(\pi - \frac{2\pi}{s}\right)\right]$$

$$= E_m \operatorname{cotan}\frac{\pi}{s} \qquad (12)$$

and

$$E_{max} = E_m\left[\sin\frac{\pi}{s} + \sin\frac{3\pi}{s} + \ .\ .\ .\ .\ .\ + \sin\left(\pi - \frac{\pi}{s}\right)\right]$$

$$= E_m \operatorname{cosec}\frac{\pi}{s} \qquad (13)$$

The percentage variation from minimum to maximum, in terms of the minimum value, is

$$\frac{\operatorname{cosec}\frac{\pi}{s} - \operatorname{cotan}\frac{\pi}{s}}{\operatorname{cotan}\frac{\pi}{s}} \times 100$$

and the magnitude of this quantity, for various values of s, is shown in the following table:

s	Per cent. variation
2	∞
4	41.0
6	15.4
10	5.17
20	1.24
30	0.56
60	0 13

In other words, with the winding divided into 30 or more sections (the field structure being bipolar), the fluctuations are quite insignificant.

Bearing in mind that the instantaneous value of the total e.m.f. varies from a mininum when $\theta = 0$ to a maximum when $\theta = \frac{\pi}{s}$, and that it then again falls symmetrically to a minimum when $\theta = \frac{2\pi}{s}$, the average e.m.f. will be

$$E_{aver.} = \frac{1}{\pi/s}\int_0^{\frac{\pi}{s}}(e_1 + e_2 + e_3 + \ . \ . \ . \ . \ + e_{s/2})d\theta$$

$$= ZB_m ld \frac{n}{60} \times 10^{-8}\int_0^{\frac{\pi}{s}}\left\{\sin\theta + \sin\left(\theta + \frac{2\pi}{s}\right) + \ . \ . \ . \ . \ . \ . \ + \sin\left[\theta + \left(\pi - \frac{2\pi}{s}\right)\right]\right\}d\theta$$

$$= ZB_m ld \frac{n}{60} \times 10^{-8} \qquad (14)$$

But when the flux density is so distributed that it follows the equation

$$B = B_m \sin\theta$$

its average value over each half of the armature surface is

$$B_{aver.} = \frac{1}{\pi}\int_0^{\pi} B_m \sin\theta \, d\theta = \frac{2}{\pi}B_m \qquad (15)$$

and the flux per pole is

$$\Phi = B_{aver.} \times \text{area of one-half of armature surface}$$

$$= B_{aver.} \times \frac{\pi d}{2} \times l = B_m dl \qquad (16)$$

Hence

$$E_{aver.} = \Phi Z \frac{n}{60} \times 10^{-8} \qquad (17)$$

which agrees with equation (7) since in the case here considered $p = 2$ and $a = 2$.

38. Resistance of Armature Winding.—In an armature having a paths, the total armature current i_a will divide equally between them, provided all paths have the same resistance. If the total resistance of all the wire on the armature is R_a ohms, the resistance per path will then be R_a/a ohms, and since all of these a paths are connected in parallel, the actual resistance of the armature, as measured between brushes, will be $R_a/a^2 = r_a$ ohms. The drop of potential due to the entire current i_a flowing through the resistance r_a, or $i_a r_a$ volts, is, of course, equal to the drop of potential through any one of the paths, or $i_a/a \times R_a/a = i_a r_a$ volts.

39. Construction of Dynamos.—The dynamo consists essentially of an electric circuit and a magnetic circuit placed in

inductive relation to each other. The electric circuit consists of the armature winding and the commutator. The magnetic

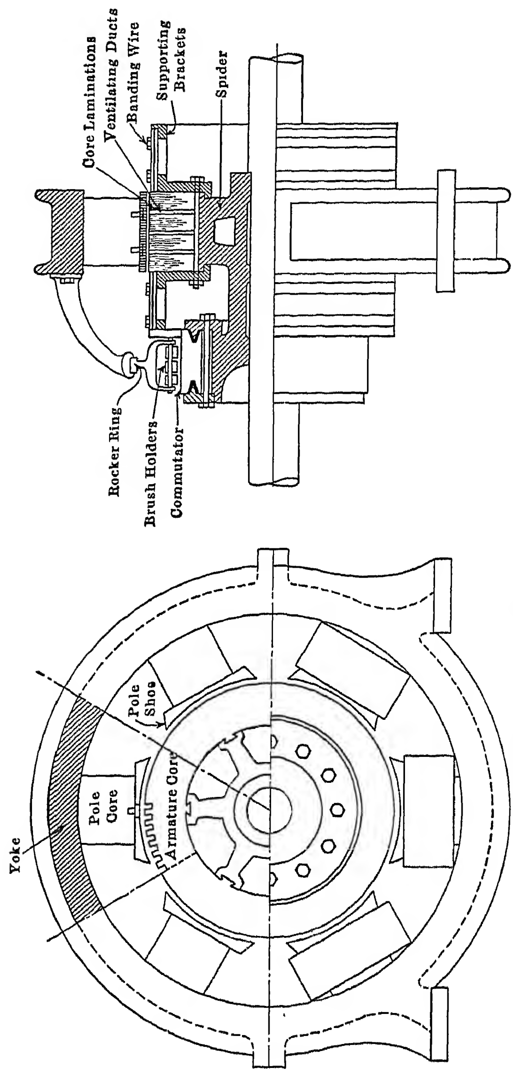


FIG. 46.—Diagrammatic view of multipolar dynamo.

circuit is made up of the yoke, pole cores and pole shoes, and the armature core. The annular space between the revolving armature and the stationary field structure is called the air-gap.

Other parts of the machine are the field winding, the brushes, brush-holders and the rocker-arm, the armature spider and the bearings. Fig. 46 shows a common arrangement of these parts in the *open type* of construction. Fig. 47 shows a *semi-enclosed type*, and Fig. 48 a *totally enclosed* motor. The principal structural features of the various parts of the machine, with the exception of the armature winding, are described in the following articles. The subject of armature windings is taken up in detail in Chap. III.

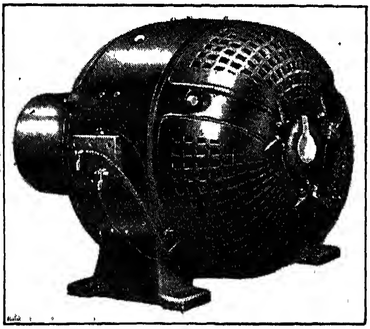


Fig. 47.—Semi-enclosed motor. (*Sprague.*)

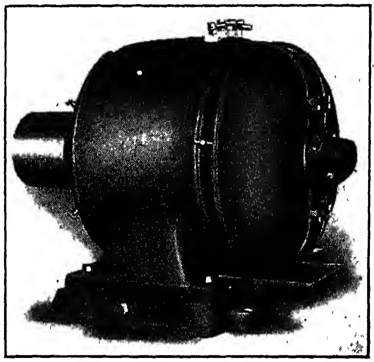


Fig. 48.—Totally enclosed motor. (*Sprague.*)

40. Bipolar and Multipolar Machines.—Although for the sake of simplicity much of the preceding discussion has been based on the assumption of a bipolar field structure, this type of field is seldom used except in machines of the smallest size. The actual number of poles generally varies from four to a maximum (in direct-current machines) of twenty to twenty-four, the number increasing with the capacity, though not at all regularly. The explanation of the principles underlying the choice of the number of poles in any given case must be deferred to a later section; in general, however, the choice of the number of poles depends upon the consideration that the magnetic reaction of the armature, when carrying current, cannot exceed definite limits without impairing the operating characteristics of the machine. Further, with an armature core of given dimensions, and with pole pieces that cover a definite percentage of the armature surface, the field frame becomes more compact, up to a certain limit, as the number

of poles is increased beyond two. The optimum limit occurs when the peripheral spread of the pole faces is approximately equal to the axial length of the pole face. A compact field frame is advantageous in that comparatively little of the field flux leaks from pole to pole without entering the armature core.

41. The Commutator.—The commutator is built up of wedge-shaped segments of drop-forged or hard-drawn copper insulated from one another by accurately gauged thin sheets of insulating material, such as mica. The process of assembling a large number of segments into a rigid structure is an interesting one. The segments, separated from one another by the mica insulation, are placed around the inner periphery of a sectored steel ring, as in

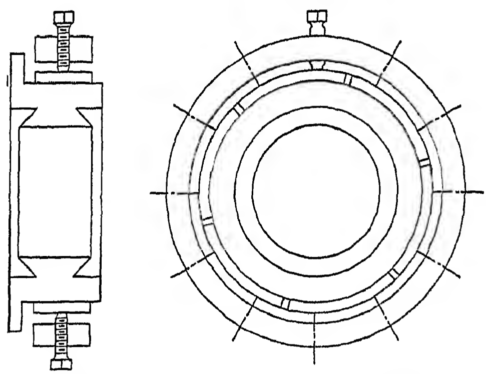


FIG. 49.—Construction of commutator.

Fig. 49, and the copper segments are then wedged together to form a rigid circular arch by means of cap-screws tapped radially through the outer steel ring. The V-shaped grooves are then turned out and the commutator spider bolted into place, after which the auxiliary steel clamping rings are removed and the external surface turned to true cylindrical form.

The insulation between the commutator and the supporting hub consists of molded mica cones and cylinders. The completed commutator must be given a high voltage test to insure the thorough insulation of each segment from the others and from the spider. The insulation between adjacent segments does not have to be as heavy as that between the segments and the commutator spider, for the latter must withstand the full terminal voltage of

the machine while the former is only called upon to withstand the smaller voltage between segments. The average voltage between adjacent segments should not exceed 10 to 15 volts in lighting and railway generators which are not of the commutating pole type, and from 20 to 25 volts in the case of railway motors. These limiting values of average voltage between segments are imposed by the requirements of sparkless commutation, and they determine the minimum number of segments in the completed commutator. For example, if a 6-pole, 600-volt railway generator is to have not more than 10 volts between adjacent segments, there must be at least 60 segments between adjacent brushes of opposite polarity, or not less than 360 segments in the entire commutator. The minimum diameter of the commutator is then determined if the minimum peripheral width of a segment is known; this minimum width is rarely less than $3/16$ in. for two reasons: first, because the taper of the segments would result in too thin a section at the inner periphery if a smaller external width were used; second, because some allowance in the radial depth of the segments must be made to permit turning down the surface in case of pitting, blistering or wear. The thickness of the insulation between segments varies from 0.02 in. in low voltage machines up to about 0.06 in. in high voltage machines. The material must be so selected that its rate of wear is the same as that of the copper bars. Amber mica is largely used because it meets this requirement. Commutators are sometimes built in such manner that the insulation does not come quite flush with the surface, thereby obviating the necessity of selecting the material for a definite rate of wear.

Commutators must be designed to have a sufficient amount of exposed peripheral surface to radiate the heat caused by brush friction and the loss due to brush contact resistance. The design must provide sufficient mechanical strength to withstand the centrifugal force. In the case of turbo-generators running at high speed the diameter is limited by the consideration that the peripheral velocity shall not exceed 8000 ft. per minute, hence, to secure sufficient radiating surface the commutator must have a considerable axial length. To prevent springing of the segments they are held in place by steel rings shrunk over the segments, and thoroughly insulated therefrom, as shown in Fig. 59.

42. The Armature Core. Eddy Currents.—The armature core not only carries the magnetic flux from pole to pole, but revolves through it in exactly the same manner as the conductors of the armature winding. If the core were solid it might be thought of as made up of a very large number of metallic filaments running parallel to the armature conductors and all connected together; in such a case each filament would be the seat of a generated e.m.f., and currents would circulate in the mass of the core in the manner sketched in Fig. 50. The e.m.f. will obviously be greatest near the surface where the peripheral velocity and the active component of the flux are likewise greatest. To minimize these *eddy* or *Foucault currents*, which, if unchecked, would result in excessive heating and loss of power, the core must be *laminated* in such a manner as to preserve the continuity of the flux path and to break up the current paths. The plane of the laminations must be at all points perpendicular to the direction of the generated e.m.f. at those points; or, by Fleming's rule, parallel to the direction of the flux and to the direction of motion. Accordingly, in machines of the usual radial pole type, Fig. 46, the armature core is built up of thin sheet steel punchings insulated from each other; sometimes the insulation consists of a coating of varnish on one side of each disk, but generally the oxide, or scale that forms on the sheets, is relied upon to provide the necessary insulation; in some designs a layer of paper is inserted at intervals of an inch or two. Laminating the core does not completely eliminate eddy currents, but the loss due to them decreases as the square of the thickness of the sheets; the sheet steel ordinarily used in armature cores is 0.014 in. thick. Armatures of the now obsolete disk type, Fig. 51, with active conductors arranged radially, had cores built up of concentric hoops, or, more practically, of thin strap iron wound as a flat spiral.

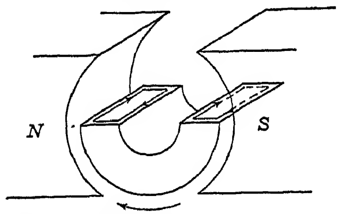


Fig. 50.—Eddy current paths solid armature core.

Core punchings up to a diameter of about 16 in. are generally made in one piece, as in Fig. 52. The disks are first blanked out and the slots are then punched by a special punch press which

cuts one or more slots at a time. Core punchings of this sort are generally keyed directly to the shaft, and are sometimes provided with holes near the shaft to form longitudinal ventilating passages. Cores of large diameter are built up of segments which are

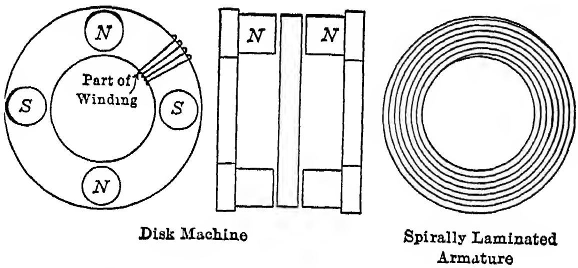


FIG. 51.—Lamination of disk armature.

attached to the spider by means of a dove-tail joint, as in Fig. 46; the joints between segments are staggered from layer to layer in order to preserve the continuity of the magnetic circuit. The core punchings are held together by end flanges which, in the case of small machines, are supported by lock nuts screwed directly to the shaft; in larger machines the end plates are held together by bolts passing through the laminations, but insulated therefrom, and the end plates are shaped to provide a support for the end connections of the armature winding (see Fig. 46).

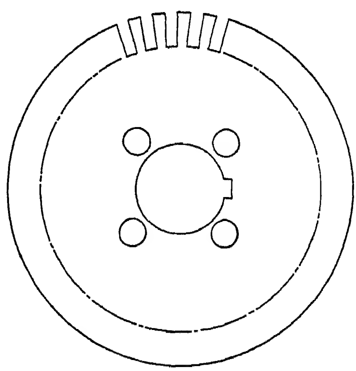


FIG. 52.—One-piece armature punching.

Ventilating ducts through the core are formed by means of spacing pieces placed at intervals of from 2 to 4 in. along the axis of the core. The spacing pieces are generally made by riveting brass strips, on edge, to a punching of heavy sheet steel, as illustrated in Fig. 53; or they may be made by pressing spherical depressions into a thick punching; or by spot welding steel strips, on edge, to the sheet steel punching. The ventilating

ducts vary in width from ¼ to ⅜ in. The spacing pieces should be so designed as to support the teeth as well as the body of the core, in order to prevent vibration and humming.

43. Shape of Teeth and Slots.—Fig. 54 illustrates typical forms of teeth and slots for direct-current machines. Smooth core armatures are used only in special machines. Open slots

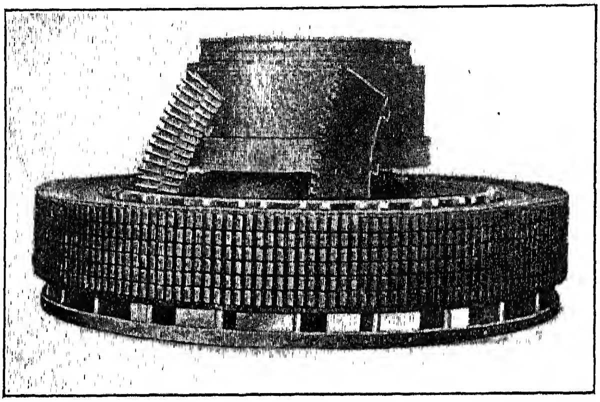


FIG. 53.—Armature core assembly, showing spacing pieces.

with parallel walls are generally used, except in the case of very small machines, for the reason that they permit the use of insulated, formed coils that can be readily slipped into place. Where semi-closed slots are used, the coils may be formed on a winding jig, but the wires of each side of a coil must be slipped into the slot one at a time. The coils are held in place in open

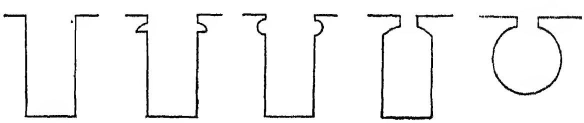


FIG. 54.—Typical shapes of teeth and slots.

slots either by steel or bronze banding wires, or by wooden or fiber wedges driven into the recesses at the tips of the teeth.

The embedding of the armature winding in the slots serves a double function; the air-gap, or distance from the pole face to the iron of the armature core, is less than it would be in a smooth-core construction having the same amount of armature copper, and so

reduces the amount of field copper necessary to produce the flux; and the armature conductors are supported by the teeth when subjected to the tangential forces caused by the reaction of the armature current upon the field flux. When the armature conductors are thus embedded in the slots they are apparently shielded from the inductive effect of the field flux, since the latter in large measure passes around the slots by way of the teeth. At first sight, therefore, it seems surprising that the fundamental equation for the generated e.m.f. is the same for a slotted armature as for a smooth-core armature. It must be remembered, however, that a line of force which at a given instant crosses the air-gap from the pole face to a given tooth tip, must later, by reason of the motion of the armature, be transferred from this tooth-tip to the following tooth. The line of force holds on, as it were, to the first tooth in the manner of a stretched elastic thread, until the increasing tension causes it to snap back suddenly to the next tooth. The increased velocity of cutting of the lines of force by the conductors exactly compensates for the reduced value of the field intensity in the slot.

44. The Pole Cores and Pole Shoes.—The pole cores are generally made of cast steel. When cast steel is used, the poles usually have a circular cross-section because this results in minimum length and weight of the copper wire in the field winding. Laminated poles of course require a rectangular cross-section. Solid poles are commonly bolted to the yoke. Laminated poles may be secured in place either by a dovetail joint or may be cast into the yoke.

The flux density in the body of the pole core, running as high as 110,000 lines per sq. in., is considerably greater than can be economically produced in the air-gap. The average flux density in the air-gap should not exceed 62,000 lines per sq. in., hence the pole faces must have greater area than the pole cores. This increased area is secured by means of pole shoes bolted or dovetailed to the core in the case of solid poles, or by means of projecting tips or horns punched integrally with the sheets composing a laminated pole. The pole faces or shoes are almost always laminated, even when solid poles are used, in order to reduce the loss and heating due to eddy currents set up in the pole faces by the armature teeth; for, as shown in Fig. 55, the flux passing between

the pole face and armature core tends to tuft opposite the teeth, and as the teeth move across the pole face these tufts are drawn tangentially in the direction of rotation until the increasing tension along the lines of force causes them to drop back to the next following tooth. The tufts of flux are therefore continuously swaying back and forth, and if the pole face is considered as built up of thin filaments, as at *P* in Fig. 55, each of the filaments will be cut by these swaying tufts first in one direction, then in the other, thereby inducing an alternating e.m.f. directed parallel to the shaft. To minimize the flow of current the pole face must therefore be laminated in planes parallel to those of the armature laminations, though the laminæ of the pole shoes do not have to be made as thin as those of the armature core.

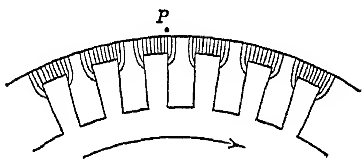


FIG. 55.—Tufting of flux at tips of teeth.

These pole-face eddy current losses will obviously be reduced by so proportioning the dimensions of teeth and slots as to prevent appreciable lack of uniformity in the distribution of the flux along the pole face. The determining factors in this proportioning are the ratio of slot opening to air-gap and the length of the air-gap itself.

45. The Yoke.—The yoke is that part of the field structure which carries the flux from pole to pole, and at the same time serves as a mechanical support for the pole cores. It is made of cast iron in small machines and of cast steel in larger sizes, or whenever saving in weight is important. In a line of machines made by the Westinghouse Electric and Mfg. Co., the yoke is made from flat steel slabs which are first heated and then rolled into circular form, the butt joint between the two free ends being placed on the axis of one of the pole cores so that the break may not introduce additional reluctance into one of the magnetic circuits. In machines of moderate or large size the yoke is usually split on a horizontal diameter for convenience in assembling and repairing. In machines of moderate size the yoke is cast as an integral part of the bed plate; in larger sizes it is cast separately, but with lugs for bolting to the bed plate.

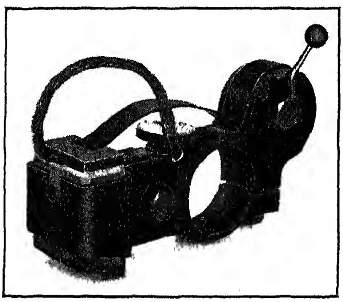
46. Brushes, Brush Holders and Rocker Ring.—The connection between the revolving armature and the external circuit is made through the *brushes*, which are usually made of graphitic carbon, except in the case of low-voltage machines when they may consist of copper or copper gauze. In automobile lighting generators and starting motors the brushes are generally made of a mixture of carbon and metallic copper. Carbon brushes are made of varying degrees of hardness to suit the requirements of commutation, as discussed in a later chapter. The graphite in the brush serves to partially lubricate the commutator, which, when fitted with brushes of the proper composition, takes on a polished surface of dark brown color. The width of the brush in the tangential direction is generally from three to five times the width of a commutator segment, so that several armature coils are simultaneously short circuited. The carbon brush must have sufficient resistance to limit the current in the short circuited coils to a value below that which would result in sparking when the short circuits are opened.

The brushes are commonly set at a trailing angle with respect to the direction of rotation, though in machines designed to run in both directions, such as railway motors, they are set radially.

When the tangential width of the brush has been decided upon, the total axial length of the brushes constituting a set is determined by the consideration that there must be a contact area of 1 sq. in. for every 30 to 50 amperes to be carried by the brush set, though this current density may be exceeded in the case of interpole machines. The individual brushes of a set must not be too large in cross-section, otherwise there would be difficulty in making and maintaining a good contact over its entire contact surface. The subdivision of the set offers the additional advantage of allowing the individual brushes to be trimmed one at a time without interfering with the operation of the machine when under load. Single brushes are used only in the case of machines of small current output.

The individual brushes are supported in metal *brush holders* which are in turn supported by studs attached to, but insulated from, the *rocker ring*, as illustrated in Fig. 56. The brush holders serve as guides for the brushes, and should allow the brush to slide freely in order that the brush may follow irregularities in the commutator surface. The construction of the brush holders

must be such that there will be no vibration of the brushes, this being a common cause of sparking. The brushes are held against the commutator surface by adjustable springs attached to the holder, but in such a manner that the springs do not carry any current. The tension of the springs is adjusted until the brush presses against the commutator with a force of from 1.5 to 2 pounds per sq. in. of contact area. Increasing the brush pressure above this limit does not materially lower the contact resistance, but increases the sliding friction and, therefore, results in increasing loss of power and heating of the commutator. The connection between the brush and the brush holder is made through a flexible lead of braided copper wire, called a *pig-tail*, which is attached to the outer end of the brush by means of a metal band clamped tightly around the carbon. The carbon is generally copper-plated at its outer end to insure good contact.



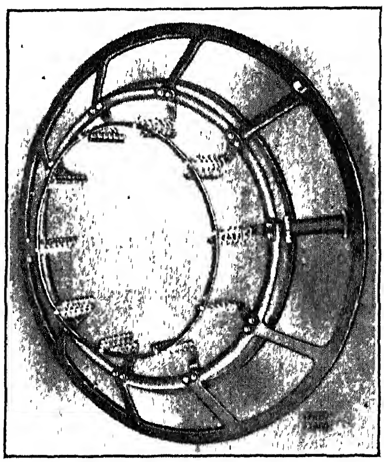


FIG. 56.—Brush holder and rocker ring.

47. Motor-generator. Dynamotor.—It is frequently necessary to convert direct current at one voltage into direct current at some other voltage, higher or lower than the first. For this purpose a *motor-generator* is used. As ordinarily constructed, a motor-generator set consists of two separate machines, a motor and a generator, directly connected to each other, and mounted on a common bed plate, as illustrated in Fig. 57. Motor-generators are also used to convert direct current into alternating cur-

rent, or vice versa. This type of machine has the advantage that the voltage of the generator end may be controlled independently of that of the motor end of the outfit. The over-all efficiency of the set is equal to the product of the efficiencies of the motor and generator. The power rating of the motor must in general be sufficiently greater than that of the generator to allow for the losses that occur in the double transformation of the energy.

Instead of using two separate machines, as in a motor-generator set, to convert the current from one voltage to another, it is possible to combine the two into a single unit, called a *dynamotor*, having a single field structure and a single armature core. By providing the armature of such a machine with two separate

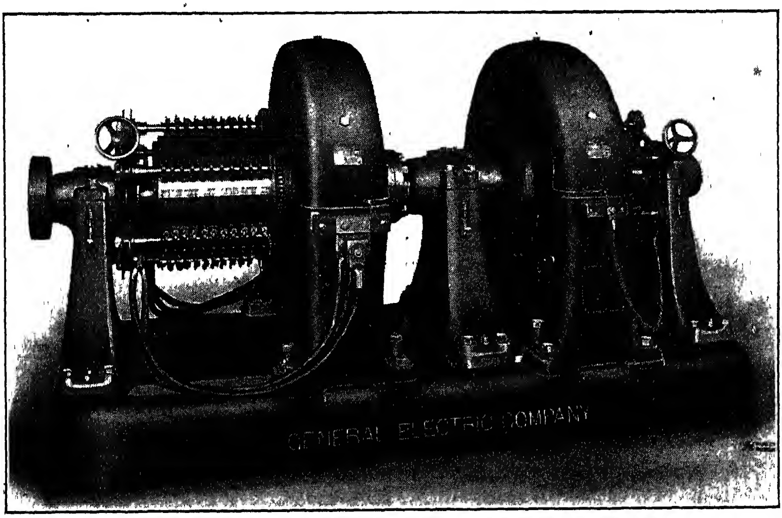


FIG. 57.—Motor-generator set.

windings, each with its own commutator and brushes, current may be introduced into one of the windings, thereby causing motor action, while the other winding will then generate an e.m.f. This type of machine is built in small sizes only. It is open to the objection that the voltage at the generator terminals cannot be independently regulated, but is fixed by the voltage impressed upon the motor terminals. The truth of this statement can be seen from the following reasoning: If the voltage impressed upon

the field flux Φ will generate in the motor armature winding an approximately equal and opposite e.m.f. (Art. 13); if there were no losses in the motor, this counter e.m.f. would be equal to V_m, hence by equation (7)

$$V_m = \frac{p\Phi Z_m n}{a_m \times 60 \times 10^8}$$

Since the generator winding rotates through the same field as the motor winding and at the same speed, the generator e.m.f. is

$$E_g = \frac{p}{a_g}\,\frac{\Phi Z_g n}{60 \times 10^8}$$

or

$$\frac{E_g}{V_m} = \frac{a_m}{a_g} \cdot \frac{Z_g}{Z_m} = \text{constant} \qquad (18)$$

The disadvantage of the fixed ratio of voltage transformation is offset by the reduced cost of construction made possible by the single armature and field structure. The dynamotor has in addition a higher efficiency than a motor-generator, and is practically free from trouble due to armature reaction.

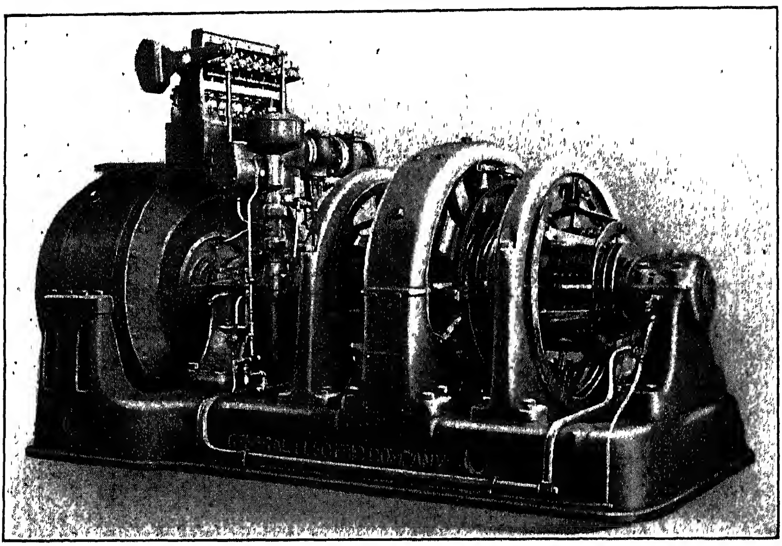


FIG. 58.—Turbo-generator set.

48. Turbo-generators.—Generators for direct connection to steam turbines must be designed for high speed of rotation since the steam turbine develops its maximum efficiency under this

withstand the centrifugal forces and to provide sparkless commutation. The end-connections of the armature winding are held in place by metal end-shells in place of the usual banding wires, and the commutator segments are prevented from springing by a steel ring or rings shrunk over them. To provide for satisfactory commutation these machines are provided with *interpoles* (Art. 49) whose function it is to generate in the coils undergoing commutation an e.m.f. of the proper magnitude and direction to reverse the current in the short time required for the segments to pass across the brush. Fig. 58 represents a 300-kw.,

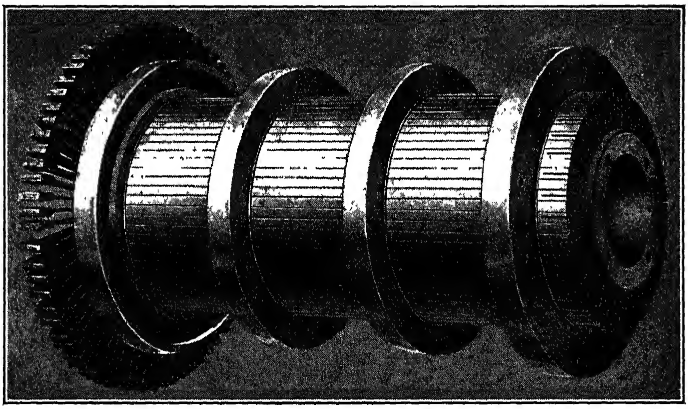


Fig. 59.—Commutator of high-speed generator.

125-volt, 1500-r.p.m. set manufactured by the General Electric Company. This machine has a double commutator, and the interpoles are clearly seen between the main poles. Fig. 59 shows the steel rings around the commutator of a 125-volt, 125-kw. machine.

Turbo-generators require a high grade of brushes to insure satisfactory commutation. The brushes wear down quite rapidly and must be adjusted with great care.

49. Commutating Pole Machines.—A full discussion of the function of commutating poles must be deferred to a later chapter. The commutating poles, also called auxiliary poles or interpoles, are small poles placed midway between the main poles; they are wound with coils through which the armature current, or a fractional part thereof, is made to flow. Interpoles

are used in machines where sparkless commutation would otherwise be difficult or impossible of attainment, as in turbo-generators, adjustable speed motors, etc.

50. The Unipolar or Homopolar Machine.—In the type of armature described in the preceding sections, the individual coils have generated in them alternating e.m.fs. which are then rectified by the commutator; the latter plays much the same part as the valves of a double-acting reciprocating pump. In the centrifugal pump, on the other hand, the developed pressure acts continuously in one direction, thereby obviating the necessity for the rectifying valves, and the electrical analogue of the centrifugal pump is found in the so-called unipolar, or homopolar, or acyclic generator, shown in section in Fig. 60. In principle, this

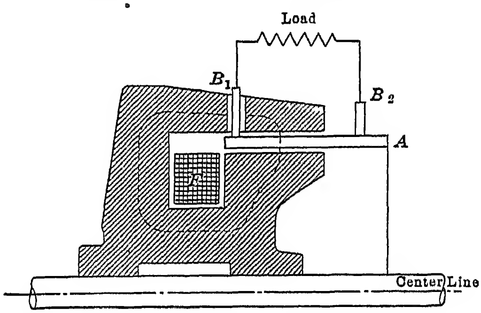


FIG. 60.—Homopolar or acyclic generator.

machine consists of a conductor so disposed in a magnetic field that the cutting of the lines of force is continuously in one direction; it is a true continuous-current machine. The armature consists of a metal cylinder A of low resistance, insulated from the shaft, and upon whose edges the two sets of brushes B_1 and B_2, make sliding contact. The armature rotates in a magnetic field produced by the exciting winding F, the path of the flux being indicated by the dashed line. The lines of force pass radially across the air-gap all around its periphery.

If the intensity of the magnetic field in the air-gap is B lines per sq. cm., the axial length of the active part of the cylinder l cm., and its peripheral velocity v cm. per second, the generated e.m.f. will be $e = Blv \times 10^{-8}$ volts. The maximum e.m.f. obtainable with this type of machine is determined mainly by the consideration that B and v may not exceed definite limits; the length l is likewise limited by such mechanical features as rigidity, freedom

from vibration, etc. At the high rate of rotation required for any reasonable value of e.m.f., difficulty is experienced in securing good brush contact. Thus if $B = 15{,}500$ (100,000 lines per sq. in.), $l = 60$ cm. (about 2 ft.), and $v = 5000$ cm. per second (about 10,000 ft. per minute), $e = 46.5$ volts. Because of the fact that the armature consists of a single conductor of large cross-section, the machine is adapted for heavy currents at relatively low voltage. Unfortunately, however, the magnetizing action of the large armature current when the machine is under load so weakens the field produced by the exciting coil F that the voltage drops considerably.

The analogy between the homopolar machine and the centrifugal pump suggests the idea that, just as high pressures may be obtained with the latter by using several stages, higher voltages may be obtained with the former machine by using several inductors in series. Such a machine has been built by the General Electric Co.[1] for 300 kw. at 500 volts and 3000 r.p.m.; and the Westinghouse Electric and Manufacturing Company[2] has built one for 2000 kw. and 260 volts, running at 1200 r.p.m.

51. Field Excitation of Dynamos.—In every dynamo-electric machine the generation of the armature e.m.f. depends upon the motion of the armature inductors through a magnetic field. In the earliest types of machines this magnetic field was produced by permanent magnets; such machines are called magneto-electric machines or, briefly, magnetos. Their use is now confined to small machines intended for ringing call-bells in small telephone systems, for gas-engine igniters and for testing purposes. The field excitation of all other generators and motors is accomplished by means of electromagnets. The following types of field excitation may be recognized:

Separate excitation

Self excitation $\left\{\begin{array}{l}\text{Series excitation}\\ \text{Shunt excitation}\\ \text{Compound excitation}\end{array}\right.$

52. Separate Excitation.—In this type of field excitation the field winding is traversed by a current supplied from a source

[1] J. E. Noeggerath, Trans. A.I.E.E., Jan., 1905.

[2] B. G. Lamme, Trans. A.I.E.E., June, 1912.

external to the machine itself, such as a storage battery or another generator. The most prominent examples of this type are alternating-current generators and certain kinds of low-voltage direct-current generators used for electroplating. Fig. 61 represents diagrammatically the connections of such a machine.

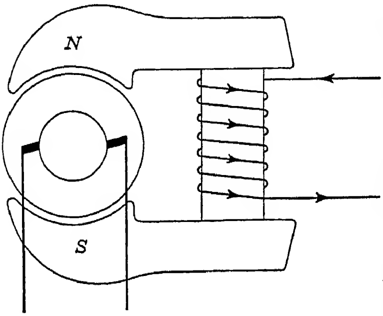


Fig. 61.—Diagram of separately excited machine.

53. Self-excitation.—The use of electromagnets, separately excited for the production of the magnetic field, was introduced by Wilde in 1862. A great step in advance was made in 1867 when Werner Siemens discovered the principle of self-excitation, whereby the armature current, in whole or in part, was made to traverse the field winding, thus causing the machine to develop its own magnetic field. Self-excited machines may be divided into three classes, depending upon the connections of the field winding to the other parts of the circuit; these classes are *series* excitation, *shunt* excitation and *compound* excitation.

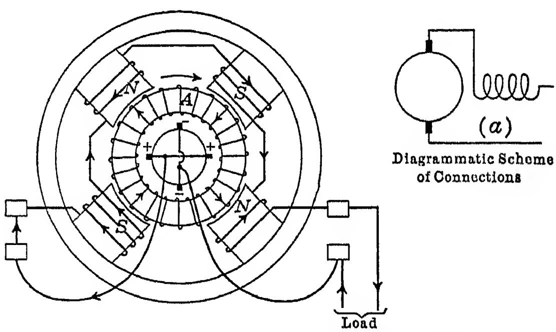


Fig. 62.—Connections of series generator.

54. Series Excitation.—In Fig. 62, *A* represents the armature and *N*, *S*, *N*, *S*, the field structure of a four-pole machine. All of the current taken by the external circuit passes through the field winding and the armature, since all these parts of the circuit are in series. The arrows indicate the direction of the current in the

case of generator action and for clockwise direction of rotation of the armature.

If the field structure of such a machine is originally unmagnetized, rotation of the armature cannot generate e.m.f., hence there can be no current in the circuit. In order that the machine may self-excite, it is necessary that there be some residual magnetism in the field poles due to previous operation, or, in the case of a new machine, produced by sending current through the field winding from some suitable external source. Assuming, then, that residual magnetism is present, a small e.m.f. will be generated when the armature is rotated, and, upon closing the external circuit through the load, a small current will flow. This current will further excite the field structure, thereby developing more e.m.f. and a still greater current, and so on. This gradual increase of both e.m.f. and current will continue until a condition of equilibrium is reached, this being determined by the degree of saturation of the field magnet and by the resistance of the circuit, in a manner that will be discussed fully in the chapter on operating characteristics.

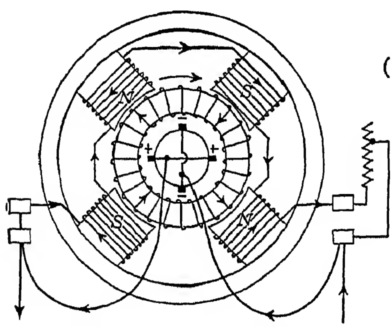
It is important to note, however, that if the field terminals are reversed the machine will refuse to "build up" as described above. For in this case the generated e.m.f. will send a current through the circuit in such a direction as to neutralize the remanent magnetism. Further, if the resistance of the circuit exceeds a critical value, the resultant flow of current may be insufficient to produce the requisite magnetizing force.

From the above description of the process of building up of a series generator, it is obvious that such a machine when running on open circuit (the receiving circuit disconnected) will develop only the small e.m.f. caused by residual magnetism; and that with increasing current, as the external resistance is lowered, the generated e.m.f. likewise increases, though not in general proportionally.

The field winding of series machines consists of relatively few turns of coarse wire. Since the entire current, i, delivered by the machine to the receiver circuit must flow through the resistance, r_f, of the field winding, there occurs a loss of power equal to i^2r_f watts in this part of the circuit. Obviously, this loss must be kept as small as possible in order that the efficiency of the dynamo

may not be seriously impaired, and since the magnitude of the current i is fixed by considerations of the load to be supplied, it follows that r_f must be kept as small as possible; hence the conclusion that the wire of the field winding must have large cross-section and moderate length.

Another way of looking at the matter is as follows: The definite choice of the cross-section of the conductors comprising the field winding depends upon the factors of equation (26), p. 107, one of these factors being the number of ampere-turns per pair of poles (or per pole) required to produce the magnetic flux necessary to develop the desired e.m.f.; this number of ampere-turns depends upon the magnitude of the flux, as well as upon the dimensions and materials of the magnetic circuit, in the manner treated in detail in Chap. IV. If, then, a given armature and field frame are to be assembled to produce a specified voltage, the number of field ampere-turns will be the same no matter what type of excitation is to be used; from which it follows, in accordance with equation (26), that the cross-section of the wire of the field winding must be in inverse proportion to the drop of potential through that winding. Since this drop



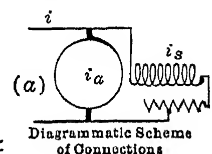


FIG. 63.—Connections of a shunt generator.

must be small in the case of series windings in order to minimize the loss of power, the cross-section must be correspondingly large.

55. Shunt Excitation.—Fig. 63 shows the same armature and field frame as Fig. 62, but provided with a shunt field winding. Fig. 63a represents the connections in a simple diagrammatic manner. It is evident that the exciting current now depends

upon the difference of potential at the brushes and upon the ohmic resistance of the field winding; it is not dependent upon the resistance of the receiver circuit in the same sense as in the previous case, but only to the extent that variations of the external resistance affect the brush voltage. If the external circuit is entirely disconnected, the remaining connection between armature, shunt winding and field regulating rheostat is precisely the same as that of a series generator. On open circuit, therefore, a shunt generator will build up just as a series generator does under load conditions; if it fails to do so, it is usually because of one or the other of the reasons discussed in the preceding section.

It is clear, therefore, that a shunt generator, unlike one of the series type, develops full terminal voltage on open circuit, that is, when no current is being supplied to the receiver circuit. Suppose, now, that the external circuit is closed through a considerable resistance so that a small load current, i, is drawn from the generator. The armature current, which was originally equal to i_s alone, now becomes $(i + i_s)$, and the effect of this increased current through the ohmic resistance of the armature is to cause a drop of terminal voltage; this in turn results in a decrease of the exciting current, i_s, and consequently also of the magnetic flux and the generated e.m.f. As the load current becomes greater and greater the terminal voltage therefore becomes less and less. It is clear that the drop of voltage will be minimized if the resistance of the armature is kept low. The drop of voltage under load conditions is also affected by armature reaction and by the degree of saturation of the magnetic circuit. A complete discussion is given in Chap. VI.

The field winding of a shunt machine consists of many turns of fine wire, for the following reason: If the terminal voltage of the machine is V volts, the shunt field current, i_s, will be $\frac{V}{r_s}$, and the power loss in the winding will be $i_s^2 r_s = \frac{V^2}{r_s}$; since V is fixed by other considerations, it follows that r_s must be as large as is feasible (in order to keep down i_s and the loss of power) hence the use of wire of small cross-section and considerable length.

Referring to equation (26), it will be seen that the cross-section of the field winding must be inversely proportional to the voltage consumed in the field winding, and as this is of the order of the full terminal voltage in the case of a shunt machine, it is easy to see why the field winding must in that case be made of much smaller wire than in the case of a series winding, where the drop through the field winding is only a small percentage of the terminal voltage.

The relation between the armature current i_a, the line current i and the shunt field current i_s, in the case of *generator* action, is given by the equation

$$i_a = i + i_s \tag{19}$$

In the case of *motor* action the relation is obviously

$$i = i_a + i_s \tag{20}$$

It should be remembered that the armature and field currents of a shunt motor do not divide in the inverse ratio of their respective resistances, for the reason that the armature, when running, is the seat of a counter-generated e.m.f. The field current is given by $i_s = \dfrac{V}{r_s}$, but the armature current is $i_a = \dfrac{V - E_a}{r_a}$, where E_a is the counter e.m.f.

56. Compound Excitation.—In some of the most important applications of direct-current machinery, such as incandescent lighting, street-railway operation, and the like, it is necessary to maintain a constant difference of potential between the supply mains no matter what the load may be. Since the center of the load is usually at a distance from the generator, it follows that the potential difference between the generator terminals should rise as the external current increases, in order to compensate for the drop of potential in the supply mains. Field windings adapted to give this characteristic are called compound windings, illustrated in Fig. 64 and diagrammatically in Figs. 65*a* and 65*b*. They are combinations of shunt and series field windings. Connections made in accordance with Fig. 65*a* result in a *short-shunt* winding, those of Fig. 65*b* in a *long-shunt* winding. The shunt winding of itself would produce a "drooping" characteristic, that is, one in which the terminal voltage falls with increasing current, as explained in the preceding section; but the series winding contributes field excitation which increases with increasing current,

hence the resultant effect will depend upon the relative magnitudes and directions of the magnetizing actions of the two field windings. By properly proportioning them, the voltage-current curve may rise, in which case the machine is said to be *over-compounded;* or the voltage may remain very nearly constant for all permissible values of current, as in a *flat-compounded*

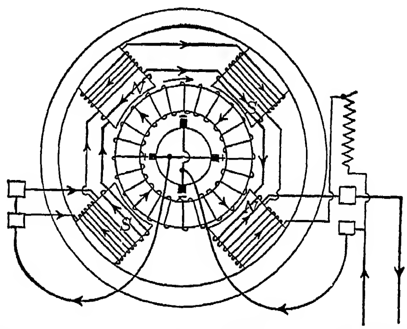


FIG. 64.—Connections of a compound generator.

machine; or it may fall at a greater or lesser rate than with the shunt winding alone, giving rise to the classification of *under-compounded* machines.

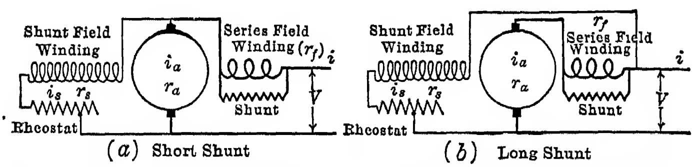


FIG. 65 —Diagrammatic scheme of connections of compound machines.

In the short-shunt compound-wound generator the relation between armature current i_a, line current i, and shunt-field current i_s is given by

$$i_a = i + i_s$$

The terminal voltage V, and the generated e.m.f., E_a, are related by the equation

$$E_a = V + ir_f + i_ar_a \tag{21}$$

and the shunt-field current is given by

$$i_s = \frac{E_a - i_a r_a}{r_s} = \frac{V + i r_f}{r_s} \tag{22}$$

In the long-shunt compound-wound generator these relations become

$$i_a = i + i_s$$
$$E_a = V + i_a r_f + i_a r_a \tag{23}$$
$$i_s = \frac{V}{r_s} \tag{24}$$

57. Construction of Field Windings.—In designing the field windings of shunt, series and compound machines, the selection of the correct number of turns and the cross-section of the conductors follows from a knowledge of the number of ampere-turns per pole required to produce the flux Φ, and from the dimensions of the pole core. The calculation of these latter quantities depends upon principles that are discussed in detail in Chap. IV. Assuming that the number of ampere-turns per pole and the dimensions of the pole core are known, the determination of the size of wire to be used in the shunt field winding is as follows: Let

i_s = current in shunt winding
V = terminal voltage at no load
v_r = voltage consumed in regulating rheostat, varying from 10 per cent. to 20 per cent. of V.

The object of the field rheostat is to permit an increase of i_s by cutting out a part or all of the regulating resistance, thereby raising the generated e.m.f.

The resistance of the winding per pole is then

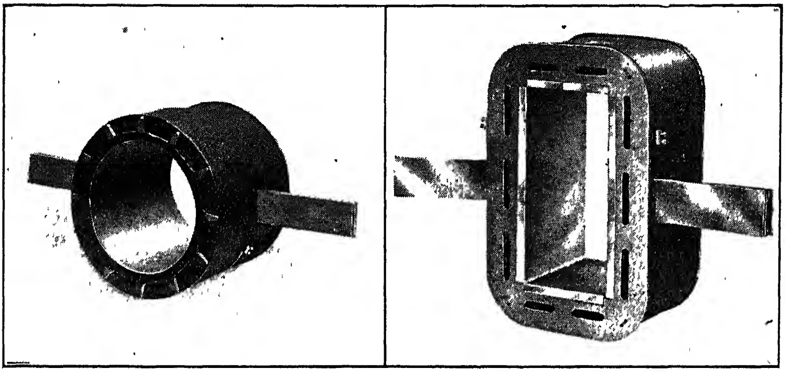
$$r_s' = \frac{V - v_r}{i_s p} = \rho \frac{\frac{1}{2} n_s l_t}{A} \tag{25}$$

where

ρ = specific resistance of copper at the working temperature of the winding (about 75° C.)
n_s = number of shunt turns per pair of poles
l_t = mean length of a turn
A = area of cross-section of conductor.

If lengths are expressed in feet and cross-sections in circular mils, $\rho = 12.6$ at 75° C. Hence

$$A = \frac{6.3(n_s i_s)l_t p}{V - v_r} \text{ circular mils.} \tag{26}$$



(a) (b)

Fig. 66.—Ventilated field coils.

In general, without regard to whether the field winding is to be of the series or shunt type, the relation between the size of wire and the other determining factors is given by

$$A = \frac{6.3 \times \text{amp.-turns per pair of poles} \times \text{mean length of turn}}{\text{drop of potential per pole}}$$

The mean length of a turn, l_t, is found by assuming a depth of winding of from 1 to 3 in. If the cross-section of the pole core is rectangular, l_t will be approximately equal to the perimeter of the core plus four times the winding depth; if the pole core is circular, of diameter d_c, $l_t = \pi(d_c + \text{winding depth})$. The winding depth must not exceed a definite limit, otherwise the heat generated in the interior of the core cannot readily be conducted to the surface. As a check on the calculations, it must be ascertained that the power lost in the coil ($i_s^2 r_s'$) does not exceed approximately two-thirds of a watt per sq. in. of exposed radiating surface.

Shunt coils are usually made of cotton-covered wires, of either round or rectangular section. Sometimes they are wound on metal frames arranged to slip onto the pole cores; sometimes they are wound on removable winding forms, the coils being held in shape by suitable insulating materials and dipped in, or painted with, moisture-proof varnish. When metal frames are used, they are frequently made with a double wall to allow the circulation of air between pole core and winding, as shown in Fig. 66. The coils of series-wound railway motors are usually impregnated with insulating compound, then taped and varnished. The series coils

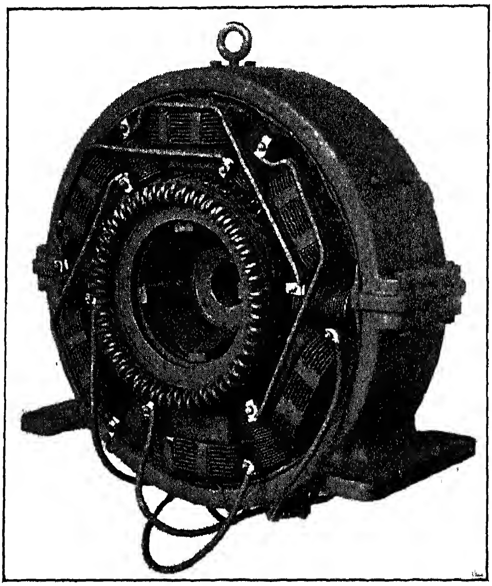


Fig. 67.—Interpole machine, edge-wound copper strap coils.

of compound and interpole machines are often made of copper strap, wound on edge, the turns being separated by distance pieces of insulating material, as shown in Fig. 67.

In order that connections may be easily made between the coils of adjoining poles, the terminals of the coils are brought out on opposite sides, so that the number of turns per coil is an integer, plus one-half.

58. Field Rheostats.—To permit regulation of the voltage of shunt and compound generators, the current in the shunt-field

winding must be under control. To this end a variable resistance, or *field rheostat*, is inserted in series with the shunt winding, as shown diagrammatically in Figs. 63 and 64. This resistance is arranged in the manner shown in Fig. 68, taps being brought out from the high resistance wire or ribbon composing the resistor to a series of insulated studs over which moves an adjustable contact arm. The terminals are always brought out in such a way that clockwise rotation of the regulating handle increases the resistance in circuit and so throttles the current in the manner of an ordinary valve.

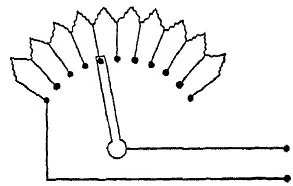


Fig. 68.—Diagram of connections of field rheostat.

Field rheostats are generally arranged to be mounted on the back of the switch-board, with the regulating handle on the front of the board. Fig. 69 represents a field rheostat made by the General Electric Company. Field rheostats for machines of large capacity are made of cast-iron grids, as shown in Fig. 70.

In shunt and compound generators of large capacity the energy stored in the magnetic field is very considerable, amounting to

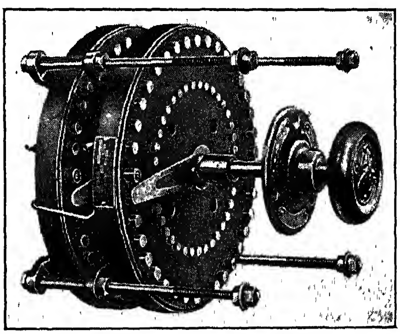


Fig. 69.—Field rheostat, back connection.

$\frac{1}{2} L_s i_s^2$, where L_s is the inductance of the shunt winding. The inductance may have a value of several hundred henrys. For instance, if $L_s = 600$ and $i_s = 4$, the energy stored in the field is 4800 joules. If the field circuit is abruptly broken, this energy will have to be dissipated in the arc formed on breaking the circuit; if, for example, the current were made to disappear in

one-half second, the average rate of energy dissipation would be $4800 \div \frac{1}{2} = 9600$ watts, and the average voltage induced by the collapse of the magnetic field would be $L_s\frac{di_s}{dt} = 600 \times 8 = 4800$ volts. In this case the arc would be very destructive, and the

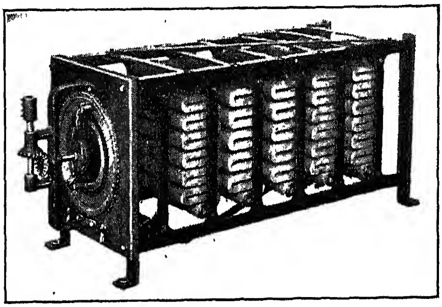


FIG. 70.—Large field rheostat.

high induced voltage would be likely to puncture the insulation of the winding. To obviate this danger the field current must be gradually reduced before breaking the circuit. In large machines the reduction of field current is accomplished by allow-

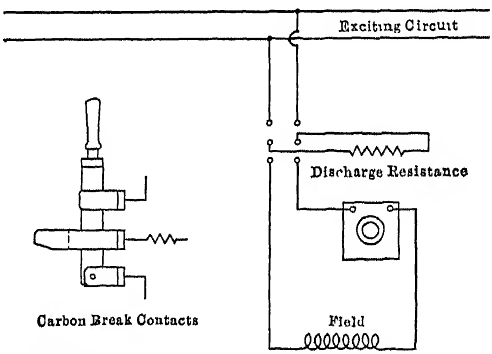


FIG. 71.—Diagram of connections of field discharge resistance.

ing the field windings to discharge through a *field discharge* resistance, connected in the manner shown in Fig. 71.

59. Polarity of Generators.—In order that a self-exciting generator of any of the types already described may be operative, it is necessary that there be some remanent magnetism in the field system; further, that the initial flow of current through the excit-

ing winding have such a direction that it will strengthen the remanent field. In other words, the polarity of the machine is determined by that of the remanent magnetism.

For example, consider the conditions existing in the shunt-wound generators illustrated in Figs. 72*a* and 72*b*, respectively. The machines are identical except that the remanent magnetism of the second is reversed with respect to that of the first.

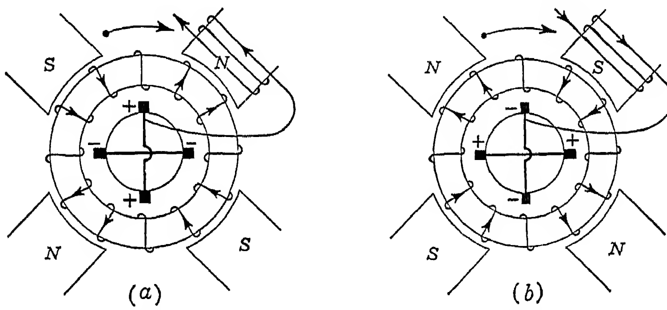


FIG. 72.—Effect of reversal of residual magnetism.

In each case the machine will build up if the direction of rotation is clockwise, but with the polarity of the terminals of the one opposite to that of the other. With the connections as shown, counter-clockwise rotation would set up a field current which would wipe out the remanent magnetism, but with counter-clockwise rotation the machines would again become self-exciting if the terminals of the field winding are interchanged.

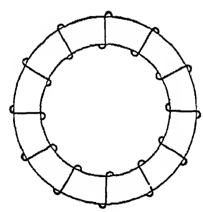


FIG. 73.—Left-handed ring-wound armature.

In Figs. 72*a* and 72*b*, the armature winding is *right-handed,* that is, it is wound around the core in the manner of a right-handed screw thread. If *left-handed* armature windings had been used (Fig. 73), other conditions remaining the same, annulment of the remanent magnetism would again be the result. Finally, it is clear that the direction of the winding around the poles plays a similar rôle.

There are therefore four elements which affect the polarity of such a machine: The *sense* of the windings of armature and pole pieces, respectively; the direction of rotation; and the order of connections of the field winding terminals to the armature termi-

nals. With a given remanent magnetism, the machine will operate only when there is a definite relation between them. Assuming that the conditions for operation are satisfied, a change in any one of these four elements will cause the machine to counteract its residual magnetism, but a change in any two of them will not affect the operation. Thus, a right-handed armature rotating clockwise in a given field flux will yield the same brush polarity as a left-handed armature rotating counter-clockwise in the same field. In general, a change in an *odd* number of the four elements will disturb conditions if they were previously correct, while a change in an *even* number of them will not affect the operation.

60. Direction of Rotation of Motors.—The same types of field windings and connections as are used for generators find equal

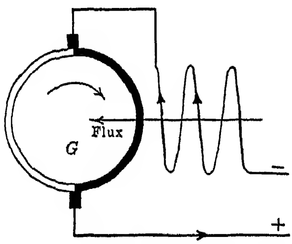


FIG. 74.—Diagrammatic sketch of series generator.

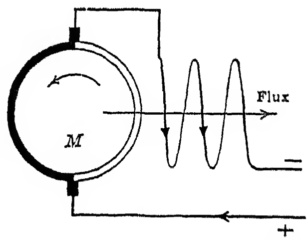


FIG. 75.—Diagrammatic sketch of series motor.

application in the case of motors. Series motors, when supplied with constant terminal voltage, fall off rapidly in speed as the load increases, or, to put it in another way, "race" as the load is removed; this characteristic of variable speed at constant terminal voltage is a sort of "mirror" image of the series generator characteristic, namely, variable voltage at constant speed. The speed characteristic of the series motor adapts this machine to street railway and hoisting service. Again, the shunt motor, when supplied with constant terminal voltage, operates at practically constant speed at all loads, just as the shunt generator delivers a nearly constant terminal voltage (within limits of machine capacity) when driven at constant speed. An over-compounded generator if sufficiently compounded used as a motor will rise in speed with increasing load, if supplied from constant potential mains (see Chap. VII), thus again emphasizing the

reciprocal relationship between generators and motors, since in an over-compounded generator operated at constant speed the voltage rises as the load increases.

Let Fig. 74 represent diagrammatically a series machine used as a generator, the shaded half of the armature circle representing a belt of current flowing into the plane of the paper and the unshaded half representing current of opposite direction. If this machine is now connected to mains of the polarity indicated in Fig. 75, and is operated as a motor, its direction of rotation will be reversed as may be seen by applying (the left-handed) Fleming's rule. This means that a series generator supplying a network fed by other generators may reverse its direction of

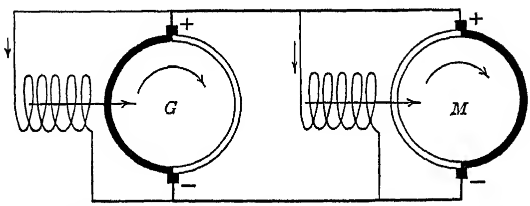


Fig. 76.—Showing direction of rotation of shunt generator and motor.

rotation and so buckle the connecting rod of the driving engine. The fundamental reason for the reversal of direction of rotation is that both armature current and field current reverse simultaneously at the same time that the machine changes from generator to motor action. Considering these changes from the point of view of Fleming's right- and left-hand rules, it is seen that there have been *three* changes in all, namely: (1) A change in the direction of the armature current, represented by the middle finger; (2) a change in the direction of the magnetic field, represented by the forefinger; and (3) a change from generator to motor action, involving the use of the left hand instead of the right.

Turning now to the case of two identical shunt machines, one used as a generator, the other as a motor, as in Fig. 76, the direction of rotation is found to be the same in both; consesequently, a shunt generator supplying a network fed by other generators is not subject to reversal of its direction of rotation in case its prime mover is disconnected or shut down, but will

continue to run in the original direction, as a motor. Examining these conditions in the light of Fleming's rules, it is clear that there have been *two* changes: (1) That of the middle finger, indicating the direction of the armature current; (2) the change from right hand to left hand because of the transition from generator to motor action.

These considerations may be generalized by observing that there are four factors to be considered in applying Fleming's rules: The direction of the armature current, the direction of the magnetic field, the direction of rotation, and the nature of the operation of the machine as generator or as motor. A change in an *odd* number of any of these four factors will change one of the remaining factors; whereas a change in any *even* number of them will not change the remaining ones.

Thus, a reversal of the polarity of the mains supplying a motor, whether it be of the series or shunt type, will change two factors, the direction of the armature current and the direction of the magnetic field; hence the direction of rotation will not be affected; it is for this reason that both types will run as alternating current motors. If a motor is to have its direction of rotation reversed, only one change must be made, either in the direction of the armature current or in that of the magnetic field, but not in both.

PROBLEMS

1. A concentrated rectangular coil of 100 turns, measuring 30 cm. by 60 cm., is rotated at a uniform speed of 600 rev. per min. about an axis passing through a diagonal of the rectangle. If the coil is in a uniform magnetic field of 200 gauses, whose direction makes an angle of 30 deg. with the axis of rotation, what are the maximum and average values of the generated e.m.f.? What are the positions of the coil with respect to the direction of the field when the instantaneous e.m.f. has (*a*) its maximum value, (*b*) ~~a~~ value equal to the average e.m.f.? $E_A = 3.6$ volts. $E_{max} = 14.03$ volts

2. If the rectangular coil of Problem 1 is replaced by a circular coil having the same number of turns and a diameter such that it encloses the same area, what will be the average e.m.f.? Compare the average e.m.f. per unit length of wire in Problems 1 and 2. $E_A = 3.6$ volts. $E_1 = 3.6/18000$ $E_2 = 3.$

3. The 8-pole alternator of Fig. 34 has a field flux of 4.5 × 10⁶ lines per pole, distributed sinusoidally around the periphery of the stationary armature. Each of the 8 slots contains 20 conductors, all conductors of the entire winding being connected in series. If the speed of rotation is 900 r.p.m., what are the average and maximum values of the generated e.m.f.?

4. The alternator of Fig. 34 is provided with pole shoes that cover 65 per cent. of the armature surface, and they are so shaped that the flux of 4.5×10^6 lines per pole crosses the air-gap along uniformly distributed radial lines. If the number of armature conductors and the speed are the same as in Problem 3, what are the average and maximum values of the generated e.m.f.? Construct a curve showing the variation of the e.m.f. from instant to instant. Avr. = 864 vol. E_{MX} = 1330 volt.

5. A ring-wound armature like Fig. 43 has 800 conductors distributed uniformly around its periphery, and rotates in an 8-pole field structure that produces a flux of 1.5×10^6 lines per pole. At what speed must the armature rotate to develop an e.m.f. of 120 volts?

6. If the total amount of wire on the armature of Problem 5 consists of 800 ft. of No. 16 B. and S. wire, which has a resistance of 4.085 ohms per thousand feet at 75° F., what is the resistance of the armature, measured between its terminals, at 85° C.?

7. The commutator of a machine which runs at 900 r.p.m. has a diameter of 16 in. There are four sets of brushes, each set consisting of four brushes; each individual brush has a width of 1.5 in. and a contact arc of 0.25 in. The contact pressure is 2 lb. per sq. in. of brush contact area, and the coefficient of friction between carbon and copper is 0.3. What is the brush friction loss, expressed in watts?

8. A series-wound generator has a normal rating of 115 volts and 10 amperes, and its field winding has a resistance of 1.5 ohms. The machine is to be operated as a separately excited generator and there is available a 220-volt supply circuit. How should the machine be connected to secure normal field excitation?

9. A 115-volt shunt motor takes a field current of 2.8 amp. and, when running without load, an armature current of 3.0 amperes. When the armature is blocked, full-load armature current of 50 amp. is produced through it by an impressed voltage of 5.5 volts. What are the resistances of the field and armature windings, and what is the counter e.m.f. generated in the armature when the machine is running under no-load conditions?

10. The field structure of the motor of Problem 9 has 4 poles and the winding is made of No. 15 B. and S. wire which has a resistance of 3.88 ohms per thousand feet at the working temperature. Each field coil is wound on a cylindrical bobbin and has a mean diameter of 7 in. Find the number of turns per coil.

CHAPTER III

ARMATURE WINDINGS

61. Types of Armatures.—Armatures, considered as a whole, may be divided into three classes according to the shape of the core and the disposition of the winding upon it. These three classes are:

1. Ring Armatures.
2. Drum Armatures.
3. Disk Armatures.

The *ring armature* is one in which a ring-shaped core is wound with a number of coils, or elements, each of which winds in and out around the core in helical fashion, as in Figs. 41, 42 and 43. In these windings the coils are usually connected successively to each other so as to form a continuous circuit, the end of each element being connected to the beginning of the next adjacent element, but this particular feature is not essential to the definition. The characteristic feature of ring windings is that there are conducting wires inside the ring which do not cut lines of force, and which do not, therefore, contribute to the e.m.f.

The *drum armature* differs from the ring armature in that no part of the winding threads through the core; the entire winding is external to the core. Each active wire, wound on the outer surface in a direction parallel to the shaft, is connected to another active wire by means of connecting wires which do not thread through the core, in the manner shown in Fig. 90. The only reason for having any opening in the core at all is to permit ventilation and cooling. In bipolar machines the end connections run across the flat ends of the core and join conductors which are nearly diametrically opposite. In multipolar machines they join conductors separated by an interval approximately equal to the pole pitch, so that the e.m.fs. in the conductors thus connected may be additive.

The drum armature may be thought of as evolved from the ring type by moving the inner connections of the winding

elements to the outer surface, at the same time stretching the coil circumferentially until the spread of the coil is approximately a pole pitch.

The *disk armature* differs from the other two types in that the active conductors, instead of lying on the outer cylindrical surface of the core, are disposed radially on the flat sides of a disk. The disk revolves between a number of pairs of poles of opposite signs, so that the wires on both faces of the disk are active (see Fig. 51). Disk armatures are seldom used in modern practice.

Of the three types of armatures described above, the drum armature is used practically to the exclusion of the others. One of the reasons for its original development was the desire to increase the percentage of active wire in the winding as a whole, the active wire being that part of the winding which cuts lines of force and so contributes to the total generated e.m.f. But the principal advantage of the drum winding is that it obviates the necessity of hand winding required in ring armatures, and therefore reduces the cost of manufacture; also, since the coils are wholly outside the core, they may be wound on formers, or winding jigs, and can be thoroughly insulated before being slipped into place.

62. Types of Windings.—All armature windings, for both direct- and alternating-current machines, belong to one or the other of the two types, *open-coil* and *closed-coil* windings. An open-coil winding is one in which, starting with any conductor and tracing progressively through the winding, a "dead-end" is finally reached; whereas, in a closed-coil winding, the starting point will finally be reached after having passed through all, or some sub-multiple, of the conductors. The use of open-coil windings is at present confined to alternating-current machines and need not, therefore, be considered here. Open-coil windings were at one time used to a large extent in direct-current series arc-lighting generators, such as the Brush and Thomson-Houston machines.[1]

CLOSED-COIL WINDINGS

63. Ring and Drum Windings.—In designing the armature of a generator or motor, the number of armature conductors is

[1] See Dynamo Electric Machinery, S. P. Thompson.

determined by the fundamental equation for the e.m.f. (equation 7, Chap. II). The problem is then so to connect the various conductors into a closed winding that their individual e.m.fs. will add together to produce the desired total e.m.f., and in such a way that the winding as a whole will be at all times symmetrical with respect to the brushes.

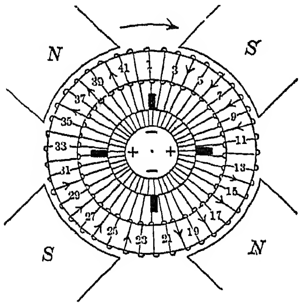
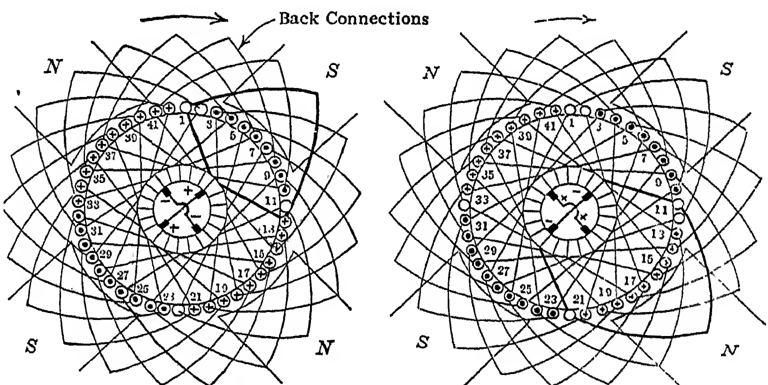


Fig. 77.—Ring-wound armature.

In Figs. 77 and 78 there are shown three distinct types of closed-coil windings for a four-pole machine having 42 active conductors. Fig. 77 is a simple ring armature, while the two parts of Fig. 78 represent *drum* armatures. In these drawings the small circles represent the cross-sections of the active conductors lying on the cylindrical surface of the armature core; the *end connections* which serve to connect up the active conductors at the back, or pulley, end of the armature are drawn for convenience outside the bounding surface of the core,



(*a*) Lap Winding (*b*) Wave Winding

Fig. 78.—Drum armature windings.

although in reality these end-connections lie on a cylindrical surface that forms an extension of the iron core and which has a diameter slightly less than that of the core, in the manner shown

in Fig. 90; similarly, the front, or commutator end connections are shown in Fig. 78 inside the bounding surface of the core, though these, too, rest on a cylindrical surface. The arrangement of the windings shown in Fig. 78 is made clearer by resorting to

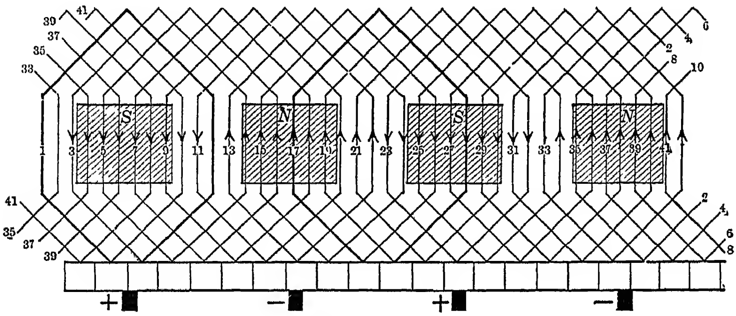


Fig. 79.—Developed lap winding.

the developed diagrams of Figs. 79 and 80, which are derived from those of Fig. 78 by rolling out the cylindrical surface of the armature core into a plane.

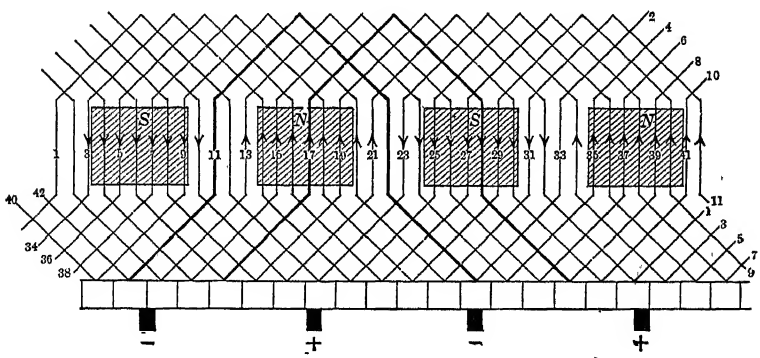


Fig. 80.—Developed wave winding.

A distinctive difference between the ring winding of Fig. 77 and the drum windings of Fig. 78 is that in the former the brushes occupy positions on lines midway between the pole tips while in the latter they are placed almost directly under the middle of the pole faces. In all cases, however, the elements or coils that

are short-circuited by the brushes during the commutation period are so located that the coil edges are passing through the neutral zone between pole tips. The position of the brushes themselves depends entirely upon the shape of the end-connections which join the coil edges to the commutator segments. In most commercial machines the individual coils are shaped like those of Figs. 79 and 80, but they might conceivably be formed like those of Fig. 81, and the commutator connections of a ring winding might be made in the manner shown in the left-hand diagram of that figure; under these conditions the brush position of the ring winding might be anywhere under the poles, while in the drum windings the brushes would be midway between the pole tips.

64. Winding Element.—It will be seen that in each case the winding consists of a number of identical *elements* which are shown in heavy lines in Figs. 78 to 80, inclusive. An element

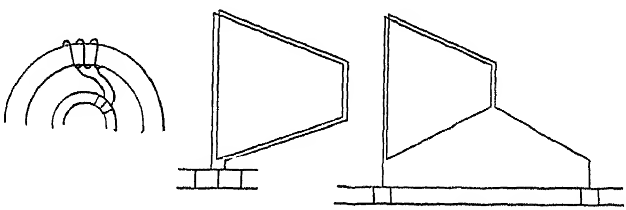


FIG. 81.—Winding elements having more than one turn.

may be defined as that portion of a winding, which, beginning at a commutator segment, ends at the next commutator segment encountered in tracing through the winding. It will be evident at once that an element may consist of more than one turn, *i.e.*, of more than two active conductors; for instance, Fig. 81 represents elements of windings similar to those of Figs. 77 and 78, but with two turns each, instead of one.

Small machines for relatively high voltage, railway motors for instance, frequently have as many as five turns per element; but in machines of large capacity there is, as a rule, only one turn per element for the purpose of improving commutation. Every time an element passes though the neutral zone of the magnetic field the current which it has been carrying must be reversed in

direction; hence its self-inductance must be kept as small as possible in order that the reversal of the current may not be impeded, and as the coefficient of self-induction increases as the square of the number of turns, the number of turns should be a minimum, or unity, for best results.

65. Ring, Lap and Wave Windings.—The three windings of Figs. 77 and 78 belong, respectively, to the *ring*, *lap*, and *wave* types of closed-coil windings. The derivation of the terms lap and wave winding will be evident from an inspection of Figs. 79 and 80; in the former, the successive elements lap back over each other, while in the latter they progress continuously in wave fashion around the periphery of the armature.

It will be noted that in both lap and wave windings the two sides of a coil or element are subjected to the influence of adjacent poles of opposite polarity, so that the e.m.fs. generated on the two sides add together. In a simple lap winding, the end of any element, say the xth, connects to the beginning of the $(x + 1)$st element, and the beginning of the $(x + 1)$st element lies under the same pole as the beginning of the xth element; in a wave winding, however, although the end of the xth connects to the beginning of the $(x + 1)$st element, the latter is not under the same pole as the beginning of the xth element, but is separated from it by a double pole pitch.

The study of the arrangement of the windings shown in Figs. 79 and 80 is greatly facilitated by preparing winding tables, in the manner illustrated below. Thus, taking the lap wound arma- first, the order of connections of the conductors is, starting conductor number 1, 1–12–3–14–etc., or in tabular form,

1	12
3	14
5	16
7	18
9	20
11	22
13	24
15	26
17	28
19	30
21	32
23	34

25	36
27	38
29	40
31	42
33	2
35	4
37	6
39	8
41	10
1 (winding closes)	

The gradual advance or creep of the winding around the periphery may be emphasized by arranging the numbers of the table in accordance with the following plan, where the letters S, N, S, N are spaced apart at a distance representing the pole pitch, the letters themselves being at the center points of the pole faces.

S	*N*	*S*	*N*
1	12		
3	14		
5	16		
7	18		
9	20		
11	22		

Limitations of space prevent the completion of the entire table on the printed page.

Proceeding in the same way with the wave winding of Fig. 80, the winding table is

1	12	23	34
3	14	25	36
5	16	27	38
7	18	29	40
9	20	31	42
11	22	33	2
13	24	35	4
15	26	37	6
17	28	39	8
19	30	41	10
21	32	1 (winding closes)	

If this table is arranged like the one immediately above, so as to show the creep of the winding, it would appear in part as follows:

S	*N*	*S*	*N*
1	12	23	34
3	14	25	36
5	16	27	38
7	18	29	40
9	20	31	42
11	22	33	2
13	24	35	
4	15	26	37
6	17	28	39

etc.

An examination of the directions of the current flow in Figs. 77, 79, and 80 will show that in the case of the first two diagrams there are four separate and distinct paths for the current through the winding ($a = 4$); each of these paths will carry one-fourth of the entire current supplied to the external circuit in the case of generator action, or supplied from the line in the case of motor action. In Fig. 80, on the other hand, though there are four poles as in the other machines, there are only two paths through the winding ($a = 2$). Other things being equal, therefore, the wave winding shown in the diagram will generate twice the e.m.f. of either of the other two in accordance with the fundamental equation

$$E = \frac{p}{a}\frac{\Phi Z n}{60 \times 10^8}$$

or, what amounts to the same thing, the same e.m.f. will be generated with only half the number of conductors required by a ring or lap winding. Furthermore, the diagrams show that four brushes are required in the cases of the ring and lap windings, while two will suffice in the case of the wave winding. These two facts in conjunction explain the reason for the use of wave windings in the case of direct-current railway motors, where the combination of the cramped space and the moderately high voltage require a minimum number of conductors; and no less important, considerations of accessibility for inspection and repairs limit the number of brush sets to two.

Lap and wave windings are often referred to as *parallel* and *series* windings, respectively.

66. Number of Brush Sets Required.—Inasmuch as the current in an element must undergo commutation once for each pas-

sage of the element through a neutral zone, it follows that the element may be short-circuited by a brush at each such reversal. Since the number of neutral zones and consequent reversals is equal to the number of poles, the number of permissible brush sets may in all cases be the same as the number of poles. In lap windings and in simple ring windings of the type shown in Fig. 77, the number of brush sets *must* be equal to the number of poles. But in wave windings, though p brushes *may* be used, two brushes will suffice irrespective of the number of poles. Thus, in Fig 82, which shows a wave winding for a 6-pole machine having thirty-two armature conductors, any two of the three negative brushes a, b, and c may be omitted, as b and c (provided that a corresponding pair of positive brushes are removed at the same

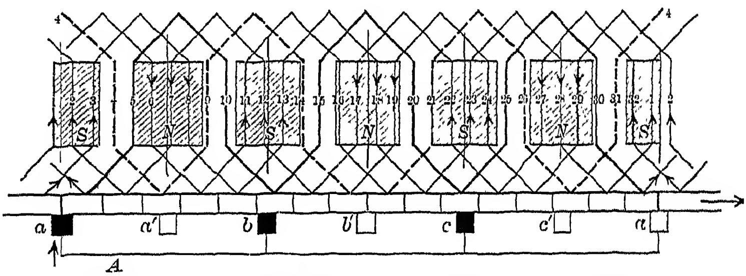


FIG. 82.—Six-pole wave winding, showing elements short-circuited by brush.

time), in which case the remaining brush, for example, brush a in Fig. 82, will short-circuit three elements in series when it spans two adjacent commutator segments. The three elements thus short-circuited by brush a are shown in heavy lines; in the position shown in the figure, brush b', if it is present, will short-circuit the three elements shown as dashed lines. Fig. 82 also makes it clear why two brushes instead of six, will suffice to collect the current, for it will be observed that brushes a, b, and c are connected not only by the external conductor A but also by the short-circuited elements shown in heavy lines; these elements are in the neutral zone, consequently have little or no e.m.f. generated in them and are, therefore, equivalent to additional dead conductors joining the three brushes; hence conductor A and any two of the brushes a, b, and c may be omitted. But if brushes b and c are retained, it will be observed that brushes

a, *b*, and *c*, which are connected together to form one terminal of the machine, operate in pairs to short-circuit single elements. This reduces the e.m.f. of self-induction to one-third of the value that would otherwise have to be handled, thereby improving commutation conditions.

67. Simplex and Multiplex Windings. Degree of Reëntrancy.—If two identical ring-wound machines are connected in parallel as indicated in Fig. 83, the combined current output will be double that of either machine separately. The same result may be attained, together with economy in the use of material, by placing two independent windings on the same armature core, subjected to the magnetizing action of a single field structure, as indicated in Fig. 84*a*. Here both the winding elements and the commutator segments of the independent windings are "sand-

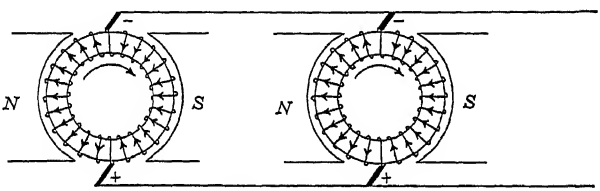


Fig. 83.—Armatures in parallel.

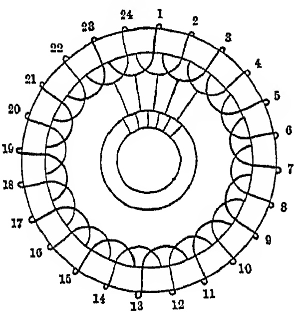
wiched" or imbricated. The same result might also be secured by using two independent commutators, one at each end. Windings of the type of Fig. 84 are called *duplex* windings as distinguished from the *simplex* windings of Fig. 83 and those preceding it. Obviously, there is nothing to prevent the multiplication of independent windings so as to form *triplex*, *quadruplex*, etc., windings.

Drum windings, both of the lap and wave varieties, may be treated in the same way as has here been described for the case of ring windings. It will be noted that the interleaving of the commutator segments of the component windings requires the use of brushes of sufficient width to collect the current from each pair of circuits at a neutral point.

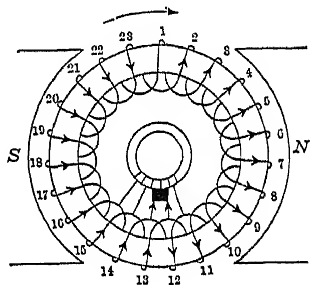
A multiplex winding is equivalent to two or more simplex windings in parallel with one another. Thus, a duplex winding is equivalent to two simplex windings in parallel; a triplex wind-

ing is equivalent to three simplex windings in parallel, a quadruplex to four, and so on.

It has been pointed out in connection with Fig. 77 that simple ring windings necessarily have as many armature circuits in parallel as there are poles; the same thing is true in simple lap windings of the kind illustrated in Fig. 79, as is easily understood when it is considered that the only difference between ring and lap windings is that in the latter the successive turns lie on the surface of the core instead of looping through it. But in wave windings, of the type shown in Figs. 80 and 82 there are only two paths through the armature irrespective of the number of



(a)



(b)

FIG. 84.—Duplex armature windings.

poles. Because of these facts lap windings are often called *parallel* or *multiple* windings, while wave windings are called *series* or *two-circuit* windings.

From what has been said above, it follows that a duplex lap winding has $2p$ armature circuits in parallel, a triplex lap has $3p$ parallel circuits, and an x-plex winding has xp parallel circuits. Similarly, a duplex wave winding has 4 parallel circuits, a triplex wave has 6, and an x-plex wave has $2x$ parallel circuits, independently of the number of poles.

The arrangement illustrated in Fig. 84a shows two independent windings each containing twelve elements, or twenty-four in all. Suppose, now, that one of the twenty-four elements is omitted, and that the remaining twenty-three elements, uniformly spaced, are connected alternately, as in Fig. 84b. Instead of having two independent windings, each closed upon itself, as in Fig. 84a,

there is now but a single closure; but a study of the direction of current flow, indicated by the arrowheads, will reveal the interesting fact that there are still four paths through the armature from brush to brush, just as in Fig. 84*a*. In other words, both drawings of Fig. 84 illustrate duplex windings, but the former is *doubly reëntrant* while the latter is *singly reëntrant*. The meaning of these terms will be clear when it is considered that a closed winding may be thought of reëntering upon itself; thus, if in tracing through a winding, all of the conductors are encountered before coming back to the starting point, there is but one closure or reëntrancy, and the winding is therefore singly reëntrant. But if, on arriving at the starting point after tracing through the given connections, it is found that only half of the total number of conductors have been encountered, it is necessary to begin to trace through the remaining half before a second closure results, in which case there are two separate closures or reëntrancies, and in that case the winding is doubly reëntrant. The *degree of reëntrancy* of a winding is, therefore, numerically equal to the number of independent, separately closed windings on the armature. Thus, it is possible to design windings as triplex, triply reëntrant; triplex, singly reëntrant; quintuplex, singly reëntrant, etc.

It should be understood that all of these conclusions apply with equal force to lap and wave drum windings, the ring type having been used in the above discussion solely for the sake of simplicity.

68. General Considerations.—The first systematic analysis of the relations to be satisfied in order that a symmetrical closed winding might result was the work of Professor E. Arnold of Karlsruhe, who published the result of his studies in 1891. The following derivation of the fundamental formulas is based upon that of Professor Arnold.

Probably the first questions that will present themselves to the student examining diagrams like those of Figs. 79, 80, and 82 are: How does one know in advance the number of coil edges to be stepped over in joining the end of one bundle of wires to the beginning of the next? Thus, in Fig. 82, the order is 1–6–11–16, etc.; would not some other order of connection do equally well? And what would be the effect of changing the total number of coil edges from 32 to some other number? The answer to these

and related questions is implicitly involved in a general equation covering all kinds of closed windings; this equation is derived in a succeeding article.

69. Number of Conductors, Elements, and Commutator Segments.—Without regard to the number of turns per element, ring windings usually have only one active coil edge per element, while drum windings have as a rule two active coil edges per element. Further, in accordance with the definition of an element, there must be as many commutator segments, S, as there are elements. Consequently, in ring windings the number of commutator segments is equal to the number of active coil edges, while in drum windings the number of commutator segments is usually equal to half the number of coil edges. S must of course be an integer, but it may be either even or odd; therefore in ring windings which have one turn per element and in which S is odd, the number of peripheral conductors may also be odd; but the number of elements in ring windings is usually made even, and more particularly in simplex windings a multiple of the number of poles in order that each branch path of the armature may be at all times identical with all of the others, in which case the number of conductors will be even. In drum windings, no matter whether S is even or odd, and irrespective of the number of turns per element, the number of conductors must be even.

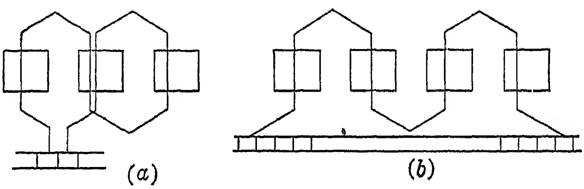


FIG. 85.—Elements having four active coil edges.

Since the number of active conductors, Z, must be a simple multiple of the number of commutator segments, S, it follows that the study of the arrangement of conductors may be reduced to one involving the order of connections of the elements to the commutator, so that the quantity S is the factor of importance.

In certain drum windings it is desirable to reduce the number of commutator segments to a value smaller than that which corre-

sponds to one segment for each pair of active coil sides. This can be accomplished in the manner indicated in Fig. 85, where each element has four active edges.

70. Winding Pitch, Commutator Pitch and Slot Pitch.—In Fig. 79 it will be observed that the back, or pulley, end of coil edge No. 1 is connected to the corresponding end of coil edge No. 12, and the front or commutator end of No. 12 is connected to the front end of No. 3. The number of coil edges passed over in this way is called the *winding pitch;* thus in Fig. 79, the *back pitch,* which will be designated by y_1, is $+ 11$, while the *front pitch,* or y_2, is $- 9$. In Fig. 80 both front and back pitches are positive and equal to 11.

Again, in Fig. 79, the beginning and end of each element are connected to adjacent commutator segments, whose numbers differ by unity. Similarly, in Fig. 80, the terminals of the elements are connected to segments which differ numerically by 11. This numerical difference between the terminal segments of an element is called the *commutator pitch,* y.

In slotted armatures the number of slots spanned by a coil or element is called the *slot pitch.*

Lap windings are right-handed or left-handed, respectively, depending upon whether y_1 is numerically greater or less than y_2. In other words, if one faces the armature at the commutator end, the winding is right-handed if it progresses clockwise from segment to segment of the commutator in tracing through the circuit. On the other hand, wave windings are right- or left-handed according to whether one arrives at a segment to the right or left, respectively, of the starting point after tracing through $p/2$ elements, where p is the number of poles. Thus, in Fig. 82, the winding is left-handed.

The algebraic sum of the front and back pitches is a measure of the total advance or retreat per element in tracing through the winding. Thus, in the case of the simplex lap winding of Fig. 79, the back pitch is 11 coil edges, while the front pitch is $- 9$ coil edges, representing a net advance of 2 coil edges per element. At the same time the ends of the element are separated by 1 segment, hence the net advance in terms of commutator segments is only half as great as the advance in terms of coil edges; obviously this is due to the fact that the number of commutator segments

is only half as great as the number of coil edges. In general, in simplex *lap windings*,

$$\Sigma y = y_1 - y_2 = 2y \tag{1}$$

In the case of the wave winding of Fig. 80, the front and back pitches are both equal to 11, so that the net advance per element is 22 coil edges; but the advance per element, in terms of commutator segments, is only half as great, or 11, since here again there are only half as many segments as coil edges; hence in simplex *wave windings*,

$$\Sigma y = y_1 + y_2 = 2y \tag{2}$$

In windings where there are more than two coil edges per element, as in Fig. 85, say n coil edges per element,

$$\Sigma y = ny$$

or

$$y = \frac{\Sigma y}{n} = \frac{y_1 + y_2 + y_3 + \cdot \cdot \cdot + y_n}{n} \tag{3}$$

This case rarely or never occurs in practice, nearly all windings being designed with only two coil sides per element.

71. Field Displacement.—Reference to Figs. 79 and 80 will show that the terminals of each element of a winding are connected to commutator segments which do not occupy exactly corresponding positions with respect to the axes of the pole pieces. There is a *field displacement*, or creep of the winding, between them which may be expressed in terms of the number of commutator segments, m, by which they fail to occupy homologous positions. Thus, in Fig. 86, which represents a portion of the lap winding of Fig. 79, the field displacement between the ends of an element is one segment, whence $m = 1 = y$; in the ring windings of Fig. 84, $m = 2 = y$.

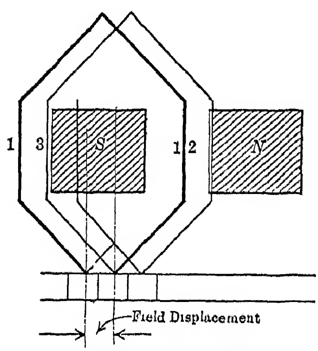


Fig. 86.—Field displacement in lap winding.

In wave windings there is a somewhat similar state of affairs. Thus, Fig. 87 represents a portion of the wave winding of Fig. 80, from which it is clear that while the terminals of an element are

separated by an interval approximately equal to a double pole pitch, so that the ends of an element are very nearly similarly placed with respect to poles of the same sign, the actual interval differs from the double pole pitch by an amount which is again a measure of the field displacement, or creep of the winding. It is clear that if this creep did not exist in the case of a wave winding, the winding would close upon itself after traversing a number of coil edges equal only to the number of poles.

In both Figs. 86 and 87 the field displacement is positive in sign, that is to say, the winding creeps ahead in a right-handed direction. In Fig. 86, if the front and back pitches were $+11$ and -13, respectively, instead of $+11$ and -9, the field displacement would be $m = -1$, and the winding would retrogress in the left-hand direction. In the same way, it is possible to connect up the 42 coil edges of Fig. 80 so as to form a wave winding by using a front pitch of $+11$ and a back pitch of $+9$, in which case also m would be negative, and the winding would be left-handed.

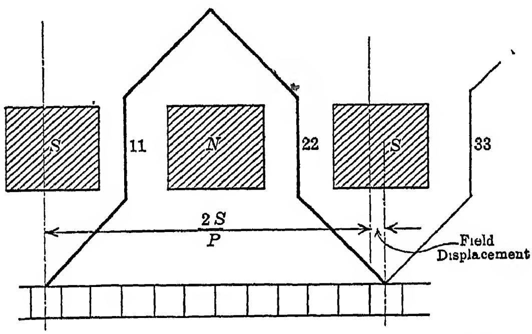


FIG. 87.—Field displacement in wave winding.

In wave windings in general, referring to Fig. 87, it will be seen that the commutator pitch is equal to the double pole pitch (expressed in terms of commutator segments) plus or minus the field displacement (also expressed in commutator segments). Since the number of commutator segments in a double pole pitch is $\frac{2S}{p}$, it follows that

$$y = \frac{2S}{p} \pm m \qquad (4)$$

In lap windings, as Fig. 86 plainly shows,

$$y = \pm m \tag{5}$$

so that in general

$$y = \frac{fS}{p} \pm m \tag{6}$$

where $f = 2$ for ordinary wave windings, and $f = 0$ for lap windings. The quantity f may be called the *field step* of the elements, the term field step meaning the nearest whole number of pole pitches between the ends of an element; thus, in wave windings the ends of an element are separated by nearly two pole pitches, so that $f = 2$; whereas in lap windings the two ends of an element are under the influence of the same pole, so that $f = 0$.

72. Number of Armature Paths.—In the simple ring winding of Fig. 77, where the ends of each element are separated by one commutator segment, so that $m = 1$, it is easy to see that there are as many circuits or paths through the winding as there are poles. But in Fig. 84, both parts of which are ring windings for a bipolar machine, the field displacement is two segments ($m = 2$), and, as has already been pointed out, the number of armature paths is four, or twice as many as there are poles. It may therefore be inferred that there is a definite relation between the field displacement m and the number of armature circuits.

Again referring to Fig. 77, it is easy to see that if we start with an element like number 1, in contact with a *negative* brush, and follow in order through elements 2, 3, 4, etc., the successive field displacements (in this case each equal to 1 segment) become cumulative and ultimately amount in the aggregate to $\frac{S}{p}$ segments; and that then the advance has brought us to the next *positive* brush and one complete armature path or circuit will have been passed over.

Precisely the same remarks apply to the lap winding of Fig. 79 and the wave winding of Fig. 80. There is no difficulty in appreciating the truth of this statement in the case of the lap winding; and such difficulty as may exist in the case of the wave winding is easily resolved by actually tracing through such a drawing as Fig. 80 or Fig. 82, noting carefully the commutator segments as they are encountered in the passage from a brush of one polarity to a brush of the opposite polarity.

In general, therefore, if we trace through a winding beginning, say, at a commutator segment in contact with a negative brush, there will have been a field displacement of m segments by the time the next segment, in order, has been reached; advancing through the second element to another segment, the total field displacement will amount to $2m$, and so on until the total field displacement amounts to $\frac{S}{p}$ segments, when a complete path will have been traced out. But in this process there will have been encountered a total of, say, S' segments, and since the field displacement per element is m segments, the total displacement in tracing through one complete path will be mS' segments; hence

$$mS' = \frac{S}{p}$$

or

$$\frac{S}{S'} = mp \tag{7}$$

Since S' segments are encountered per path, the total number of paths must be

$$\frac{S}{S'} = a$$

which is necessarily an integral number, hence

$$mp = a$$

or

$$m = \frac{a}{p} \tag{8}$$

Thus, in ordinary ring or lap windings, where the number of paths equals the number of poles ($a = p$), the field displacement is $m = 1$; in duplex ring or lap windings, which have twice as many paths as poles, $m = \frac{a}{p} = 2$; or, in general, m equals the degree of multiplicity of lap or ring windings. In wave windings, on the other hand, m is generally fractional; thus in a simplex wave winding, where a is always 2, $m = \frac{1}{2}$ in four-pole machines (see Fig. 80), $m = \frac{1}{3}$ in six-pole machines (see Fig. 82), $m = \frac{1}{4}$ in eight-pole machines, etc.; in duplex wave windings, where $a = 4$, it follows that $m = \frac{4}{p}$, and in triplex wave windings $m = \frac{6}{p}$.

73. General Rules.—In the case of ordinary lap windings it has now been shown that

$$y = \frac{y_1 - y_2}{2} = \pm m = \pm \frac{a}{p} \tag{9}$$

while in the case of wave windings it is

$$y = \frac{y_1 + y_2}{2} = \frac{2S}{p} \pm m = \frac{2S \pm a}{p} \tag{10}$$

From these equations there may be deduced certain convenient rules for determining the order of connections of the coil edges, thereby fixing the design of the winding elements.

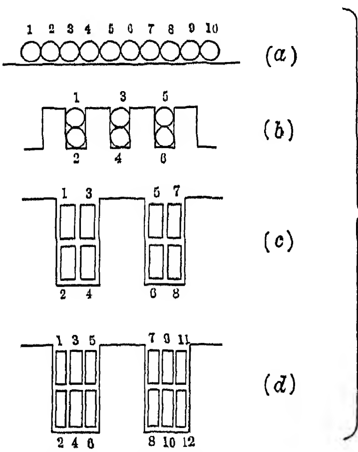


FIG. 88.—Standard numbering of coil edges.

It has been pointed out in a previous section that the number of coil edges (2S) of a drum winding is necessarily even. If, then, the coil sides are numbered, half of them will bear even numbers and the other half odd numbers. Since each coil side proceeding outwardly from a commutator segment must have a return path through another coil edge, the numbering may be so arranged that the even numbers will constitute the outgoing group while the odd numbers will comprise all of the return group. This means that even-numbered coil edges will be connected to odd-numbered coil edges at both ends, therefore, *front and back pitches must be odd.* This is a general rule for all drum windings provided the numbering is carried out in accordance with the system indicated in Fig. 88.

1. *Lap or Parallel Windings.*—An examination of the formula $y = \pm \frac{a}{p}$ shows that there are no restrictions upon the number of elements, which may, accordingly, be even or odd. In the great majority of commercial windings there are only two coil sides per element ($n = 2$), so that

$$y_1 - y_2 = 2y = \pm 2\frac{a}{p} = \pm 2m$$

from which it follows that *the pitches must differ by twice the degree of multiplicity* in addition to their being odd. There remains the further condition that both *y_1 and y_2 must not differ too greatly from the pole pitch,* $\frac{2S}{p}$, as otherwise the e.m.fs. of the connected sides will not be effectively additive. It is not essential that the average pitch approximate $\frac{2S}{p}$ so far as mere closure is concerned, and in certain so-called *chord* windings or *fractional pitch* windings the average pitch is purposely made smaller than this value.

As an example of these rules, it may be observed that in Fig. 79

$$Z = 42 \qquad S = 21 \qquad p = 4 \qquad a = 4$$

$$y = m = \frac{a}{p} = 1 \qquad y_1 - y_2 = 2y = 2 \qquad y_1 = 11 \qquad y_2 = 9$$

Had the pitches been made 9 and 7, respectively, or 7 and 5, the winding would close, but it would be an exaggerated form of chord winding.

Since $m = \frac{a}{p} = y$, it follows that in an *m-plex lap winding the commutator pitch equals the degree of multiplicity.* Thus, in a simplex lap winding the ends of an element are connected to adjacent segments; in a duplex winding they are separated by one segment, etc.

2. *Wave or Series Windings.*—The general formula

$$y = \frac{fS \pm a}{p}$$

reduces to $y = \frac{y_1 + y_2}{2} = \frac{2S \pm a}{p}$ for most commercial windings of this type. It is clear that the choice of S, and therefore of the number of active conductors, is not unlimited as in lap windings. In Fig. 82, for instance, which represents a simplex wave winding for a 6-pole machine, $a = 2$, $p = 6$, $2S = 32$, hence $y = \frac{32 \pm 2}{6} = 5$ or $5\frac{2}{3}$. The latter value of y being impossible, we must take $y = 5$. Since the pitches must approximate $\frac{2S}{p} = 5\frac{1}{3}$,

select $y_1 = y_2 = 5$, though values of 7 and 3 would result in a closed chord winding.

The restriction upon the number of elements in wave windings frequently causes the use of "dummy coils." Suppose, for example, it is necessary to design a simplex 4-pole wave winding to be placed on an armature core having 65 slots, each slot being of sufficient size to accommodate four coil sides, in the manner of Fig. 88c. This means that $Z = 260$, assuming each coil edge to consist of a single conductor, and of course this value of Z must accord with the fundamental equation (7), Chap. II. Summarizing, $2S = 260$, $a = 2$, $p = 4$; whence

$$y = \frac{260 \pm 2}{4} = 64\tfrac{1}{2} \text{ or } 65\tfrac{1}{2}$$

But since y must be an integer, the value of $2S$ nearest to 260 that will satisfy the equation is 258 ($2S = 262$ is impracticable be-

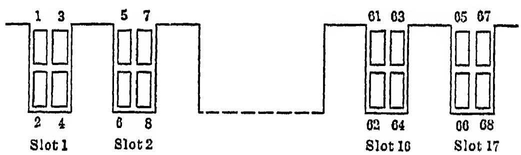


FIG. 89.—Numbering of coil edges.

cause the maximum number of coil edges that can be placed in the slots is 260). Taking $2S = 258$, it follows that there must be one element, consisting of two conductors, that is not a part of the winding; it is put in simply to fill up the space in the two slots which contain only three active conductors each. Therefore, $y = \dfrac{258 \pm 2}{4} = 64$ or 65. Since y_1 and y_2 *must be odd*, and further $\dfrac{y_1 + y_2}{2} = y$, the following pairs of pitch values are possible:

$$\begin{cases} y_1 = 65 \\ y_2 = 65 \end{cases} \quad \begin{cases} y_1 = 63 \\ y_2 = 67 \end{cases} \quad \begin{cases} y_1 = 67 \\ y_2 = 63 \end{cases} \quad \begin{cases} y_1 = 65 \\ y_2 = 63 \end{cases} \quad \begin{cases} y_1 = 63 \\ y_2 = 65 \text{ etc.} \end{cases}$$

Practical considerations in this particular case dictate the use of $y_1 = 65$. For if the coil edges are numbered in accordance with Fig. 88, it will be seen from Fig. 89 that the back ends of conductors 1 and 3 may then be joined to conductors 66 and 68,

respectively, thereby allowing the conductors to be insulated together in pairs and facilitating the placing of the bundles in the slots.

In wave windings the field displacement is given by $m = \frac{a}{p}$, so that after tracing through $p/2$ elements, corresponding to one circuit of the periphery, the total displacement is $\frac{p}{2} \times \frac{a}{p} = \frac{a}{2}$ commutator segments. Therefore, in simplex windings ($a = 2$) the end of the $p/2$th element connects to a segment adjacent to the starting segment; in duplex windings it connects to the next but one, etc.

3. *Series-parallel Windings.*—The ordinary wave winding results in but two paths ($a = 2$) through the armature irrespective of the number of poles. But it is possible to secure any multiple of this number of paths by a suitable choice of S in the general formula. Wave windings having more than two paths are called *series-parallel* windings. Thus, if a 6-pole armature has 154 conductors wound to form 77 elements, it may be arranged as a 4-circuit (duplex) wave winding; substituting $f = 2$, $S = 77$, $a = 4$, and $p = 6$ in the equation $y = \frac{fS \pm a}{p}$, there results $y = 25$.

74. General Rule for the Degree of Reëntrancy.—If, in the general formula,

$$y = \frac{fS \pm a}{p}$$

the two sides of the equation have a common factor q, we have

$$\frac{y}{q} = \frac{f\frac{S}{q} \pm \frac{a}{q}}{p}, \quad \text{or } y' = \frac{fS' \pm a'}{p} \tag{12}$$

which means that the original winding is really made up of q independent windings, each of which has $S' = \frac{S}{q}$ elements and a commutator pitch of y', the latter counted with respect to the S' segments. That is, the winding will be multiplex and multiply reëntrant of the qth degree in the event that y and S have a common factor q; it will be singly reëntrant if y and S are prime to each other.

In ordinary duplex wave windings ($f = 2$)

$$y = \frac{2S \pm 4}{p} = \frac{2(S \pm 2)}{p}$$

from which it follows that if y is even, that is, contains 2 as a factor, S must also be even because $\frac{S \pm 2}{p}$ must be an integer and p is always an even number. This leads to the simple rule that *a duplex wave winding is doubly reëntrant if y is even;* and to the corollary that it is singly reëntrant if y is odd.

In triplex wave windings in which $f = 2$,

$$y = \frac{2S \pm 6}{p} = \frac{2(S \pm 3)}{p} \tag{13}$$

Suppose now that y contains 3 as a factor, in which case $y = 3x$, where x is an integer; then from (13)

$$3x\frac{p}{2} = S \pm 3$$

$$\frac{p}{2}x = \frac{S}{3} \pm 1$$

Therefore, since $\frac{p}{2} \cdot x$ is integral, S must be a multiple of 3, hence the winding is triply reëntrant. Hence a *triplex wave winding will be triply reëntrant if y is a multiple of* 3, and it will be singly reëntrant if y is not a multiple of 3.

In the case of quadruplex wave windings, however, such simplifications of the general rule are not possible. Such windings may be singly, doubly, or quadruply reëntrant. Thus, if $f = 2$, $a = 8$, and $p = 6$, $S = 79$ leads to a singly reëntrant winding in which $y = 25$; $S = 82$ results in a doubly reëntrant winding, $y = 26$; and $S = 80$ gives quadruple reëntrancy, $y = 28$.

75. Recapitulation of Winding Rules.—The conditions that must be satisfied by the various windings discussed above may be summarized as follows:

A. Lap Windings.—1. The number of elements, S, may be any number, even or odd (though preferably a multiple of the number of poles), consistent with the condition that it must satisfy the relation

$$S = \frac{Z}{2 \times \text{number of turns per element}}$$

where the number of turns per element is commonly one, though it may be two or more depending upon commutation requirements; and the number of peripheral conductors, Z, must satisfy the equation

$$E = \frac{p}{a}\frac{\Phi Z n}{60 \times 10^8}$$

2. The winding pitches, y_1 and y_2, expressed in terms of the number of coil edges spanned, must be approximately equal to $\frac{2S}{p}$ and must both be odd.

3. The numerical difference between the winding pitches must be 2 in simplex windings, 4 in duplex windings, 6 in triplex windings. In general, the difference between the pitches must be twice the degree of multiplicity.

4. The commutator pitch, expressed in terms of the number of commutator segments between the ends of an element, must be 1 in simplex windings, 2 in duplex windings, 3 in triplex windings. In general it must be equal to the degree of multiplicity.

5. The number of armature circuits in parallel is equal to the number of poles times the degree of multiplicity.

6. The degree of reëntrancy is necessarily single in simplex windings; in duplex windings it will be double if S is even, single if S is odd; in triplex windings it will be triple if S is a multiple of 3, single if S is not a multiple of 3.

B. *Wave Windings.* 1. The number of elements, S, must of course satisfy the condition that

$$S = \frac{Z}{2 \times \text{number of turns per element}}$$

and where Z in turn satisfies the equation

$$E = \frac{p}{a}\frac{\Phi Z n}{60 \times 10^8}$$

and, in addition, S must satisfy the relation

$$y = \frac{2S \pm a}{p}$$

where y must be an integer.

2. The winding pitches y_1 and y_2 must be approximately equal

to $\frac{2S}{p}$, they must both be odd, and their average must be equal to the commutator pitch, y.

3. The number of armature circuits, a, to be substituted in the formula

$$y = \frac{2S \pm a}{p}$$

must be twice the desired degree of multiplicity.

4. The degree of reëntrancy is necessarily single in simplex windings; in duplex windings it will be double if y is even, single if y is odd; in triplex windings it will be triple if y is a multiple of 3, single if y is not a multiple of 3.

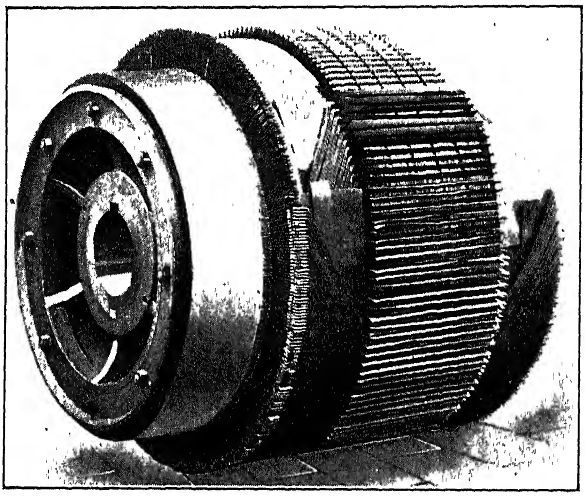


FIG. 90.—Partially wound drum armature (lap winding).

76. Two-layer Windings. Examples of Drum Windings.—An inspection of any of the windings of Figs. 79, 80, 82, etc., will show that the end-connections of successive conductors proceed alternately in opposite directions. If all of the conductors lay in the same cylindrical surface, as in the case of smooth core armatures, the crossing of the end-connections would make the actual winding process difficult of execution. But where slotted armatures are used, if the conductors are arranged in two layers, the end-connections of the upper layer may all proceed in one

direction while the end-connections of the lower layer, at the same end of the armature, may all proceed in the opposite direction, as in Fig. 90. Since the upper and lower layers include all the odd and even numbered coil sides, respectively, conductors in the top layer must connect to others in the lower layer, the transi-

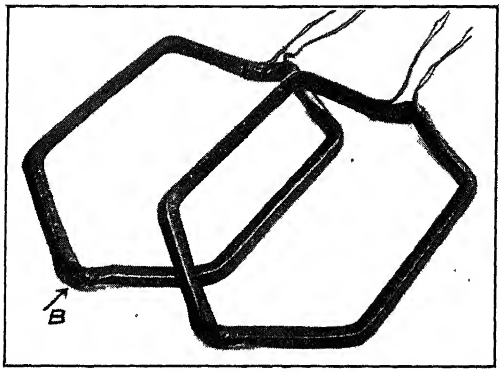


Fig. 91.—Samples of winding elements.

tion being effected by the peculiar bend in the coil shown at *B*, Fig. 91.

It is easy to recognize an armature as lap or wave wound, when the conductors are made of bars or strips of copper, by observing the relative directions of the top end-connections at the

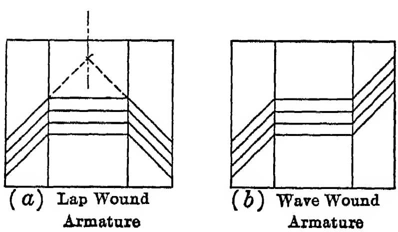


Fig. 92.—Showing direction of end connections in lap and wave windings.

two ends of the armature. Thus, if the top end-connections, when produced, meet at or near the center of the core, as in Fig. 92*a*, the winding is a lap winding; whereas if the top end-connections are parallel, as in Fig. 92*b*, the winding is a wave winding.

Figs. 93 to 96, inclusive, show clearly how a slight change in

the number of coil edges will change the winding from single reëntrancy to multiple reëntrancy. Thus, in Figs. 93 and 94, although both windings (duplex lap) have identical pitches, the former, with 62 coil edges, is singly reëntrant, while the latter, with 64 coil edges, is doubly reëntrant. Electrically, however, these windings have identical properties, except for the slight difference in e.m.f. due to the different number of active conduc-

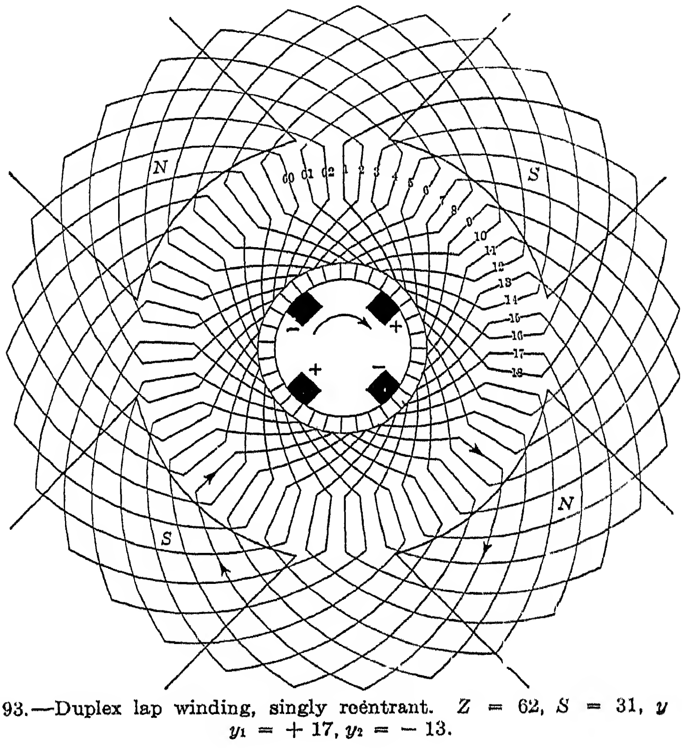


FIG. 93.—Duplex lap winding, singly reëntrant. $Z = 62$, $S = 31$, $y = 2$, $y_1 = +17$, $y_2 = -13$.

tors. Similar remarks apply to Figs. 95 and 96, which show singly- and doubly-reëntrant duplex wave windings. Note that in Fig. 95 the pitches (y, y_1 and y_2) equal 17, but that pitches of 15 would also work out correctly; and that in Fig. 96 it would also be possible to design the winding with pitches of $y = 14$, $y_1 = 15, y_2 = 13$.

77. Equipotential Connections.—Consider a parallel-wound armature like Fig. 97, which represents diagrammatically a

winding for an 8-pole machine. The eight parallel paths through the armature from terminal to terminal are shown somewhat more clearly in their relations to one another in the diagram of Fig. 98. It will at once appear that if each path is to carry its proportionate share of the total armature current, each path must at all times generate the same e.m.f. and have the same resistance as all the other paths. In any case, the current will divide between the eight paths in accordance with Kirchhoff's

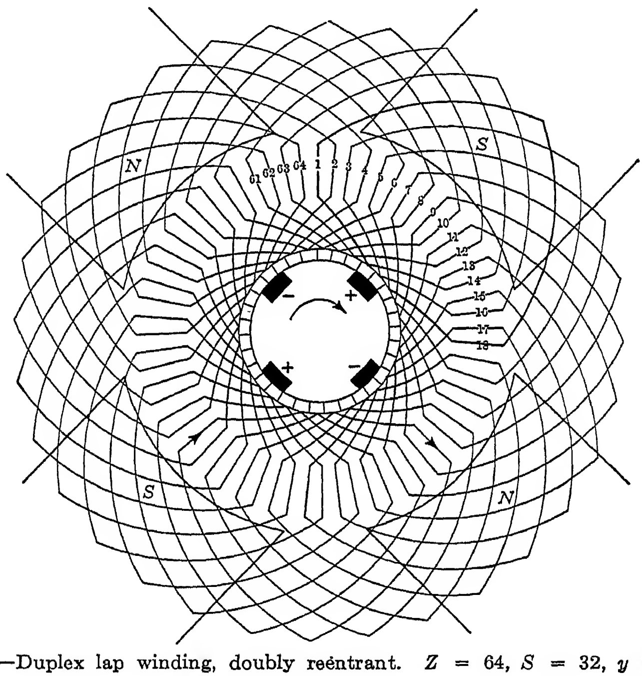


FIG. 94.—Duplex lap winding, doubly reëntrant. $Z = 64$, $S = 32$, $y = 2$, $y_1 = +17$, $y_2 = -13$.

laws for divided circuits, namely: (1) the summation of all the potential differences in each closed circuit must be zero; (2) the sum of all currents meeting at a point must be zero. If for any reason the e.m.f. generated in one path is greater than in another, for instance, if that of circuit 3–2′ is greater than that of 3–3′, the brushes 2′ and 3′ will not have the same potential and an equalizing current will flow in the lead joining brushes 2′ and 3′. Even very small differences of potential may give rise to internal

equalizing currents of large magnitude, owing to the low resistance of the circuits, so that excessive heating of the winding and sparking at the brushes may result if preventive measures are not employed.

The causes of possible unequal e.m.fs. in the various paths are as follows:

1. The armature may not be exactly centered with respect to the pole shoes, due to natural irregularities in construction or to

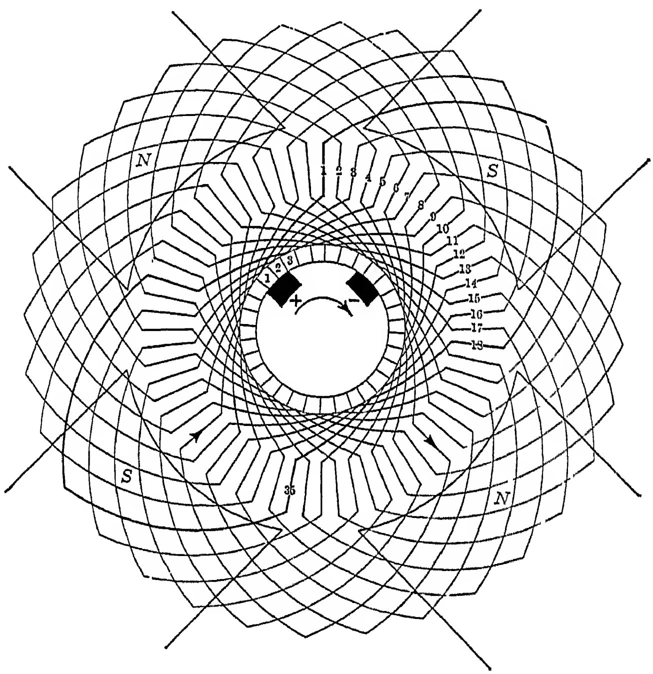


Fig. 95.—Duplex wave winding, singly reëntrant. $Z = 64$, $S = 32$, $y = y_1 = y_2 = 17$.

wear of the bearings. The air-gap is consequently not uniform, and some of the poles therefore carry more flux than others, thereby generating more e.m.f. in the coils subject to their influence than is generated in coils under the weaker poles. This cause is of importance in lap and ring windings, where each armature circuit is at any one time under the influence of one pole only; in wave windings each path is simultaneously acted

upon by all of the poles, hence this type of winding is free from the disturbing effect of non-uniform polar flux.

2. The poles may not all be identical in construction, so that their fluxes may differ even if the air-gap is uniform. Thus, the joints between the poles and the yoke, or between the pole cores and the shoes, may not all be equally good, or the magnetizing effect of the field windings may differ, especially in cases where the field coils are connected in parallel instead of in series.

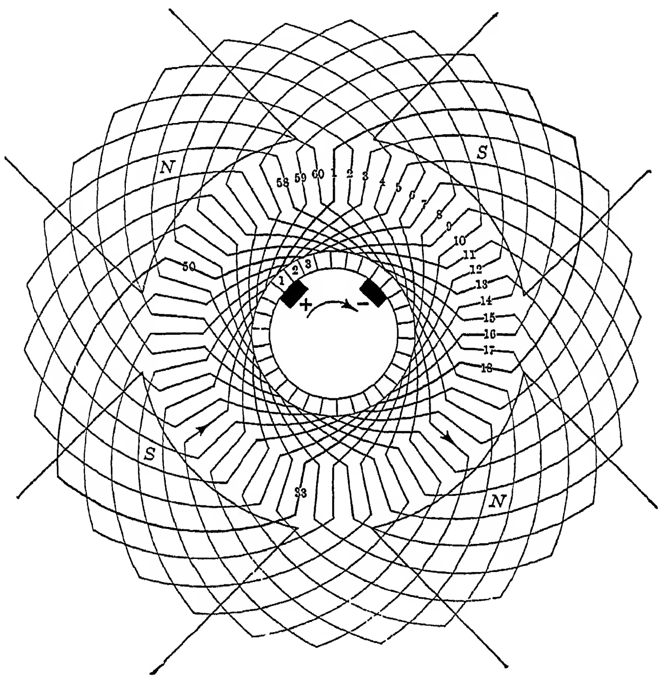


FIG. 96.—Duplex wave winding, doubly reëntrant. $Z = 60$, $S = 30$, $y = 16$, $y_1 = 17$, $y_2 = 15$.

3. The armature circuits may be unsymmetrical, due to a choice of number of elements which is not an exact multiple of the number of paths. In this connection it should be observed that in multiplex windings there is always dissymmetry between the circuits, due to the fact that the brushes short-circuit an unequal number of elements of the component windings (Fig. 99). This kind of asymmetry will give rise to equalizing currents even

if the individually generated e.m.fs. are equal, because the circuits have slightly different resistances.

The equalizing currents are a source of loss because of the extra heating caused by them. This effect can be minimized by

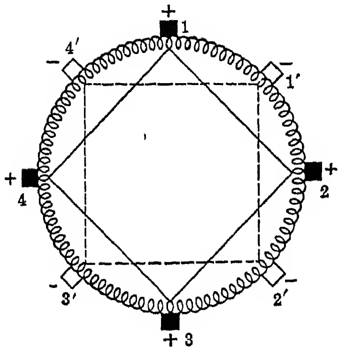


FIG. 97.—Parallel-wound armature with equalizer connections.

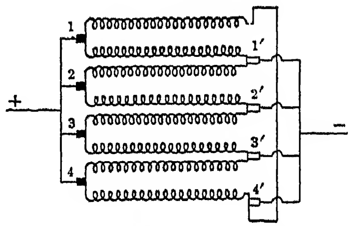


FIG. 98.—Diagrammatic scheme of connections of armature of Fig. 95.

striving for the greatest possible degree of magnetic and electrical symmetry. To obviate the remaining difficulty of sparking at the commutator, resort is had to the use of *equipotential connections,* which are low-resistance conductors joining points in the

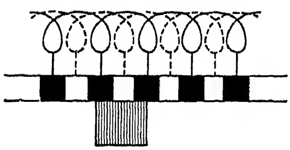


FIG. 99.—Short-circuiting of elements of multiplex winding.

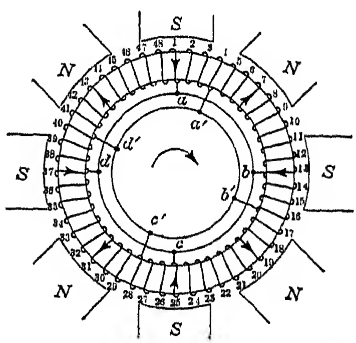


FIG. 100.—Equalizer connections in parallel winding.

winding which, under ideal conditions, would at all times have the same potential. Thus, in Fig. 100, the points a, b, c, d, and a', b', c', d', etc., are always under the influence of corresponding parts of poles of like sign. Should irregularities exist, equalizing current

will flow through these connections, relieving the brushes of the extra current and preventing sparking. The equalizing current is of course an alternating one. The equipotential connections, occasionally referred to as the equalizing rings, are sometimes placed between the commutator and the armature core under the end connections; in large drum armatures they are generally mounted on the exposed side of the core as in Fig. 101.

When the equipotential connections were first introduced by Mordey they were intended to reduce the usual number of brush sets. Thus, it is clear from Fig. 100 that two brushes might be

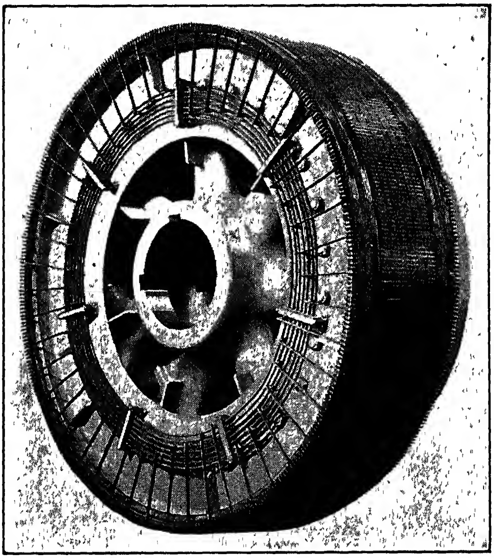


FIG. 101.—Equalizing rings of large lap-wound armature.

expected to take care of the entire current, since the commutator segments that would be touched by three of the brushes of one polarity are already connected to the fourth brush by the equalizing connections. The difficulty arising from the use of only two brushes is that all of the armature circuits are not identically situated with respect to the line terminals, the extra resistance of the equalizing connections between the remote armature paths and line being sufficient to introduce an unbalancing of the circuits. In present practice the number of brush sets is not reduced when equalizing connections are used.

PROBLEMS

1. Draw the development of a simplex lap winding for a 4-pole drum armature having 96 coil edges. Use pitches of +25 and −23. Show positions of brushes and indicate direction of current flow in each element. Prepare a winding table corresponding to the drawing.

2. Draw the development of a simplex wave winding for a 4-pole drum armature having 94 coil edges. Use (*a*) pitches of +23 and +23; (*b*) pitches of +25 and +23. Show positions of brushes and indicate direction of current flow in each element. Prepare a winding table corresponding to the drawing.

3. A drum armature has space for 450 coil edges. Find all possible lap and wave windings, suitable for a 6-pole field structure, up to and including quadruplex windings. Each element is to consist of one turn. State front, back and commutator pitches and degree of reëntrancy in each case.

4. The armature core of a 4-pole, 550-volt railway motor has 59 slots and is provided with a two-circuit winding. If the flux per pole is approximately 2×10^6 and the speed is 1200 r.p.m., how many conductors are necessary and how should they be arranged, if the number of commutator segments is 117?

5. A 4-pole generator has a rating of 15 kw. at 125 volts and 1170 r.p.m. The diameter of the armature is 12 in., the length of the armature core is 5 in., and the pole arc is 70 per cent. of the pole pitch. The armature has 47 slots, each containing 4 conductors, and there are 47 commutator segments. Knowing that the flux density under the pole faces is in the neighborhood of 50,000 lines per sq. in., specify the type of winding and compute the winding and commutator pitches.

6. The average length of an end-connection of the winding of Problem 5 may be taken as 1.5 times the pole pitch. If the winding is made of No. 5 B. and S. wire, what is the resistance of the armature at 90° C.?

7. A 220-volt motor having a simplex lap winding has a rated speed of 900 r.p.m. What changes are necessary to adapt it for operation at

(*a*) 220 volts and 1800 r.p.m.?

(*b*) 110 volts and 1800 r.p.m.?

(*c*) 110 volts and 900 r.p.m.?

CHAPTER IV

THE MAGNETIZATION CURVE. MAGNETIC LEAKAGE

78. The Magnetization Curve.—Every dynamo consists of an electrical circuit interlinked with a magnetic circuit. The armature winding is the electrical circuit in which the e.m.f. is produced in the case of a generator and in which the working current produces the torque in the case of a motor. In either case, the activity of the armature is dependent upon the magnitude of the magnetic flux, and the latter, in turn, is dependent upon the magnetizing effect of the field winding and the reluctance of the magnetic circuit in accordance with the relation

$$\text{Flux} = \Phi = \frac{\text{m.m.f.}}{\text{reluctance}}$$

Since

$$E = \frac{p}{a}\,\frac{\Phi Z n}{60 \times 10^8}$$

it follows that

$$E = \frac{p}{a}\,\frac{Zn}{60 \times 10^8} \cdot \frac{\text{m.m.f.}}{\text{reluctance}}$$

which means that E is a function of the field excitation; the graph of this function is called the *magnetization curve*, or the saturation curve, or the no-load characteristic. If the magnetic circuit had constant reluctance, the no-load characteristic would be a straight line through the origin, but since the permeability of the iron of the magnetic circuit falls off as the flux increases, the flux does not bear a constant ratio to the m.m.f.; the result is that the no-load characteristic droops below the straight line form, as indicated in Fig. 102.

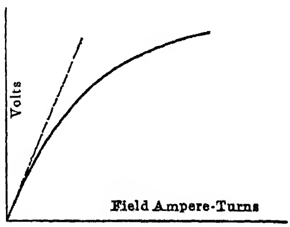


FIG. 102.—Magnetization curve of a dynamo.

The dashed lines in Figs. 103 and 104 represent the mean paths of the flux in typical forms of bipolar and multipolar ma-

chines. These lines are so drawn that they pass through the centers of gravity of the sections of the tubes of induction. It will be observed that a complete path or magnetic circuit, such as C', Fig. 104, includes the armature core, two sets of teeth, two air-gaps, two pole shoes, two pole cores, and the connecting yoke. A magnetizing winding P on one pole will set up the same flux in each of the paths C and C' (assuming perfect symmetry of construction) since these paths are in parallel. A similar winding on every *alternate* pole would then magnetize all the poles equally, hence the excitation required to drive the flux through a complete magnetic circuit is the excitation *per pair of poles.*

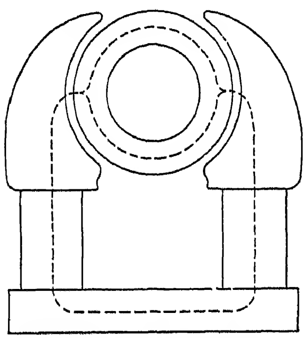


Fig. 103.—Magnetic circuit of bipolar machine.

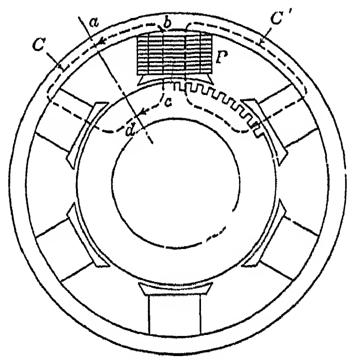


Fig. 104.—Magnetic circuits of a multipolar machine.

The excitation per pole is, therefore, that required to maintain the flux in a magnetic circuit such as *abcd*, Fig. 104, consisting of a single air-gap, one set of teeth, one pole shoe and core, and half of the connecting circuit through the armature core and the yoke. Field excitation is generally expressed in terms of the number of ampere-turns per pole; or in terms of ampere-turns per pair of poles.

The magnetization curve is of great importance. Whether the machine is to be used as a generator or as a motor, the form of the magnetization curve will largely determine its operating characteristics. Conversely, a given set of specifications will in large measure fix the form of the magnetization curve. It is, therefore, apparent that the determination of this curve is of fundamental importance. In the case of a completed machine the magnetiza-

tion curve can be determined experimentally; it can also be calculated when the dimensions of the machine and the nature of the materials used in its construction are known.

79. Experimental Determination of Magnetization Curve.—Since

$$E = \frac{p}{a}\,\frac{Zn}{60 \times 10^8} \cdot \frac{\text{m.m.f.}}{\text{reluctance}} = kn \times \text{function of field ampere-turns} \qquad (1)$$

it is clear that it is only necessary to run the machine at a constant speed n (driving it with a motor or other suitable prime mover) and to observe a series of simultaneous pairs of values of E and ampere-turns. In the above equation (1) E is the e.m.f. generated in the armature by rotation through the flux produced by the field current; therefore, to measure E directly, the armature must be without current, that is, it must be on open circuit. The machine must then be separately excited during this test, as in Fig. 105, current being supplied to the field winding from a suitable external source, controlled by a variable resistance R and measured by an ammeter A. The procedure then consists of varying the current A by means of R, and taking a reading of voltmeter V for each setting of A, the speed being kept constant throughout.

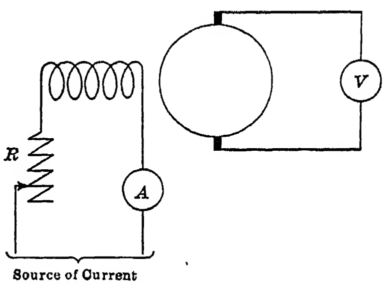


FIG. 105.—Experimental determination of magnetization curve.

Inspection of equation (1) indicates that with a fixed value of field excitation the generated e.m.f. E would be directly proportional to the speed. This is, however, not quite true; for it is possible that current may flow in those elements of the armature winding which are short-circuited by the brushes, and these short-circuit currents may easily reach values of sufficient magnitude to react upon the flux and so affect the generated e.m.f. To reduce the disturbing effect of these short-circuit currents to a minimum, it is necessary to cut down the e.m.f. which gives rise to them, and, with a given field excitation, this can be done by reducing the speed. It is best, therefore, to determine the

magnetization curve at a speed considerably below the rated speed, and then to multiply the observed voltage by the ratio of rated speed to the speed actually used. The effect of the current in the short-circuited coils of the armature is discussed in more detail in Chap. VIII.

The form of the magnetization curve obtained experimentally is not the same if the exciting current in the field winding is first gradually increased from zero to a maximum and then gradually reduced from this maximum back again to zero. The observed readings when plotted take the form of Fig. 106.

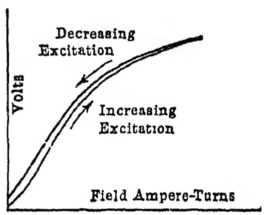


FIG. 106.—Effect of hysteresis on magnetization curve.

The difference between the two curves is due to the hysteresis of the iron part of the magnetic circuit, hysteresis being the name given to that property of iron (or other magnetic substance) by virtue of which the induced magnetism lags behind changes in the magnetizing force.

The magnetization curve of a machine being a function of the B-H curves of the materials of which it is constructed, it is impossible to devise a theoretically correct equation for it in the absence of any known relation between B and H. There is, however, an empirical formula, known as Froelich's equation, which expresses with a fair degree of accuracy the relation between flux (or the corresponding generated e.m.f.) and the excitation, and which is of the form

$$\text{flux} = \frac{\text{constant} \times \text{excitation}}{\text{constant} + \text{excitation}},$$

This equation represents a hyperbola passing through the origin, as in Fig. 102 and which has a horizontal asymptote located at a distance above the origin equal to the constant in the numerator. There is another (vertical) asymptote to the left of the origin at a distance equal to the constant in the denominator. Froelich's equation, and the use that may be made of it, is discussed at greater length in Art. 117, Chap. VI; it may be noted, however, that the equation in the form given

above fails to take the residual magnetism into account, since if the excitation is placed equal to zero the flux likewise reduces to zero value. But the equation may be modified to include the effect of residual magnetism by the simple expedient of transferring the origin to a point sufficiently to the right of the one determined by the equation as given above. Thus, calling the excitation X, and letting a and b represent the two constants, the original equation may be written

$$\Phi = \frac{aX}{b + X}$$

if now the origin is transferred to the right by an amount X_0 the equation becomes

$$\Phi = \frac{a(X'+X_0)}{b + (X' + X_0)}$$

where X' is the excitation measured from the new origin, and this equation may be written

$$\Phi = \frac{aX' + m}{b' + X'}$$

where $m = aX_0 =$ constant, and $b' = b + X_0 =$ constant, so that the revised equation has the form

$$\Phi = \frac{\text{constant} \times \text{excitation} + \text{constant}}{\text{constant} + \text{excitation}}$$

In other words, Froelich's equation may be made to take residual magnetism into account by the introduction of an additional constant term in the numerator of the right-hand expression.

80. Calculation of the Magnetization Curve.—The computation of the coordinates of points on the magnetization curve is based upon the method previously outlined in Art. 26, Chap. I. Thus, assuming a value of the e.m.f. to be generated, and computing the corresponding value of the flux from the relation $\Phi = \frac{a}{p}\frac{E}{Zn} \times 60 \times 10^8$, it is then possible to determine the average flux density in each part of the magnetic circuit since the dimensions, and therefore the cross-sections, of all parts are known. From these values of the flux density it is then possible to find from the B-H curve of the corresponding material the necessary number of

ampere-turns per unit length, and by multiplying the latter by the known length of the portion of the circuit in question, the product will be the number of ampere-turns required for that portion. Adding together all of these partial products, the sum will represent the total number of ampere-turns for the entire circuit.

Repeating this process for other assumed values of generated e.m.f., any desired number of points on the magnetization curve can be determined. Five such points will generally be sufficient to construct the curve, corresponding, say, to quarter, half, three-quarters, and full voltage, and to a value from 10 to 20 per cent. greater than full voltage.

The calculations will be facilitated and systematized by tabulating the results in the manner indicated in the following table, which also serves to show the symbols referred to in subsequent articles where details of the calculations are explained:

Part of circuit	Cross-section	Length of path	Flux density	Amp.-turns per unit length	Total amp.-turns
Armature core.	A_a	l_a	B_a	at_a	AT_a
Teeth, one set.	A_t	l_t	B_t	at_t	AT_t
Air-gap, single.	A_g	δ	B_g	at_g	AT_g
Pole shoe, single.	A_s	l_s	B_s	at_s	AT_s
Pole core, single.	A_c	l_c	B_c	at_c	AT_c
Yoke.	A_y	l_y	B_y	at_y	AT_y

Knowing the number of ampere-turns for each part of the complete magnetic circuit, the total number of ampere-turns per pair of poles will then be

$$AT = AT_a + 2AT_t + 2AT_g + 2AT_s + 2AT_c + AT_y \qquad (2)$$

in accordance with the statement of Art. 78.

Since for any path x

$$H_x = \frac{4\pi}{10}\frac{AT_x}{l_x}$$

(all quantities being expressed in c.g.s. units), the requisite number of ampere-turns for the length l_x is

$$AT_x = \frac{10}{4\pi}H_x l_x = 0.8H_x l_x$$

But

$$\frac{10}{4\pi}H_x = \frac{AT_x}{l_x} = at_x = \text{ampere-turns per cm.}$$

$$\therefore AT = at_a.l_a + 2at_t.l_t + 2at_g.\delta + 2at_s.l_s + 2at_c.l_c + at_y l_y \quad (3)$$

The magnetization (B–H) curves of magnetic materials are usually plotted in terms of B and at (the latter being simply $0.8H$), so that when the values of B for the various parts of the circuit have been determined, the corresponding values of at may be read from the curve and substituted in the expression for AT. Fig. 25 shows magnetization curves for the usual commercial materials, the coordinates being plotted in terms of metric and also of English units. If English units are used, the expression for AT becomes

$$AT = at''_a.l''_a + 2at''_t.l''_t + 2at''_g.\delta'' + 2at''_s.l''_s + 2at''_c.l''_c + at''_y.l''_y \quad (4)$$

81. Magnetic Leakage.—The flux per pole, Φ, may be designated the useful flux, since it is this flux which is involved in the production of the generated e.m.f. But the entire flux produced by the magnetizing action of the field winding does not penetrate the armature, an appreciable part of it "leaking" across from pole to pole, and in general between all points which have between them a difference of magnetic potential. This "leakage flux," φ, increases the total flux from Φ to

$$\Phi_t = \Phi + \varphi$$

The ratio

$$\frac{\Phi_t}{\Phi} = 1 + \frac{\varphi}{\Phi} = \nu \quad (5)$$

is called the leakage coefficient, or preferably, the *coefficient of dispersion*. It is always greater than unity, and in machines of the usual radial multipolar type ranges from about 1.1 to 1.25, the larger values corresponding to small machines. Since the leakage flux must traverse the poles and yokes, the cross-section of these parts must be sufficiently large to carry it as well as the useful flux, hence the necessity of keeping down leakage as much as possible. The conditions to be satisfied to attain this end are, accordingly, minimum reluctance of the main magnetic circuit and maximum reluctance of leakage paths; this means, practically, a

compact magnetic circuit made up of short poles, the interpolar spaces being wide and of small section.

The magnitude of the coefficient of dispersion is not constant for a given machine under all conditions. The leakage flux, φ, being mainly in air, is very nearly proportional to the m.m.f., while Φ is less and less proportional to the m.m.f. as the saturation of the iron is increased. In general, therefore, $\nu = 1 + \frac{\varphi}{\Phi}$ increases more or less with increasing excitation.

Methods for the calculation of the coefficient ν will be given in a subsequent section. For the present it will suffice to state that ν is a function of the dimensions of the machine. This introduces a difficulty in a new design because the flux densities, etc., cannot be determined until the dimensions have been decided upon, and the dimensions are themselves dependent upon Φ_t and, consequently, also upon ν. It is therefore necessary in such a case to assume a value of ν in accordance with previous experience, proceeding then to the calculation of Φ_t and the dimensions. The true value of ν can then be calculated and the tentative computations modified in case the discrepancy is sufficiently large to warrant a readjustment.

82.[1] Details of Calculation of Magnetization Curve.—1. Ampere-turns Required for the Air-gap.—The great permeability of iron as compared with air is responsible for the fact that the reluctance of the air-gap often constitutes from 70 to 90 per cent. of the entire reluctance of the magnetic circuit. The accurate determination of the excitation consumed in the air-gap is, therefore, of predominant importance.

Two different cases arise in practice: (*a*) smooth core armatures. and (*b*) slotted armatures.

(*a*) *Smooth Core Armatures.*—In a machine having p poles, the angle subtended by the pole-pitch is $\frac{2\pi}{p}$. The angle β subtended by the pole shoe is usually between 0.55 and 0.7 of $\frac{2\pi}{p}$; the quantity $\frac{\beta \times 100}{2\pi/p}$ is called the per cent. of polar embrace. If the flux crossed the air-gap along radial lines, the determination of B_g

[1] Arts. 82 to 89, inclusive, may be omitted without impairing continuity of treatment, in case considerations governing design are to be studied separately.

and AT_g would be very simple; actually, however, the flux spreads out beyond the pole tips, and there is a further spreading at the flanks, as illustrated in Figs. 107 and 108. *The flux always distributes itself in such a way that the total reluctance is a minimum.* The spreading of the flux is equivalent to increasing b to b' and l to l'. The mean flux density in the gap is then

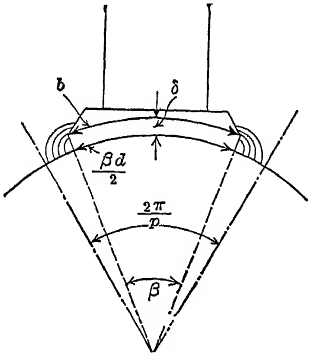


FIG. 107.—Fringing flux at pole tips.

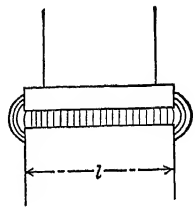


FIG. 108.—Spread of flux at flanks of pole shoes.

$$B_g = \frac{\Phi}{b'l'} \tag{6}$$

and since in air $B = H$, it follows that

$$AT_g = 0.8B_g\delta \tag{7}$$

all dimensions being in centimeters. In inch units

$$AT_g = 0.8\frac{B_g''}{(2.54)^2}(\delta'' \times 2.54) = 0.3133B_g''\delta'' \tag{8}$$

For practical purposes it is sufficiently accurate to take b' as the average of the polar arc b, and of the arc on the armature subtended by the angle β and increased by 2δ on each side; that is,

$$b' = \tfrac{1}{2}\left[b + \left(\frac{\beta d}{2} + 4\delta\right)\right] = \frac{\beta}{2}(d + \delta) + 2\delta \tag{9}$$

Similarly, l' may be taken as

$$l' = l + 2\delta \tag{10}$$

in case the axial lengths of pole shoes and armature core (between heads) are the same. If these lengths are not equal, let them be represented by l_s and l, respectively; then $b'l'$ in the above equation for B_g should be replaced by an area A_g such that

$$A_g = \frac{A'_g + A''_g}{2} \tag{11}$$

where A'_g is the area of the pole shoe and A''_g is the area on the armature core threaded by the flux. Obviously

$$A'_g = bl_s \tag{12}$$

and

$$A''_g = \left(\frac{\beta d}{2} + 4\delta\right)l \tag{13}$$

(*b*) *Slotted Armatures.*—In this case the calculation of B_g is complicated by the fact that the flux tends to tuft at the tips of the teeth, and that more or less of it enters the teeth by way of the slots, as indicated in Fig. 109. It is clear that a given difference of magnetic potential between the pole face and the armature core will produce less flux when slots are present than when the armature surface is smooth, the clearance (δ) being the same in both cases. In other words, the slots increase the gap reluctance, and this effect may be allowed for either by assuming δ to have been increased to a larger value, or by assuming a contraction of the pole arc b to a smaller value, b'.

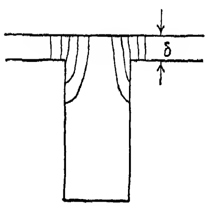


FIG. 109.—Fringing flux at tooth tip.

The problem is further complicated by the fact that the air-gap is frequently not of uniform length over the entire pole face. To improve commutation it is common to chamfer the pole tips, Fig. 110*a*, or to make the cylindrical surfaces of the armature and pole face eccentric, Fig. 110*b*. The effect of the increased gap length at the pole tips is to produce a fringing flux in the inter-

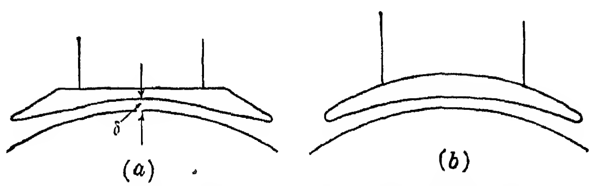


FIG. 110.—Chamfered and eccentric pole shoes.

polar space, as shown by the flux distribution curve of Fig. 111. Ordinates of this curve represent the radial component of flux density at corresponding points on the armature periphery. The ripples at the crest of the curve are caused by the slots and teeth.

Similarly, there is a fringing field at the ends of the core, as shown in Fig. 112, and if ventilating ducts are provided, there will be dips in the curve of axial flux distribution corresponding

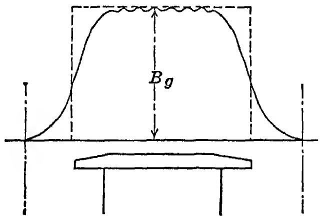


FIG. 111.

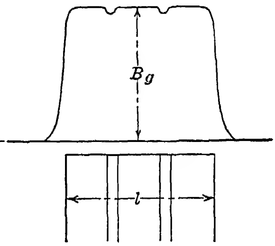


FIG. 112.

FIGS. 111 and 112.—Peripheral and axial distribution of field intensity.

to the depressions opposite the slots, just as in Fig. 111. The extra reluctance due to the ventilating ducts is equivalent to a reduction in the axial length l, and the fringing at the flanks is equivalent to an increase in l, so that the two effects tend to neutralize each other.

83.[1] **Correction to Pole Arc.**—It has been shown by F. W. Carter[2] that the presence of slots increases the effective length of the air-gap from δ to δ', where

$$\delta' = \delta \frac{t}{t - \sigma b_s} \tag{14}$$

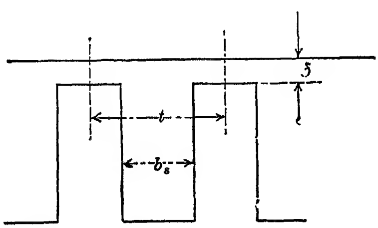


FIG. 113.—Dimensions of teeth and slots.

and where

t = tooth pitch
b_s = width of slot opening

[1] See footnote, p. 156.
[2] Air-gap Induction, Elec. World, Vol. XXXVIII, p. 884, 1901.

and

$$\sigma = \frac{2}{\pi}\left[\arc\tan\frac{b_s}{2\delta} - \frac{\delta}{b_s}\log_e\left(1 + \frac{b_s^2}{4\delta^2}\right)\right] \tag{15}$$

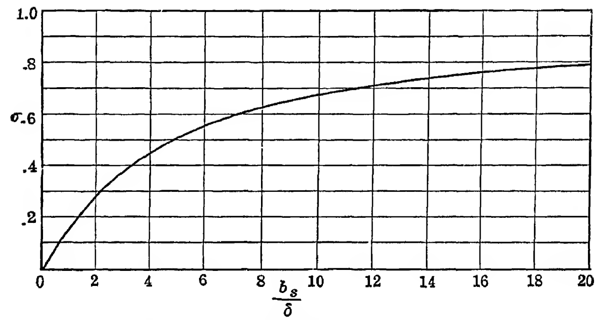


FIG. 114.—Correction factor, σ.

If, however, the effect of the slots is taken into account by reducing the pole arc instead of lengthening the gap, it follows that

$$b' = b\frac{t - \sigma b_s}{t}$$

Values of the factor σ are plotted in Fig. 114 in terms of the ratio $\frac{b_s}{\delta} = \frac{\text{slot opening}}{\text{clearance}}$.

The fringing field at the pole tips is equivalent to an increase in the value of b, but this effect is generally offset by the increased gap space at the tips. Arnold has given a method[1] for computing the increased length of pole due to fringing, but it is generally unnecessary to introduce such refined calculations.

84.[2] **Corrected Axial Length.**—The reluctance due to the ventilating ducts may be considered as reducing the axial length l to

$$l'_1 = l\frac{t_v - \sigma b_v}{t_v} \tag{16}$$

where t_v is the distance between centers of the ventilating ducts and b_v is the width of the duct (Fig. 115), and where σ is to be found from Fig. 114 using as argument $\frac{b_v}{\delta}$. The length l'_1 can be further corrected to take account of the flux which enters the sides of the core from the flanks of the pole shoes, as indicated by the dotted lines in Fig. 115; the correction takes the form of

[1] Die Gleichstrommaschine, Vol. I, p. 274, 2nd ed.

[2] See footnote, p. 156.

an additional length, l'_2, so that the equivalent axial length is

$$l' = l'_1 + l'_2 \tag{17}$$

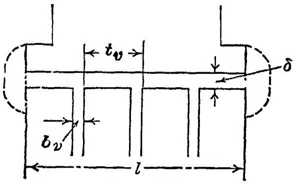


FIG. 115.

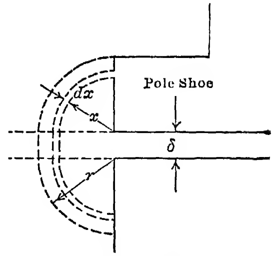


FIG. 116.

FIGS. 115 and 116.—Correction of axial length due to fringing of flux.

The value of l'_2 may be estimated as follows: Assume that the lines of force of the fringing flux are made up of quadrants of circles and of straight lines, as in Fig. 116. The permeance of an elementary tube of width dx and breadth b' is

$$dP = \frac{b'dx}{\delta + \pi x}$$

and the entire permeance of all the tubes between the limits $x = 0$ and $x = r$, on both sides of the core, is

$$P = 2\int_0^r \frac{b'dx}{\delta + \pi x} = \frac{2}{\pi} b' \log_e \frac{\delta + \pi r}{\delta}$$

But the permeance is to be made equivalent to that of a tube of force of length δ and cross-section $b'l'_2$, hence

$$\frac{b'l'_2}{\delta} = \frac{2}{\pi} b' \log_e \left(1 + \frac{\pi r}{\delta}\right)$$

and

$$l'_2 = 1.5\delta \log_{10} \left(1 + \frac{\pi r}{\delta}\right) \tag{18}$$

For values of r from 1 to 5 times δ, l'_2 varies from 0.9δ to 1.8δ. Generally it is sufficiently accurate to take $l'_2 = 1.5\delta$.

Having found b' and l', the corrected value of flux density in the air-gap is

$$B_g = \frac{\Phi}{b'l'}$$

and therefore

$$AT_g = 0.8B_g\delta$$

if metric units are used (flux density in lines per sq. cm. and air-gap in centimeters), or

$$AT_g = 0.3133\ B''_g\delta''$$

if flux density is given in lines per sq. in. (B''_g) and air-gap in inches (δ'').

85.[1] **Ampere-turns Required for the Teeth.**—The same difference of magnetic potential that maintains the flux through the teeth also produces a certain amount of flux through the slots, since the two paths are in parallel. When the teeth are not highly saturated their permeance is so considerable that the flux passing down the slots is relatively insignificant and may be neglected; but in many machines the iron of the teeth is purposely worked at high flux density in order to limit the effect of armature reaction (see Chap. V), and in such cases the permeance of the teeth is decreased to such an extent that the slot permeance becomes comparable with it. If, then, it were assumed that the entire flux per pole passed through the teeth immediately under the pole (with an allowance for the spread of the flux at the pole tips), the resultant tooth density would be higher than it is in reality, and the ampere-turns per unit length corresponding to this apparent density might be greatly in excess of the true value because of the flatness of the magnetization curve at high saturation. The actual tooth density, B_t, must therefore be distinguished from the apparent density, B'_t.

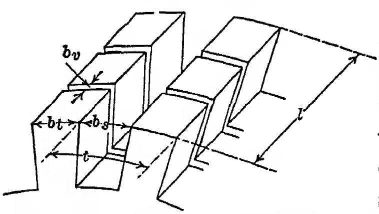


FIG. 117.—Dimensions of teeth and slots.

The conditions that determine the relation of the actual to the apparent density are (1) that the total flux per pole is equal to the sum of the flux in the iron of the teeth and in the air of the slots, ventilating ducts, and insulation space between laminæ; and (2) that the magnitudes of the flux in the iron and in the air are proportional to the permeances of the respective paths. Therefore

$$\Phi = \Phi_{iron} + \Phi_{air} \tag{19}$$

$$\frac{\Phi_{iron}}{\Phi_{air}} = \frac{\mu \times \text{cross-section of iron}}{\text{cross-section of air}} = \mu K \tag{20}$$

[1] See foot note, p. 156.

where μ is the permeability of the iron corresponding to the actual tooth density B_t.

Referring to Fig. 117,

$$\text{cross-section of iron} = b_t(l - n_v b_v)k \tag{21}$$

where k is the lamination factor (usually about 0.9), and

$$\text{cross-section of air} = b_s l + b_t . n_v b_v + b_t(l - n_v b_v)(1 - k) \tag{22}$$

From equations (21) and (22) K may be determined for a given set of dimensions. It follows that

$$\frac{\Phi}{\Phi_{iron}} = \frac{\Phi_{iron} + \Phi_{air}}{\Phi_{iron}} = \frac{1 + \mu K}{\mu K} = \frac{B'_t}{B_t} \tag{23}$$

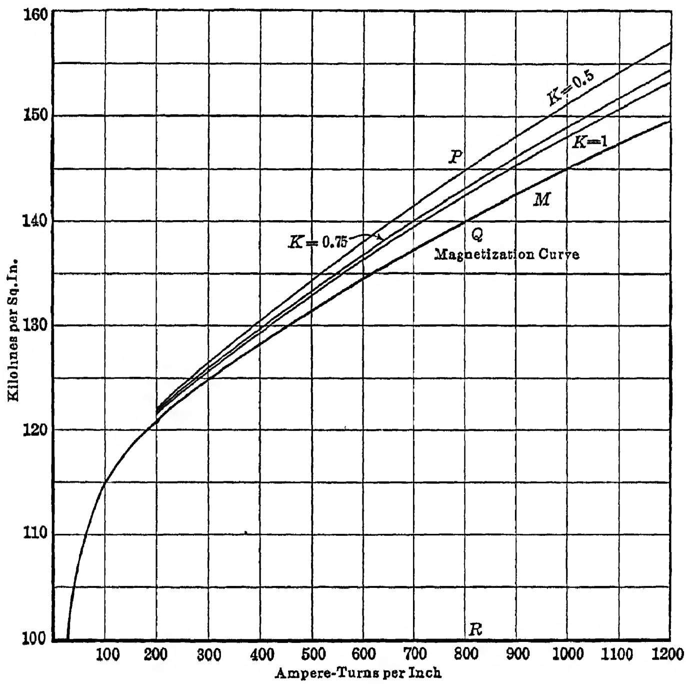


FIG. 118.

For a given value of K it is possible to compute from this equation a series of simultaneous values of B_t and B'_t by assuming values for B_t, finding the corresponding values of μ from the magnetization curve of the core material, and substituting in equation (23). Thus, in Fig. 118, curve M shows the relation between B_t and at_t, as determined by test, for commercial sheet steel. The remaining

curves give B'_t for various values of K.

The above method of determining B_t when B'_t is known has the disadvantage that K may differ from any of the values for which curves have been prepared. It is, however, possible to find B_t from B'_t, for any value of K, directly from the magnetization curve (M, Fig. 118), as follows:

From the relation

$$\frac{B'_t}{B_t} = \frac{1 + \mu K}{\mu K}$$

we have

$$B'_t = B_t + \frac{1}{K}\frac{B_t}{\mu} = B_t + \frac{H}{K} \tag{24}$$

provided B is expressed in lines per sq. cm.

In Fig. 119, let C represent the magnetization curve plotted in terms of B and H, and assume for the present that B and H are plotted to the same scale. To the left of the origin lay off any convenient scale to represent values of K, and lay off ON equal to unity to the scale of K. Then, if OM is any value of K, $\frac{ON}{OM} = \frac{1}{K} = \tan \alpha$, and if OR is drawn parallel to MN the intercept QR will equal $\frac{H}{K}$, corresponding to $H = OQ$. Therefore, $PR = OS = B'_t$ since $PQ = B_t$. That is, for a given value of B'_t, lay off OS on the axis of ordinates equal to this given value and through S draw a line parallel to MN until it intersects curve C in a point P. Ordinate PQ is then the actual tooth density (B_t) and OQ is the corresponding value of H.

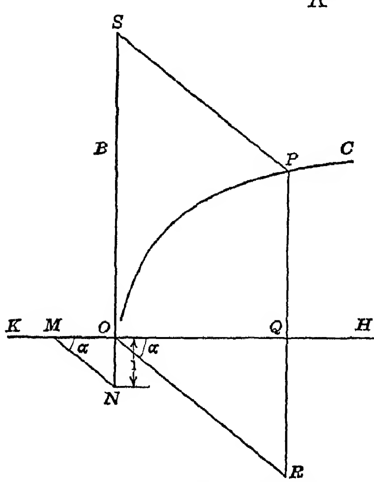


FIG. 119.—Graphical relation between apparent and actual tooth induction.

Since B and H are never plotted to the same scale, and since magnetization curves are usually drawn in terms of B and at, suitable modifications must be made in the construction. If B is plotted to represent lines per sq. cm. and at in am-

pere-turns per cm., the length ON must be made equal to $\frac{4\pi A_0}{10B_0}$ to the scale of K, where

A_0 = number of ampere-turns per cm. per unit length of horizontal axis

B_0 = number of gausses per unit length of vertical axis.

If B is in lines per sq. in. and at in ampere-turns per in., ON must be made equal to $2.54 \times \frac{4\pi A'_0}{10B'_0} = 3.19\ \frac{A'_0}{B'_0}$ to the scale of K where

A'_0 = number of ampere-turns per inch per unit length of horizontal axis

B'_0 = number of lines per sq. in. per unit length of vertical axis.

It does not immediately follow that $AT_t = at_t.l_t$, because the tapering of the teeth results in an increasing density from the tip to the root, consequently at_t changes in value from point to point along the length of the tooth; nor does it follow that the value of at to be used is that corresponding to the flux density at the middle of the tooth. If values of B_t are computed for a number of points along the length of the tooth, and the corresponding values of at_t, found from Fig. 118, are plotted, a curve like Fig. 120 will in general result. Evidently the true value of at_t is the mean ordinate of the curve; assuming that the curve is parabolic, the mean ordinate, by Simpson's rule, is

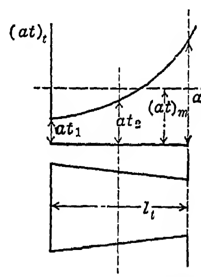


FIG. 120.—Variation of excitation along tooth axis.

$$(at_t)_{mean} = \frac{at_1 + 4at_2 + at_3}{6} \tag{25}$$

whence

$$AT_t = (at_t)_{mean} \cdot l_t \tag{26}$$

86.[1] **Ampere-turns Required for the Armature Core.**—It is clear from Figs. 103 and 104 that the iron of the core below the roots of the teeth carries half of the useful flux per pole. Therefore,

$$B_a = \frac{\Phi}{2A_a} \tag{27}$$

[1] See footnote, p. 156.

If the radial depth of the iron under the teeth is h,

$$A_a = kh(l - n_v b_v) \tag{28}$$

To the value of B_a thus determined there corresponds a value of at_a ampere-turns per unit length, whence

$$AT_a = at_a \cdot l_a \tag{29}$$

87.[1] **Ampere-turns Required for the Pole Cores and Pole Shoes.**—The flux carried by the pole cores and pole shoes varies from section to section, but it may be assumed without sensible error that the flux is uniform and equal to $\nu\Phi$. We have then,

$$B_c = \frac{\nu\Phi}{A_c} \text{ and } B_s = \frac{\nu\Phi}{A_s} \tag{30}$$

to which values of flux density there correspond unit excitations of at_c and at_s, respectively; hence

$$AT_c = at_c \cdot l_c \text{ and } AT_s = at_s \cdot l_s \tag{31}$$

88.[1] **Ampere-turns Required for the Yoke.**—The flux carried by the yoke is either equal to $\nu\Phi$ or $\frac{1}{2}\nu\Phi$, depending upon the type of machine. Fig. 103 illustrates the first case, and Fig. 104 the second case; the latter is representative of most modern machines. Then, usually,

$$B_y = \frac{\frac{1}{2}\nu\Phi}{A_y} \tag{32}$$

to which value there corresponds at_y ampere-turns per unit length, and

$$AT_y = at_y \cdot l_y \tag{33}$$

89.[1] **The Coefficient of Dispersion.**—The leakage flux φ that enters into the equation

$$\nu = 1 + \frac{\varphi}{\Phi}$$

includes the flux in all paths associated with the main flux Φ and originating in the exciting winding, but which do not close through the armature. If the leakage paths are correctly mapped out, the stray flux in each of them is equal to the m.m.f. divided by the reluctance. The calculation can be simplified, and a fair degree of accuracy attained, by assigning simple

[1] See foot note, p. 156.

geometrical forms to the leakage paths. Since the greater part of the leakage flux takes place through air, the reluctance of the path in the iron may be neglected. It must be remembered also that all of the leakage paths are not acted upon by the same difference of magnetic potential. For instance, the m.m.f. acting between the tips of adjacent poles is that required to drive the useful flux across the double air-gap, two sets of teeth, and the armature, or it is equivalent to

$$X = AT_a + 2AT_g + 2AT_t \text{ ampere-turns;} \tag{34}$$

then points on adjacent pole cores that are each half way between the yoke and the shoe will have between them a difference of magnetic potential approximately equivalent to $\frac{1}{2}X$ ampere-turns.

Let Fig. 121 represent a development of a portion of a multipolar machine. The leakage flux in any one pole P is represented by the dashed lines φ_1, φ_2, etc., and the total leakage flux per pole is

$$\varphi = 2\varphi_1 + 4\varphi_2 + 2\varphi_3 + 4\varphi_4 \tag{35}$$

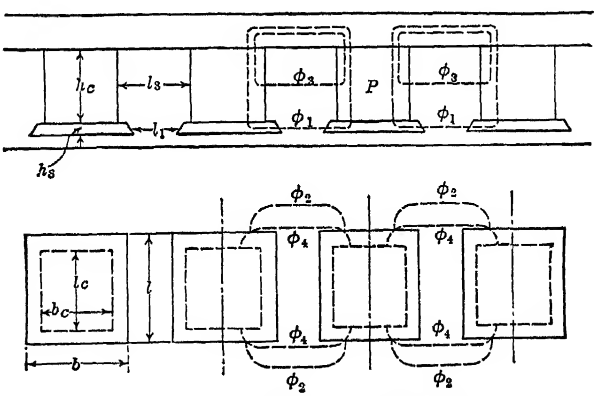


FIG. 121.—Paths of leakage flux.

Leakage between Inner Surface of Pole Shoes, φ_1.—

$$\left.\begin{aligned} \varphi_1 &= \frac{4\pi}{10} X \frac{h_s l}{l_1} \text{ (lengths expressed in centimeters)} \\ &\text{or} \\ \varphi_1 &= 3.2X\frac{h_s l}{l_1} \text{ (lengths expressed in inches)} \end{aligned}\right\} \tag{36}$$

Leakage between Lateral Surfaces of Pole Shoes, φ_2.—Assume that the lines of force are made up of straight lines of length l_1, and of quadrants of circles of radius x.

$$\left.\begin{aligned}\therefore \varphi_2 &= \frac{4\pi}{10}X\int_0^{b/2}\frac{h_s dx}{l_1 + \pi x} = 0.4Xh_s \log_e\left(1 + \frac{\pi}{2}\frac{b}{l_1}\right)\\ &\qquad\text{(lengths expressed in centimeters)}\\ \text{or}\qquad&\\ \varphi_2 &= 2.34Xh_s \log_{10}\left(1 + \frac{1.57b}{l_1}\right)\\ &\qquad\text{(lengths expressed in inches)}\end{aligned}\right\} \quad (37)$$

Leakage between Inner Surfaces of Pole Cores, φ_3.—

$$\left.\begin{aligned}\varphi_3 &= \frac{4\pi}{10}\frac{X}{2}\frac{h_c l_c}{l_3} \text{ (lengths expressed in centimeters)}\\ \text{or}\qquad&\\ \varphi_3 &= 1.6X\frac{h_c l_c}{l_3} \text{ (lengths expressed in inches)}\end{aligned}\right\} \quad (38)$$

If the pole cores are round, of diameter d_c, they may be assumed to have been replaced by square poles of equal cross-section. In that case

$$b_c = l_c = \frac{d_c}{2}\sqrt{\pi} = 0.89d_c \quad (39)$$

The above expression for φ_3 is derived on the assumption that the axes of the pole cores are parallel. This is approximately the case when the machine has numerous poles. If the poles are considerably inclined to each other, let $(l_3)_{min}$ and $(l_3)_{max}$ represent their minimum and maximum separations, at the pole shoes and yoke, respectively; then it is readily shown that

$$\left.\begin{aligned}\varphi_3 &= \frac{4\pi}{10}X\frac{l_c h_c}{(l_3)_{max} - (l_3)_{min}}\left[\frac{(l_3)_{max}}{(l_3)_{max} - (l_3)_{min}}\log_e\frac{(l_3)_{max}}{(l_3)_{min}} - 1\right]\\ &\qquad\text{(lengths expressed in centimeters)}\\ \text{or}\qquad&\\ \varphi_3 &= 3.2X\frac{l_c h_c}{(l_3)_{max} - (l_3)_{min}}\left[\frac{2.3(l_3)_{max}}{(l_3)_{max} - (l_3)_{min}}\log_{10}\frac{(l_3)_{max}}{(l_3)_{min}} - 1\right]\\ &\qquad\text{(lengths expressed in inches)}\end{aligned}\right\} \quad (40)$$

Leakage between Lateral Faces of Pole Cores, φ_4.—The leakage paths may be assumed to be made up of straight lines of length l_3 and of quadrants of circles. The average m.m.f. acting on each elementary tube of force is $\frac{1}{2}X$.

$$\therefore \varphi_4 = \frac{4\pi}{10}\frac{X}{2}l_c\int_0^{\frac{1}{2}b_c}\frac{dx}{l_3+\pi x} = 0.2Xh_c \log_e\left(1+\frac{\pi}{2}\frac{b_c}{l_3}\right)$$

(lengths expressed in centimeters) (41)

or

$$\varphi_4 = 1.17Xh_c \log_{10}\left(1+\frac{1.57b_c}{l_3}\right) \quad \text{(inch units)}$$

The coefficient of dispersion is then

$$\nu = 1 + \frac{\varphi}{\Phi} = 1 + \frac{2\varphi_1 + 4\varphi_2 + 2\varphi_3 + 4\varphi_4}{\Phi}$$

$$= 1 + \frac{X}{\Phi}\cdot \text{(function of frame dimensions)} \qquad (42)$$

If the flux Φ were directly proportional to the excitation X, the coefficient ν would be constant; but since X includes the excitation required to drive the flux through the teeth, and these are frequently highly saturated, the ratio $\frac{X}{\Phi}$ is not constant, hence ν is more or less variable. It generally increases as the load on the machine increases.

PROBLEMS

1. A 4-pole, 120-volt shunt generator is rated at 25 kw. at 900 r.p.m. The armature has a simplex wave winding of 194 face conductors, two conductors per element. The shunt field winding has 800 turns per pole and the exciting current at no load is 5.5 amperes. Assuming that there is no magnetic leakage, find the inductance of the shunt circuit and the energy stored in the magnetic circuit.

2. Construct the magnetization curve of the machine of Problem 1, given the following additional data (assuming a tentative value of 1.15 for the coefficient of dispersion):

External diameter of armature core	13.5 in.
Gross length of armature core	7.0 in.
Number of ventilating ducts	2
Width of each duct	¼ in.
Radial depth of core below teeth	2¾ in.
Number of slots	49
Slot dimensions	0.4 × 1.25 in.
Conductors per slot	4
Air-gap (clearance)	3/16 in.
Ratio of pole arc to pole pitch	0.7
Diameter of core (cast steel)	5.5 in.
Radial length of core	7.0 in.
Depth of pole shoe	1 in.
Yoke (cast steel)	1.5 × 10 in.
Diameter of commutator	9 in.

3. Compute the coefficient of dispersion of the machine specified in Problem 2.

CHAPTER V

ARMATURE REACTION

90. Magnetizing Action of Armature.—In the foregoing discussion of the behavior of a dynamo, it was tacitly assumed that the armature was currentless. Under this no-load condition the magnitude and distribution of the magnetic flux are dependent only upon the excitation due to the field winding and upon the shape and materials of the frame. But, under load conditions, the current in the armature conductors gives rise to an independent excitation which alters both the magnitude and distribution of the flux produced by the field winding alone. This magnetizing action of the armature is called *armature reaction*.

For the sake of simplicity, let us examine first the conditions in a bipolar machine. If the armature is currentless, the flux due to the field excitation will be symmetrically distributed in the manner illustrated in Fig. 122. The line *ab*, drawn through the center of the shaft at right angles to the polar axis, is the *geometrical neutral* axis; it is an axis of symmetry of the flux under no-load conditions. Armature conductors on opposite sides of the geometrical neutral axis will then be the seat of oppositely directed e.m.fs. If, under no-load conditions, the brush axis coincides with the geometrical neutral, the winding elements lying in the neutral axis will be short-circuited by the brushes during a very brief interval in which only a small e.m.f. is generated in them, hence the short-circuit is harmless.

If the field excitation is now removed and the armature is supplied with current from some external source, there will result a magnetic field whose distribution is approximately as shown in Fig. 123. Magnetic poles will be developed in the line of the brush axis. Most of the flux will be concentrated in the region covered by the pole shoes, since the reluctance there is much less than in the interpolar gap.

Under load conditions, the armature current and the field excitation exist simultaneously, and the resultant flux can then

be thought of as compounded of the two fields shown separately in Figs. 122 and 123, at least as a first approximation.[1] The form of the resultant field is shown in Fig. 124, which serves equally well for the cases of generator and motor action. It will be observed that in the case of the generator *the field is strengthened at the trailing tips, A and A′, and weakened at the leading tips, B and B′*; whereas in the case of the motor the exact reverse is true. Moreover, the neutral axis (that in which the winding elements are not cutting lines of force) has been shifted to the

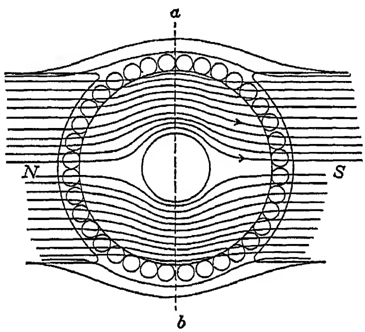


Fig. 122.—Distribution of magnetic field, armature currentless.

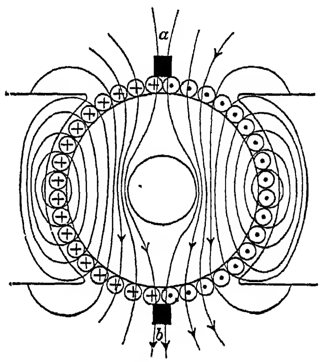


Fig. 123.—Magnetic field due to armature current, field magnets not excited.

position $a'b'$; the effect is the same as though the flux had been twisted or skewed in the direction of rotation in the case of the generator, and in the opposite direction in the case of the motor.

As a result of the shift of the neutral axis, the brushes (assumed to be still in the axis ab) short-circuit elements which are cutting lines of force and in which an active e.m.f. is being generated. Currents of large magnitude may therefore flow in such elements

[1] Note.—It is not exactly true that the resultant field is made up of the separate fields of Figs. 122 and 123 as components. What actually happens is that the windings of the field structure and of the armature each produce a definite m.m.f., and that these m.m.fs. then combine to form a resultant m.m.f., which in turn produces the resultant flux. The composition of the separate fields would give correct results only if the flux were at all points proportional to the m.m.f., and this condition is, of course, not satisfied in the presence of iron cores, especially if the iron is worked at a flux density at or near the knee of the magnetization curve. (See § 98.)

because of the low resistance of the circuit which includes the short-circuited winding elements and the brush contacts; and the rupture of this circuit, as the commutator segments pass from under the brush, may cause sparking and perhaps blistering of the commutator. Furthermore, the machine will not develop its full e.m.f.: for of the $Z/2$ conductors in series, say on the left-hand side of the armature of Fig. 124, those between b and b' will generate an e.m.f. opposite in sign to that of the e.m.f. due to conductors between b' and a. Both of these effects are objectionable, the former because it reduces the life of the commutator and lowers the efficiency, the latter because it unnecessarily reduces the available output of the machine. Obviously, the remedy for both troubles is to make the distortion or skewing

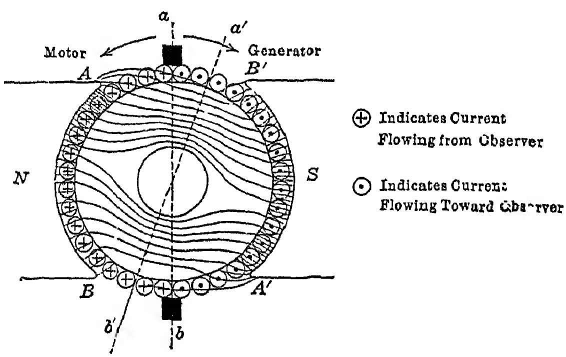


FIG. 124.—Distribution of magnetic field under load conditions.

so small that the neutral axis is not appreciably shifted; or if this cannot be accomplished, to shift the brushes until they are in (or near) the neutral axis; but when the brushes are shifted, the armature field (Fig. 123) moves with them in such a way that the resultant polarization of the armature coincides with the brush axis.

The net result is that the resultant field tends to skew more and more as the brushes are moved toward the neutral axis. Fortunately, however, the piling up of the flux in the pole tips A and A' results in their saturation, so that further skewing becomes insignificant and the brush axis may even pass the neutral.

91. Commutation.—It is desirable at this point to examine the phenomena occurring during commutation somewhat more in

detail than has yet been done, in order to settle in a general way the conditions that must be satisfied by the brush position. Fig. 125 represents three elements, *a*, *b* and *c*, of a ring winding operating as a *generator*. It is evident that element *a* will occupy successively the positions of *b* and *c*, and that during the *b* position its current must change from the value existing in conductors to the left of the brush, to the equal and opposite value existing in conductors on the right. This change cannot occur instantaneously; in the ideal case the current would change uniformly from the initial to the final value in exactly the time required for the element to pass under the brush, during which time the element is short-circuited in the manner of coil *b*. This is represented diagrammatically in Fig. 126, where $+i$ and $-i$ are, respec-

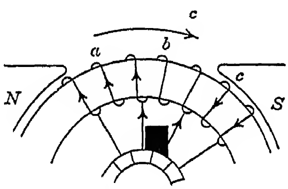


FIG. 125.—Reversal of armature current during commutation.

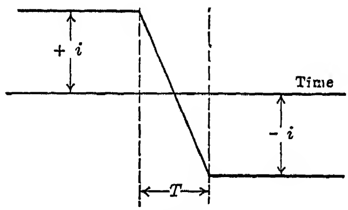


FIG. 126.—Ideal variation of current in coil undergoing commutation.

tively, the initial and final values of the current in the element, and T is the duration of the short-circuit, or the period of commutation.

Now, the self-inductance of the element undergoing commutation tends to keep the current at its original value and in the original direction, and in order to counteract this tendency, the e.m.f. of self-induction must be balanced by an opposing e.m.f. If, then, the brushes were exactly in the neutral axis, no e.m.f. would on the average be generated in the short-circuited coil, the effect of self-induction would not be opposed, and the reversal of the current could not be satisfactorily completed in the time T. Since the direction of the e.m.f. required to cancel the e.m.f. of self-induction must be the same as the final direction of the current (see coil *c*, Fig. 125), it follows that the *short-circuited coil must be under the influence of the pole in advance of the neutral axis*, in the direction of rotation in the case of a generator.

Similar considerations applied to the case of a motor will show that the brushes must likewise be displaced from the neutral axis, but in the backward direction with respect to the direction of rotation. The displacement of the brush from the neutral axis will be nearly the same in both cases, for a given machine used both as generator and motor, but tends to be slightly less in the motor than in the generator. This is due to the fact that each slot is generally occupied by several coils which are commutated successively, and as a rule the conditions obtaining in the last coil to be commutated are the most important; with a given axis of commutation, the last coil to be commutated in the case of the generator will be nearer the neutral axis than in the case of the motor, hence at that moment it is in a weaker field; and if the brush shift is then correct for the generator, the reversing field will be too strong for proper reversal of the current in the motor coil, and the brush must be moved slightly nearer the neutral in the case of the latter machine.

When the angular displacement of the brushes is in the direction of rotation, as in a generator, the angle is called the angle of *brush lead;* when in the direction opposite to the rotation, as in a motor, it is called the angle of *backward lead.*

92. Components of Armature Reaction.—Imagine the brushes of the armature of Fig. 123 supplied with constant current from some external source, the field being unexcited. If the brushes are rocked forward or backward, the armature m.m.f. will follow, and it will remain constant in magnitude. It may, therefore, be represented by a line of constant length, *OA*, Fig. 127, in line with the brushes. If the fields are now excited, their magnetomotive force may be represented by a line *OF* (the direction of current flow in armature and field windings being taken the same as in Fig. 124). Resolving *OA* into the components *OC* and *CA*, it is seen that the armature magnetizing action is equivalent to a *cross-magnetization* due to *OC* (so called because it acts across the main m.m.f. *OF*), and a *demagnetization* due to *CA*, which directly opposes the main excitation, *OF*. The demagnetizing action of the armature is a direct consequence of the rocking of the brushes to the position most favorable for commutation.

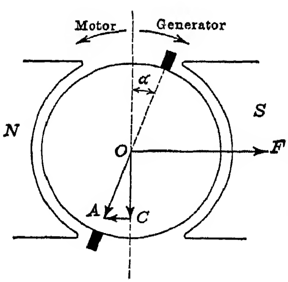


FIG. 127.—Components of armature m.m.f.

It will be clear from Fig. 127 that if the brushes of a generator have a backward lead, the armature will assist in magnetizing the field, that is, the demagnetizing component becomes magnetizing. Similarly if the brushes of a motor are given a forward lead; but in both cases the commutator will spark viciously.

Were it not for the fact that a negative brush lead affects commutation unfavorably, the armature reaction might be purposely exaggerated to such an extent as to self-excite the fields. This feature is taken advantage of in the Rosenberg type of generator for train lighting (see Chap. XI), but in general it requires special auxiliary devices to take care of the commutation difficulties.

93. Cross-magnetizing and Demagnetizing Ampere-turns.—The resolution of the armature m.m f., *OA*, in Fig. 127, into components is not a strictly accurate proceeding; it is qualitative rather than quantitative. But it leads directly to the conclusion that the entire armature winding of Fig. 128 may be considered as made up of two distinct "belts" of conductors, namely, those between *AD* and *CB*, and those between *CA* and *BD*. The former conductors, when grouped in pairs in the manner indicated by the horizontal lines, constitute a number of turns whose magnetizing effect is directly across that of the main exciting winding; they are called the *cross-magnetizing turns*. The remaining conductors, grouped in vertical pairs, constitute the *demagnetizing turns*, since their effect is in direct opposition to the main exciting winding. It follows, therefore, that in a *bipolar* machine the demagnetizing turns per pair of poles are equal to the number of armature conductors within the double angle of lead, 2α; and the *demagnetizing* (or back) *ampere-turns* per pair of poles, AT_d, equal this number multiplied by the current per conductor.

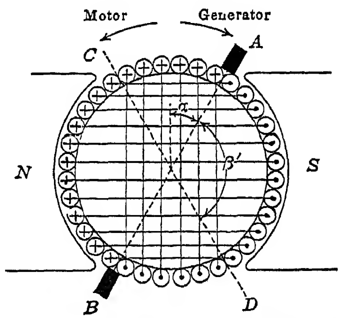


FIG. 128.—Cross-magnetizing and demagnetizing belts of conductors.

$$\therefore \ \mathrm{AT}_d = \frac{2\alpha Z}{360}\frac{i_a}{2} = \frac{\alpha Z i_a}{360} \qquad (1)$$

Similarly, the *cross-magnetizing ampere-turns* are given by

$$AT_c = \frac{\beta' Z}{360} \cdot \frac{i_a}{2} = \frac{\beta' Z i_a}{720} \tag{2}$$

where

$$\beta' = 180 - 2\alpha$$

It is important to realize that the demagnetizing turns are a consequence of the cross-magnetizing action of the armature; for if the brushes are originally in the geometrical neutral axis, the entire magnetizing action of the armature is across the main field, thereby causing a distorted resultant field. The shifting of the brushes to a position near the resultant neutral axis then brings the demagnetizing turns into existence.

94. Cross-magnetizing and Demagnetizing Effect in Multipolar Machines.—In the foregoing discussion of the case of bipolar machines, a ring-wound armature was tacitly assumed.

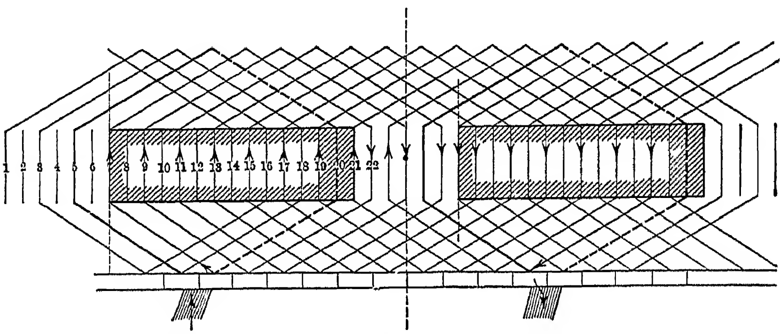


FIG. 129.—Reduction of demagnetizing action caused by fractional pitch of winding.

Accordingly, the brush axis and the axis of commutation coincided, and no distinction was made between them. It must be remembered, however, that the end connections of lap and wave windings are generally so shaped that the brushes are opposite the middle of the poles when the sides of the coil undergoing commutation are in the geometrical neutral.

An extension of the principles developed for the case of the bipolar machine leads to the generalization that all of the conductors lying within the double angle of lead have a demagnetizing

effect upon the field, while the remaining conductors produce a cross-magnetization. This conclusion holds accurately for all windings of the ring type, and for lap and wave windings of full pitch. But in short-chord windings it will be found that the conductors occupying the space between pole tips carry currents which are partly in one direction and partly in the other, thereby partially neutralizing the demagnetizing effect. For example, assume a 4-pole simplex lap winding having 80 conductors; that is

$$p = 4$$
$$a = 4$$
$$Z = 80$$
$$m = 1$$

Take $y_1 = 15$ and $y_2 = -13$. On tracing through the winding diagram, a portion of which is shown in Fig. 129, it will be found that the current in the interpolar region is alternately in opposite directions.

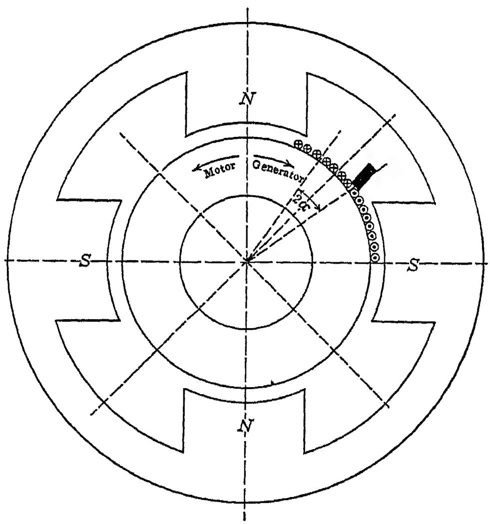


Fig. 130.—Demagnetizing belt of conductors in multipolar machine.

Omitting from consideration the special case of short-chord windings, the number of demagnetizing ampere-turns per pair of poles can be determined as follows (see Fig. 130):

1

The total number of conductors lying within the demagnetizing belts is

$$\frac{Z}{360} \cdot 2\alpha . p$$

and, therefore, the number of *demagnetizing ampere-turns per pair of poles* is

$$AT_d = \tfrac{1}{2} \cdot \frac{Z}{360} \cdot \frac{2\alpha p}{p/2} \cdot \frac{i_a}{a} = \frac{\alpha Z i_a}{180a} \tag{3}$$

The cross-magnetizing ampere-turns produce distortion of the main field by strengthening the field at one tip of the pole and weakening it at the other, as in Fig. 131. But though all of the conductors outside of the double angle of lead contribute to this effect, those which are not in the angle β subtended by the pole exert their m.m.f. upon a path so largely in air that the flux due to them is negligible; attention may, therefore, be confined to the $\frac{\beta Z}{360}$ conductors under the pole. The m.m.f. due to these conductors is $\frac{4\pi}{10} \cdot \frac{\beta Z}{360} \cdot \frac{i_a}{a}$ gilberts, and this acts upon a path C whose reluctance is mainly due to the double air-gap and two sets of teeth; the teeth consume a m.m.f. equal to $\frac{4\pi}{10}$ $(2AT_t)$ gilberts, hence the remainder, or $\frac{4\pi}{10}\left[\frac{\beta Z}{360}\frac{i_a}{a} - 2AT_t\right]$, will produce a cross-field whose intensity at the tips is

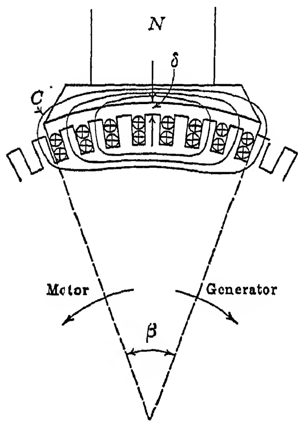


FIG. 131.—Cross field in multipolar machine.

$$B_c = \frac{4\pi}{10}\left[\frac{\beta Z}{360}\frac{i_a}{a} - 2AT_t\right]\frac{1}{2\delta'}, \tag{4}$$

where $\delta' = \delta \frac{t}{t - \sigma b_s}$ is equal to the gap length corrected to take account of the effect of the slots.

The resultant pole tip densities will then be

$$(B_g - B_c) \text{ at the } \begin{Bmatrix} \text{leading} \\ \text{trailing} \end{Bmatrix} \text{ pole tip of a } \begin{Bmatrix} \text{generator} \\ \text{motor} \end{Bmatrix}$$

and

$$(B_g + B_c) \text{ at the } \begin{Bmatrix} \text{trailing} \\ \text{leading} \end{Bmatrix} \text{ pole tip of a } \begin{Bmatrix} \text{generator} \\ \text{motor} \end{Bmatrix}.$$

Now, since commutation takes place at the weakened pole tip, and since the direction of the "commutating field" must be that of the original field, B_g, it follows that $B_c < B_g$, in order to generate in the short-circuited coil an e.m.f. of the proper direction to balance the e.m.f. of self-induction. Generally,

$$B_g - B_c = 2000 \text{ to } 3000 \text{ (lines per sq. cm.)}$$

and since B_g is usually between 6000 and 10,000, it follows that

$$B_g = (1.25 \text{ to } 2)B_c$$

Substituting this relation in the expression for B_c, and transposing

$$\delta' = \frac{(1.25 \text{ to } 2)\left[\frac{\beta Z i_a}{360a} - 2AT_t\right]}{1.6\,B_g} \tag{5}$$

from which it is possible to compute the length of air-gap necessary to prevent reversal of the field at the commutating tip. If the clearance, δ, has been fixed, the formula gives an idea of the extent of chamfer to be given to the pole tips.

The above formula also leads to a relation which serves as an approximate criterion for a successful machine. Thus, neglecting the term AT_t, we may write

$$(1.25 \text{ to } 2)\,\frac{\beta Z i_a}{360a} = 1.6B_g\delta' = 2AT_g \tag{6}$$

or

$$\frac{\beta Z i_a}{360a} = (1.0 \text{ to } 1.6)AT_g \tag{7}$$

Now, $\beta = \dfrac{360\Psi}{p}$ where Ψ, the ratio of pole arc to pole pitch, is usually about 0.7 in direct-current machines; and $\dfrac{Z}{2}\cdot\dfrac{i_a}{a}$ is the total number of armature ampere-turns. Further, AT_g will vary from 0.7 to 0.9 of the field ampere-turns per pole. Substituting these relations, it will be found that

Armature ampere-turns per pole $\lessapprox$ 1.1 field ampere-turns per pole (8)

The factor 1.1 is an upper limit that is seldom found in practice; ordinarily it will have a value of from 0.8 to 0.9.

95. Corrected Expression for Demagnetizing Effect of Back Ampere-turns.—(*a*) *Lap Windings.*—Fig. 132 is a development of that portion of a "chorded" (short-chord) lap winding embraced in a span slightly greater than the pole-pitch. It is required to determine the reduction in the value of $AT_d = \frac{\alpha Z i_a}{180 a}$ due to the fact that the coils in the neutral zone carry currents which are not all in the same direction.

In the first place, it will be evident that if the winding were of full-pitch, all of the $\frac{2S}{p}$ coil sides lying to the left of A_2 would carry current in the same direction (vertically upward in the figure).

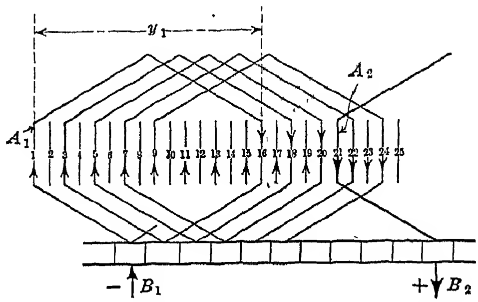


FIG. 132.—Fractional pitch lap winding.

In the second place, however short the chording may be, the current in the coil sides immediately to the right[1] of, and including, A_2 will all be in the same direction, or downward in the figure. It follows, therefore, that the reversed currents all lie in a zone to the left of A_2, the extent of this zone depending upon the difference between y_1 and the pole pitch. Similarly, there will be zones of reversed currents to the left of all the coil edges, which, like A_1 and A_2, are connected to commutator segments touched by the brushes.

If coil edge A_1 is numbered 1, the first coil edge carrying reversed current is $(y_1 + 1)$, the second is $(y_1 + 3)$, etc. The number corresponding to the last one in the group will evidently be $\frac{2S}{p} = y_1 + \left(\frac{2S}{p} - y_1\right)$.

[1] This is a consequence of the fact that the winding sketched in Fig. 132 is right-handed, *i.e.*, $y_1 > y_2$. If the winding were left-handed, $y_1 < y_2$, the words "right" and "left" would have to be interchanged.

Summarizing these results,

$$
\begin{array}{ll}
y_1 + 1 & \text{corresponds to the 1st} \\
y_1 + 3 & \text{corresponds to the 2nd} \\
y_1 + 5 & \text{corresponds to the 3rd} \\
y_1 + (2k - 1) & \text{corresponds to the } k\text{th} \\
y_1 + \left(\frac{2S}{p} - y_1\right) & \text{corresponds to the } n\text{th}
\end{array}
$$

In other words, there are n of such reversed bundles, where

$$2n - 1 = \frac{2S}{p} - y_1$$

or

$$n = \tfrac{1}{2}\left(\frac{2S}{p} - y_1 + 1\right) \tag{9}$$

Since the current in each of these n coil edges balances the demagnetizing effect of the current in n bundles whose direction is normal, the total reduction will be that due to $2n$ bundles. Since each element contains $\frac{Z}{2S}$ turns, AT_d will be less than the computed value by

$$\frac{Z}{2S}\frac{i_a}{a}\left(\frac{2S}{p} - y_1 + 1\right) \text{ ampere-turns} \tag{10}$$

It should be noted that extreme chording may cause some of the n reversed coil sides to fall outside of the double angle of lead, and, therefore vitiate the above correction But such extreme chording would not be used in practice, hence the correction may be safely used. It should be noted, further, that neither the formula for AT_d nor for the correction due to chording takes account of the number of coil edges in the neutral zone which are short-circuited by the brushes during commutation.

(*b*) *Wave Windings.*—(Fig. 133). If y_1 is the back pitch of the winding (at the pulley end), and y_2 is the front pitch (at the commutator end),

$$y = \frac{y_1 + y_2}{2} = \frac{2S + a}{p}$$

the positive sign of a indicating that the winding is right-handed. The extent to which y_1 falls short of the pole pitch is then a measure of the chording; obviously, then, $y_2 > y_1$. Full-pitch winding would result in uniform opposition of direction of current on

either side of coil sides like A_1, A_2, Fig. 133, which are in contact with the brushes.

Due to chording, however, the first conductor which carries reversed current is $(y_1 + 1)$, the second is $(y_1 + 3)$, etc. If there are n such conductors, the number of the nth conductor will be $y_1 + (2n - 1)$. All of these conductors bear even numbers,

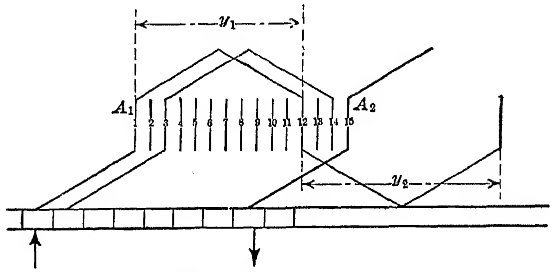


FIG. 133.—Fractional pitch wave winding.

since y_1 is necessarily odd. Now, the number of conductors (coil sides) per pole pitch is $\frac{2S}{p} = y - \frac{a}{p}$, which is a mixed number, but which may be taken as equal to y. If y is odd, the last even number in the group is $y - 1$, whence

$$y - 1 = y_1 + (2n - 1)$$

or

$$2n = \frac{y_2 - y_1}{2} \tag{11}$$

if y is even,

$$y = y_1 + (2n - 1)$$

$$2n = \frac{y_2 - y_1}{2} + 1 \tag{12}$$

hence the number of ampere-turns to be deducted from (AT_d) is

$$\left.\begin{array}{ll} \dfrac{Z}{2S}\dfrac{i_a}{a}\left(\dfrac{y_2 - y_1}{2}\right) & \text{if } y \text{ is odd,} \\ \dfrac{Z}{2S}\dfrac{i_a}{a}\left(\dfrac{y_2 - y_1}{2} + 1\right) & \text{if } y \text{ is even.} \end{array}\right\} \tag{13}$$

96. Shape of Magnetic Field Produced by Armature Current.—The current in the armature conductors lying to one side of the commutated coil has a direction opposite to that of the current in the conductors on the other side. In other words, the current

is distributed around the periphery in a series of alternately directed bands or belts, equal in number to the number of poles. This is indicated in Fig. 134 which represents a development of a 4-pole generator. The peripheral distribution of m.m.f. due to the armature current will then be represented by the ordinates of the broken line. The m.m.f. is a maximum at the points where the current reverses, and it is zero at the points midway between

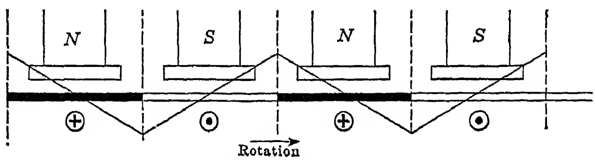


FIG. 134.—Peripheral distribution of armature m.m.f.

them. If the pole shoes completely surrounded the armature and the surface of the latter were smooth, the armature flux would be at all points directly proportional to the m.m.f., since in such a case the reluctance would be constant all around the gap (neglecting the reluctance of the flux paths in the iron in comparison with those in the air).

The number of conductors surrounded by the elementary tube of flux, P, Fig. 135, is $\frac{Z}{\pi d} \cdot 2x$ where d is the diameter of the armature.

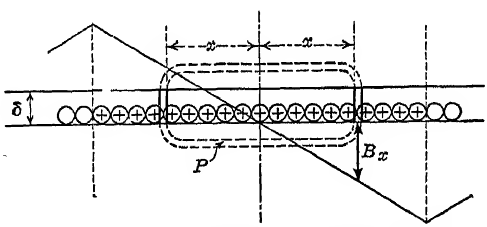


FIG. 135.—Calculation of armature flux.

ture. Since each conductor carries $\frac{i_a}{a}$ amperes, i_a being the total armature current, the m.m.f. acting on tube P is $\frac{4\pi}{10}\frac{Z}{\pi d}\frac{i_a}{a} \cdot 2x = \frac{4\pi}{10} q \cdot 2x$ where $q = \frac{Z}{\pi d} \cdot \frac{i_a}{a}$ is the number of *ampere-conductors*

per unit length of periphery. The flux density at a point distant x from the center of the pole is then

$$B_x = \frac{\frac{4\pi}{10} q \cdot 2x}{2\,\delta} = \frac{xq}{0.8\delta} \tag{14}$$

In the usual case of machines having pole shoes separated from each other by an intervening air-space, the flux distribution curve is not similar in form to the curve of m.m.f. Under the pole shoe it will closely follow the m.m.f. curve because of the practically uniform reluctance, but between the pole tips the reluctance increases at a much greater rate than the m.m.f., hence the armature flux density will be small at the point midway between them, as shown in Fig. 136.

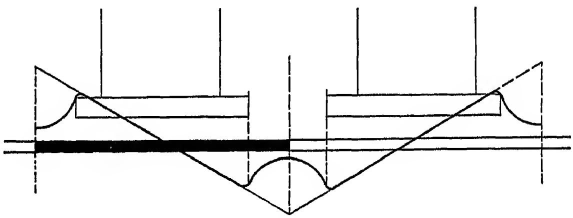


FIG. 136.—Distribution of armature flux.

97. Approximate Distribution of the Resultant Field.—Parts a, b, and c of Fig. 137 represent the effect of the armature field in modifying the magnitude and distribution of the resultant magnetic field for three positions of the brushes. In each diagram curve F shows the flux distribution due to the field excitation alone; curve A is the flux curve due to the armature, and curve R is the resultant of F and A. The diagrams are drawn for the cases of commutation:

(*a*) midway between pole tips,
(*b*) between the pole tips but nearer the leading tip,
(*c*) under the middle of the poles.

In case (*a*) the distortion of the magnetic field is clearly shown. The symmetry of curve A with respect to F means that the flux added to the trailing tip (generator action being assumed) is exactly equal to the flux removed from the leading tip, hence the flux per pole remains unaltered. In case (*b*) there is distortion

and also a resultant demagnetization, as it is clear that the flux A under a pole is more subtractive than additive. In case (c) there is no distortion, but only demagnetization, as might be expected from the fact that the brushes have been shifted to such an extent as to eliminate all the cross ampere-turns and to raise the back ampere-turns to a maximum value.

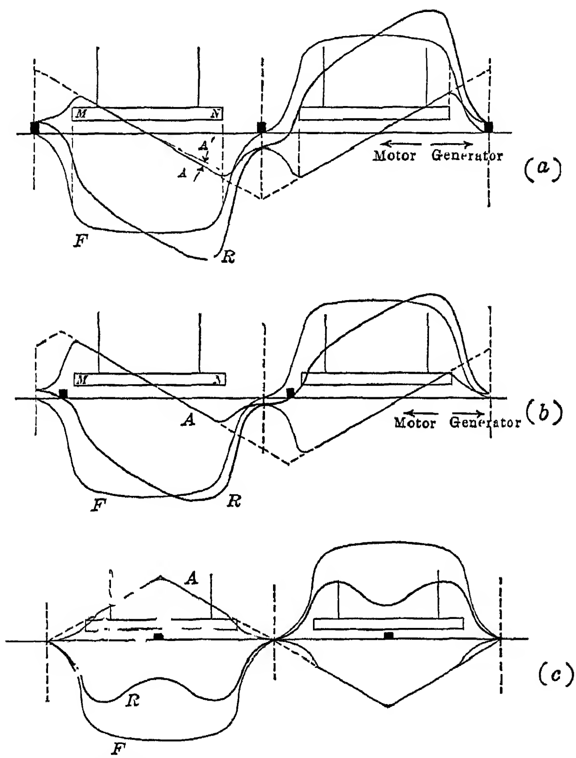


FIG. 137.—Distribution of resultant flux for various brush positions.

98. Demagnetizing Component of Cross Magnetization.—In the preceding article the shape of the resultant field R was determined on the theory that it may be considered as made up of two components: one, a field produced by the m.m.f. of the field winding acting alone: the other, a field produced by the armature m.m.f. acting alone. As a matter of fact, this theory is not strictly correct, as the following illustrative analogy will show:

Imagine a rod of cast iron acted upon simultaneously by com-

pressive and tensile stresses, and suppose that these stresses are equal. If we assume that the stresses act independently in deforming the rod, the elongation due to the tension would considerably exceed the shortening due to the compression, provided the tensile stress is beyond the elastic limit; on this basis there would be a resultant elongation. But it is quite clear from the assumed equality of the stresses that the resultant stress and, therefore, the resultant deformation are both zero, hence the absurdity of the first method.

In the case of the magnetic circuit, m.m.f. is analogous to stress, and flux to deformation. Hence we must conclude that the only correct procedure is first to combine the several m.m.fs. to form a single resultant and from the latter determine the distribution and magnitude of the resultant flux.

It will be clear from the above considerations, taken in connection with diagrams *a* and *b* of Fig. 137, that the increased m.m.f. on the side N of the pole shoe cannot raise the resultant flux on that side to the same extent as the diminished m.m.f. at M lowers the flux on that side—this because of the fact that the permeability of the iron of the pole shoe and armature teeth decreases with increasing magnetizing force. Therefore, even when commutation takes place midway between the poles, corresponding to zero brush lead and an entire absence of back ampere-turns, there is still a resultant demagnetizing action due to the cross ampere-turns; though this effect is usually not very pronounced, it is nevertheless appreciable. The general nature of the effect is indicated in Fig. 137*a*, where the broken line marked A' indicates that the increased m.m.f. due to A on the side of the pole marked N is less than proportional to A. A more detailed analysis of the magnitude of this demagnetizing component of the cross-magnetizing action is given in the following article.

99. Excitation Required under Load Conditions.—Let curve OM of Fig. 138 represent the magnetization curve of a generator and let OV be the terminal voltage, V, at rated load. The excitation required to generate this e.m.f. at no load is then $(AT)_0$ ampere-turns per pair of poles. When the armature delivers current to the load, the excitation required to maintain constant terminal voltage must be greater than $(AT)_0$ in order to compensate:

(*a*) The ohmic drop, or drop in potential, caused by the flow of the current through the resistance of the armature.

(*b*) The demagnetizing effect of the armature back ampere-turns.

(*c*) The demagnetizing component of the armature cross ampere-turns.

(*a*) If the terminal e.m.f. under load conditions is to remain the same as at no load, the total *generated* e.m.f., E_1, must be greater than V by the drop $i_a r_a$, where r_a is the resistance of the armature. Referring to Fig. 138, the excitation required to generate E_1 volts is $(AT)_1$.

(*b*) If the armature demagnetizing ampere-turns per pair of poles (corrected, if necessary, to take care of chording) amount

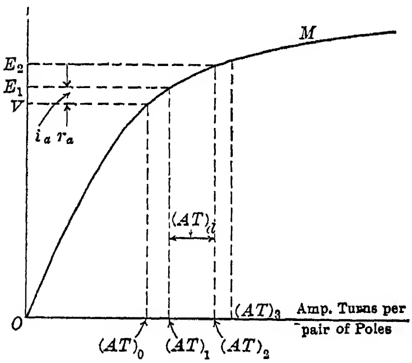


FIG. 138.—Excitation required under load conditions.

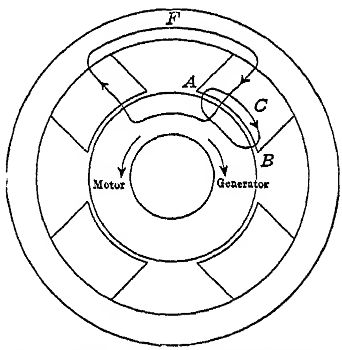


FIG. 139.—Comparison of paths of main and cross field.

to $(AT)_d$, the field excitation must be increased over and above $(AT)_1$ by $(AT)_d$, that is, to the value $(AT)_2$ corresponding to an open-circuit e.m.f. of E_2 volts.

(*c*) Because of the fact that saturation of the pole tips may reduce the flux at one pole tip more than it is increased at the other, it may be necessary to increase the field excitation still further, as from $(AT)_2$ to $(AT)_3$, Fig. 138, in order that the terminal voltage under load conditions may be maintained at its no-load value. In order to determine this increase, the following facts may be noted:

The cross field which is responsible for the demagnetization of one pole tip and the magnetization of the other is due to the

conductors lying under the pole face, and the path of this cross flux is indicated by the line marked C, Fig. 139. The number of cross ampere-turns acting around this path is $\frac{\beta Z}{360} \cdot \frac{i_a}{a}$, half of which oppose the main excitation at pole tip A, the other half reinforcing it at B. The main field excitation acts on the circuit marked F. Circuits C and F have in common the reluctance made up of the *double air-gap, two sets of teeth, the armature core* (in part), *and the pole shoes,* and these parts constitute a large percentage of the reluctance of the entire magnetic circuit. If the difference between the reluctances of circuits C and F were negligible, it would follow that a given m.m.f. acting around such a circuit as C would produce a flux equal to that produced by the same m.m.f. acting

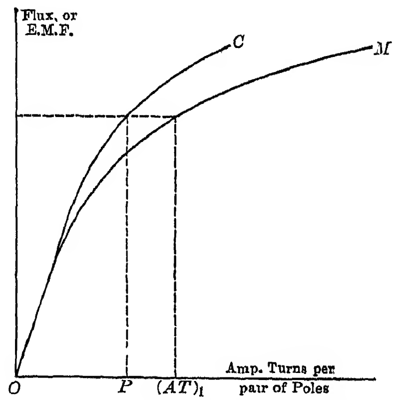


FIG. 140.—Magnetization curves.

around circuit F, in which case the relation between flux and m.m.f. would be represented by the magnetization curve of the machine. Actually, however, the relation between flux and m.m.f. in such a circuit as C, Fig. 139, will be given by the curve marked C, Fig. 140, which lies to the left of the magnetization curve, M; ordinates of this curve represent flux, or the e.m.f. generated by rotation through the flux at a given speed, while abscissas represent the ampere-turns required to maintain this flux through the double air-gap, two sets of teeth, and the small reluctance of the pole shoes and armature core comprised in circuit C, Fig. 139. Abscissas of curve C, Fig. 140, are less than those of curve M for a given flux because of the greater reluctance of the entire mag-

netic circuit. In Fig. 140 the abscissa $(AT)_1$ represents the ampere-turns per pair of poles required to develop the e.m.f. E_1, as in Fig. 138; abscissa OP then represents the number of ampere-turns required to produce a corresponding flux in the circuit C, Fig. 139. This is shown separately in Fig. 141. When the armature is delivering a current i_a, the brushes being assumed to be in the neutral axis, the excitation at the middle of the pole face MN is not affected by the cross-magnetizing action, but at the tip M

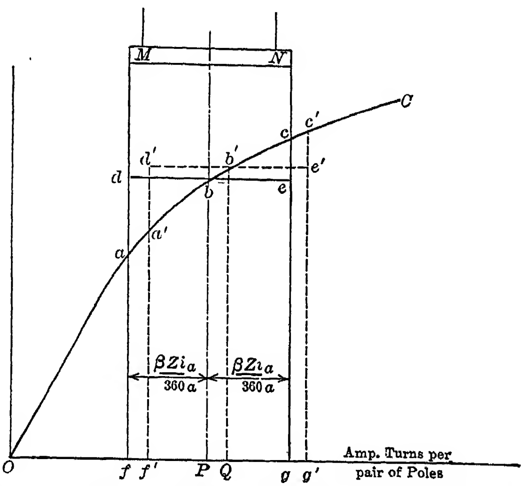


FIG. 141.—Demagnetizing effect due to saturated pole tips.

the magnetizing action is decreased by bd ampere-turns and at the tip N it is increased by be ampere-turns, where $bd = be = \dfrac{\beta Z}{360} \cdot \dfrac{i_a}{a}$. The flux density in the air-gap at M is therefore proportional to the ordinate of the point a of curve C, and that at N is proportional to the ordinate of point c, while at intermediate points the flux density is proportional to the corresponding ordinates of curve C. The total flux per pole is therefore reduced in the proportion that area $fabcg$ bears to area $fdbeg$. If then the total flux per pole shall remain unaltered, the excitation will have to be increased by an amount PQ, such that

$$\text{area } f'd'e'g' + \text{area } b'c'e' - \text{area } a'b'd' = \text{area } fdeg \qquad (15)$$

The excitation PQ is the amount that must be added to $(AT)_2$, Fig. 138, to give the total required excitation $(AT)_3$ in that figure.

100. Experimental Determination of Flux Distribution.—Since the instantaneous e.m.f. generated in an armature conductor is proportional to the radial component of the flux density at the point occupied by the conductor at the moment in question (see Art. 32), the measurement of this e.m.f. will provide data for the calculation of the flux distribution.

Consider the case of a simplex ring-wound armature (Fig. 142) provided with such a large number of commutator segments that the turns of each element may be assumed to be concentrated in a radial plane, in other words, that all the turns of the element (if there be more than one turn per element) are simultaneously in a field of the same intensity. Take a narrow strip of tough paper (sheet fiber or press-board) whose length is equal to the periphery

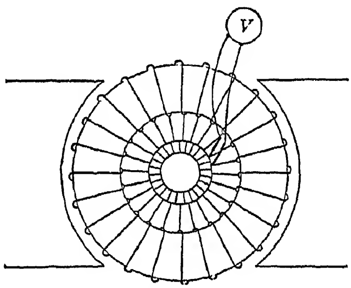


FIG. 142.—Double pilot brush.

of the commutator, and along its axis drill a series of small holes whose spacing is the same as that of the commutator bars. Wrap the strip around the commutator and fasten it to the brush studs so that the commutator may rotate within it without binding. The free ends of a pair of leads connected to a low-reading voltmeter are then to be provided with contact points made of moderately hard lead pencils. When the contact points are inserted into adjacent holes in the strip, the reading of the voltmeter will be equal to the e.m.f. generated in the element minus the ohmic (*ir*) drop due to the current flowing through the resistance of the element, assuming that the experiment is made when the machine (generator) is running under load conditions.

Instead of the perforated strip described above, there may be employed a "pilot" brush made of two thin pieces of sheet brass screwed on opposite sides of a strip of wood or ebonite, whose

thickness is such that the metal strips are separated by the distance from center to center of adjacent commutator segments.

A similar arrangement will suffice in the case of a simplex lap or a simplex wave winding, provided the elements are of full pitch; moreover, in the case of the simplex wave winding, the voltmeter reading will be due to $p/2$ elements in series, instead of only one element. If the winding is duplex or triplex, the spacing of the contact points must be two or three segments, respectively; in general, if the winding is m-plex, the distance between contacts must correspond to m commutator segments.

As stated above, the observed readings of the voltmeter are less than the true values of generated e.m.f. by an amount equal to the ohmic drop in the element (or elements) due to load current if the machine is supplying an external circuit. In order to eliminate this correction of the observed readings, an auxiliary "search" coil, of full pitch, may be wound on the armature, one of its terminals being grounded on the shaft and the other connected to an insulated stud on the end of the shaft. Connect one terminal of a voltmeter of the D'Arsonval type to the frame of the machine (or better, to a metal brush rubbing on the shaft) and the other terminal to a brush that makes contact once per revolution with the insulated stud. If the moving coil of the voltmeter has sufficient inertia and is well damped, it will give a steady reading proportional to the e.m.f. generated in the search coil at the instant when the contact is established between the brush and the rotating stud. If the brush is made capable of adjustment around the arc of a circle concentric with the shaft, the contact can be made to occur when the search coil is in any desired position under the poles.

101. Potential Curve.—In Fig. 143 the ordinates e_1, e_2, e_3, etc., represent the e.m.fs. generated in individual coils. It is evident, therefore, that the expression

$$e_1 + e_2 + e_3 + \ . \ . \ . + e_n$$

will be equal to the reading of a voltmeter one of whose terminals is connected to the main brush (B) and the other to a single pilot brush separated from B by an angle equivalent to the spread of n coils. If the pilot brush (P, Fig. 144) is moved around the periphery of the commutator and voltmeter readings (cor-

rected for drop of potential if current is flowing) are taken at various points, a *potential curve* of the kind shown in full line in Fig. 145 will result.

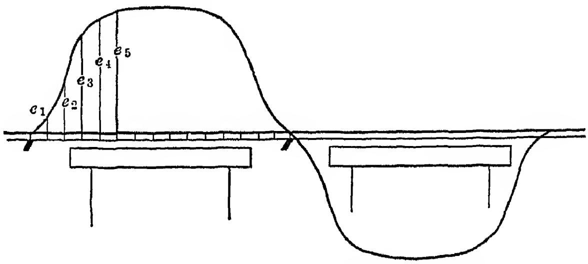


FIG. 143.—Variation of voltage per element.

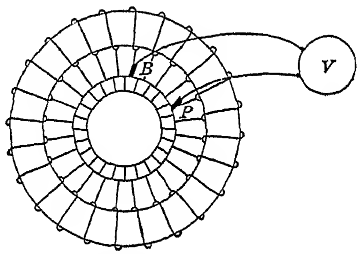


FIG. 144.—Determination of potential curve.

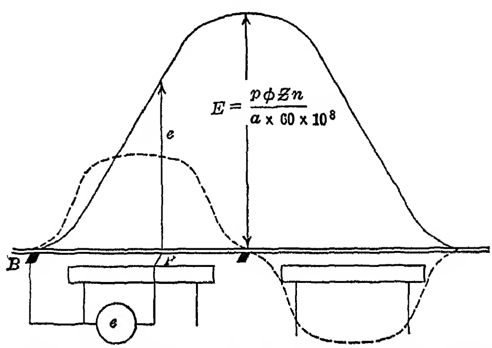


FIG. 145.—Relation between voltage per element and potential curve.

Since any ordinate, e, of this curve is the sum of the ordinates of the dotted curve (which is the same as that of Fig. 143) lying to the left of e, it follows that the first derivative of the function which represents the potential curve will represent the curve of flux distribution, provided the winding is divided into a large

number of elements. In other words, the slope of the potential curve at any point is proportional to the e.m.f. generated in the coil corresponding to that point.

102. Predetermination of Flux Distribution in the Air-gap.—The change in the distribution of the air-gap flux due to the magnetic reaction of the armature current is very important because of its effect upon the commutating characteristics of the machine, as explained in a preliminary manner in Art. 91 and in greater detail in Chap. VIII. It is therefore occasionally desirable, in designing a new machine, to be able to predetermine the curve of flux distribution due to the field excitation alone (curve *F*, Fig. 137), and also the curve of flux distribution due to the armature m.m.f. (*A*, Fig. 137). Several methods[1] for determining these curves have been developed, but all of them, except that of Carter, are approximate; and Carter's method, though mathematically correct, is derived by assuming a simple shape of pole core and pole face that is not ordinarily used in practice.

For determining the curve of field flux distribution, the method recommended by Arnold involves mapping out the paths of the lines of force in the air-gap and in the interpolar spaces in the manner illustrated in Fig. 146. This leaves much to one's judgment, but some guidance is afforded by the consideration that the lines are substantially perpendicular to the surfaces of the pole faces and of the armature where they enter or leave the iron; it is also true that the flux will distribute itself in such a manner that the total reluctance is a minimum, or, what is the same thing, for a given m.m.f. between the pole face and armature surface, the total flux will be a maximum. If, then, more than one trial is made, that one will be most nearly correct which yields the largest total flux.

[1] W. E. Goldsborough, Trans. A.I.E.E., Vol. XV, p. 515; Vol. XVI, p. 461; Vol. XVII, p. 679.

S. P. Thompson, "Dynamo Electric Machinery," Vol. II, p. 206, 7th ed.

F. W. Carter, Electrical World, Vol. XXXVIII, p. 884 (1901).

E. Arnold, "Die Gleichstrommaschine, Vol. I, p. 320, 2d ed.

T. Lehmann, Elektrotechnische Zeitschrift, Vol. XXX, pp. 996 and 1019 (1909).

B. G. Lamme, Trans. A.I.E.E., Vol. XXX, Part 3, p. 2362 (1911).

C. R. Moore, Trans. A.I.E.E., Vol. XXXI, Part I, p. 509 (1912).

Under the central part of the pole, where the air-gap (δ) is uniform, the flux density (B_g) will also be uniform and inversely proportional to δ; at any other point a tube of force of length δ_x and mean section b_x (taking a unit length along the core) will have a permeance b_x/δ_x, hence the flux density at the armature surface will be

$$B_x = \frac{\text{m.m.f.}}{a_x} \cdot \frac{b_x}{\delta_x} = B_g \frac{\delta}{\delta_x} \cdot \frac{b_x}{a_x} \tag{16}$$

If B_g is taken as 100 per cent., values of B_x can then be found from

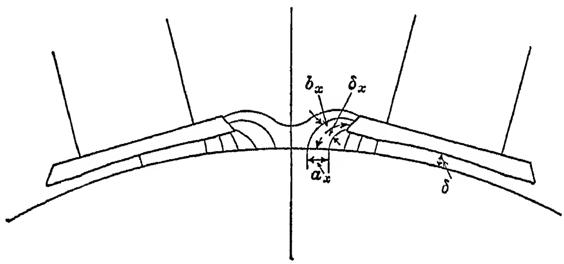


FIG. 146.—Distribution of main field.

the scaled values of δ_x, b_x and a_x, and the computed results when plotted along the developed armature surface will determine a curve like R, Fig. 147. Curves R' and R'' represent portions of similar curves for adjoining poles (of opposite polarity), so that the resultant of R, R' and R'' gives the desired curve, F. The area

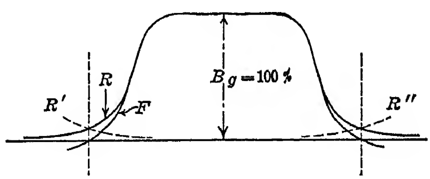


FIG. 147.—Curve of flux distribution.

of one loop of curve F, multiplied by l' (the corrected length of armature core) must then equal Φ.

The determination of the curves of flux distribution due to the armature m.m.f. (curve A, Fig. 137) is more difficult than in the case of the field flux, but the same general method is applicable. Thus, in Fig. 148 are indicated the paths of the lines of force

emanating from the armature. At any distance x from the brush the permeance of the tube of force of unit depth parallel to the shaft will be equal to $\frac{b_x}{\delta_x}$, and if the peripheral distribution of permeance is plotted, as curve P of Fig. 149, and the ordinates of curve P are multiplied by the corresponding ordinates of the curve of armature m.m.f. (M.M.F.), the resultant values will give

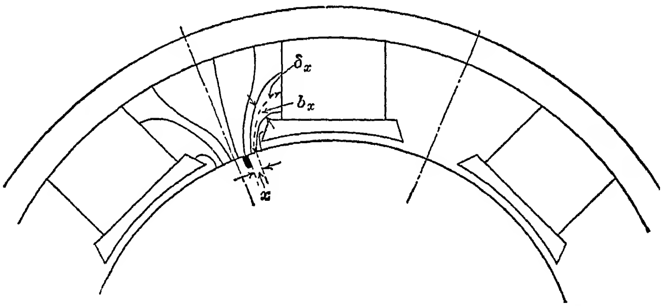


FIG. 148.—Magnetic lines of force due to armature current.

the curve of armature flux (A). The field due to the armature m.m.f. has greatest influence in the axis of commutation, shown at B in the figure; in the paper by Lamme (*loc. cit.*) it is recommended that the mean path of the flux issuing from the axis of commutation be taken as the arc of a circle extending to the middle point of the pole core, and intersecting the surfaces of armature and pole core at right angles.

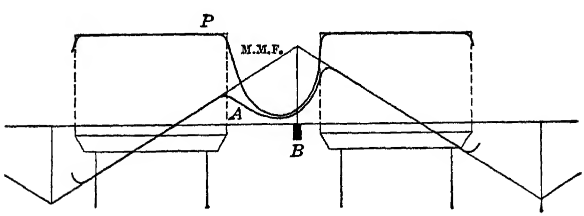


FIG. 149.—Calculation of flux distribution due to armature m.m.f.

PROBLEMS

1. The machine specified in Problem 2, Chap. IV, has a brush lead of two commutator segments. How many ampere-turns per pole are necessary to compensate the demagnetizing action of the armature at full rated load?

2. Compute the field intensity at the pole tips due to the cross-field of the armature of the machine referred to in Problem 1 when it is delivering full-load current. Assume that the field excitation is sufficient to give an average air-gap density of 40,000 lines per sq. in.

3. Under the conditions assumed in Problem 2, what additional field excitation is necessary to neutralize the demagnetizing component of cross-magnetizing action?

4. The armature resistance of the machine specified in Problem 2, Chap. IV, is 0.0245 ohm. When the machine is delivering full-load current, a voltmeter connected to a double pilot brush spanning adjacent commutator segments gives a reading of 7.5 volts. What is the field intensity in the region occupied by the conductors corresponding to the segments touched by the pilot brushes?

CHAPTER VI

OPERATING CHARACTERISTICS OF GENERATORS

103. Service Requirements.—The lamps, motors, or other translating devices supplied with electrical energy from a distribution circuit may be connected to the supply mains in *parallel*, in *series*, or in *series-parallel*, as shown diagrammatically in Fig. 150*a*, *b*, and *c*.

Parallel connection (*a*) is used when the individual units constituting the load on the system are designed to operate with a constant difference of potential between their terminals, and in such a system any individual load unit can be disconnected

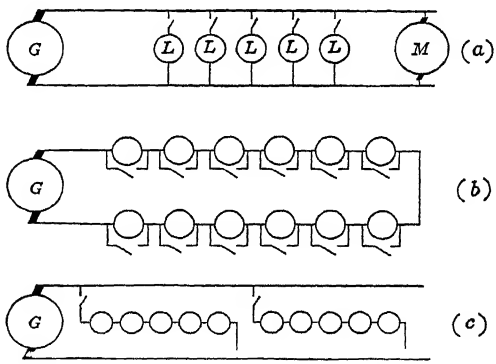


FIG. 150.—Parallel, series and series-parallel circuits.

without interfering with the operation of the remaining units; as examples of this class of service may be mentioned the use of incandescent lamps for interior lighting, and street railways operating with constant difference of potential between trolley and rail.

Series connection (*b*) is used principally in arc-lighting and in series-incandescent lighting of streets and alleys, where each lamp requires the same current. If an individual load unit in this system is to be cut out of service, it cannot be disconnected

as in the parallel system, but must be short-circuited by a "jumper" connection, as in Fig. 150*b* above, in order to preserve continuity of service in the remaining units.

Series-parallel distribution (*c*) is merely a combination of the other two, and requires no special consideration, once the principles underlying the series and the parallel systems are understood. A common example of series-parallel connection is found in the lighting circuits of a trolley car, where several strings of five 110-volt lamps in series are each connected across the 550-volt supply circuit.

If in a constant potential (parallel) system there are N lamps (or other load units) each taking i amperes, the total current supply is Ni amperes, and the power consumed, neglecting the loss in the line, is NiV watts, where V is the line voltage. In a constant current (series) circuit, if there are N lamps each requiring v volts and i amperes, the total impressed voltage must be Nv volts and the power consumed will be Nvi watts, again neglecting the line loss. In the first case (parallel system) the line conductor must have a cross-section at the supply end capable of carrying Ni amperes, and gradually tapering off as the end of the line is approached. In the second case (series system) the line conductor will be of uniform cross-section from end to end, since the current is everywhere the same, but the difference of potential between the supply lines will be large at the generator end (G) and will gradually decrease as the distance from the generator increases. The parallel system requires a much greater weight of copper in the line than the series system, but this disadvantage is offset by the fact that the high voltage required in a series circuit of any considerable power limits the use of series circuits to outdoor service. Thus, if 125 arc lamps, each consuming approximately 50 volts, are connected in series, the total voltage consumed by the lamps will be 6250 volts; adding to this the voltage consumed in overcoming the resistance of the line, the e.m.f. required at the generator will be of the order of 7000 to 8000 volts, or much too high for safety in indoor service.

Although the parallel system of distribution is ordinarily called the constant-potential system, it will be readily apparent that the difference of potential between the conductors will vary more or less from point to point, becoming less as the distance from

the generator increases, because of the drop of potential due to the resistance of the line wires. This drop of potential due to ohmic resistance may be reduced to any desired amount by increasing the cross-section of the conductor, but it is clear that a limit is set by the rapidly increasing cost of the conductors. If the lamps (or other translating devices) are grouped at a distance from the generator, the voltage at the lamps may be kept constant, irrespective of the current in the feeder circuit, provided the voltage of the generator is raised, as the load increases, to a sufficient extent to compensate the drop of potential in the line.

104. Characteristic Curves.—In view of the various types of service requirements described above, it becomes important to investigate the characteristic behavior of the usual types of generators in order to determine the kind of service to which each is adapted. Probably the simplest way to study and compare the several kinds of machines is to construct *characteristic curves* which show the relations between the variables involved in the operation of the machine. For example, the *external characteristic* of a generator is a curve showing the relation between terminal voltage, plotted as dependent variable, and external (line) current, plotted as independent variable. Other characteristic curves are discussed in following articles.

105. Regulation.—In the case of a generator, the terminal voltage at full load is generally different from that at no load. The difference between the two values is then a measure of the closeness with which the machine regulates for constant voltage; the difference is called the voltage *regulation*. In order to make this measure a perfectly definite one, so that machines of different makes and sizes may be compared, the Standardization Rules of the American Institute of Electrical Engineers define the *percentage* voltage regulation (or simply the regulation) as the *difference between the full-load and no-load voltages, divided by the full-load voltage, and multiplied by* 100.[1]

[1] The Standardization Rules define percentage regulation as "the percentage ratio of the change in the quantity occurring between the two loads to the value of the quantity at either one or the other load, taken as the normal value." Inasmuch as the full-load voltage is usually considered the normal voltage, it would be used as the divisor in obtaining the percentage regulation, hence the definition given above.

Similarly, if a machine is designed to regulate for constant current, as an arc-light generator, its regulation would be computed in like manner by dividing the difference between full-load and no-load currents by the full-load current. The speed regulation of a motor, engine, turbine, etc., would be similarly defined in terms of speeds at full-load and at no-load.

106. Characteristic Curves of Separately Excited Generator.—The following symbols will be used:

E = generated e.m.f.
V = terminal voltage
r_a = resistance of armature, including brushes and brush contacts
r_f = resistance of field winding
R = resistance of external load circuit
i = current taken by load
i_f = current in field winding
n_f = field turns per pair of poles
n = speed of rotation, r.p.m.

No-load Conditions.—Under no-load conditions, the armature being driven at its rated speed, the relation between the e.m.f. generated in the armature winding and the exciting current in the field winding is given by the magnetization curve discussed in Chap. IV (see Fig. 102). Since the generated e.m.f. is given by the equation

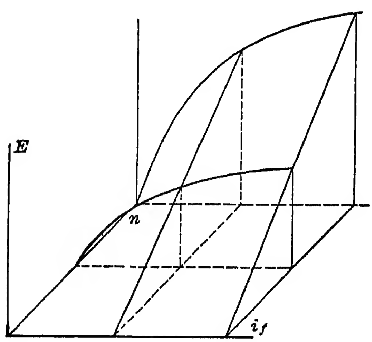


FIG. 151.—Effect of variation of speed upon magnetization curve.

$$E = \frac{p}{a} \cdot \frac{\Phi Z n}{60 \times 10^8}$$

it follows that if Φ is kept constant (by keeping i_f constant), the generated e.m.f. will be directly proportional to the speed. If, then, E, i_f, and n are plotted along three axes of coordinates, there will result a surface of the kind illustrated in Fig. 151. Sections of the surface cut by planes parallel to the (E, n) plane are straight lines whose slope increases as the distance of the section from the (E, n) reference plane increases

—at first rapidly, then more slowly. Sections cut by planes parallel to the (E, i_f) plane are magnetization curves corresponding to the speed represented by the distance of the secant plane from the origin of coordinates.

External Characteristic. Load Conditions.—With the connections shown in Fig. 152, let the machine be driven at its rated speed, the field excited by a constant current i_f, and the brushes set with an angular lead α most favorable for good commutation. The line current (which is here the same as the armature current)

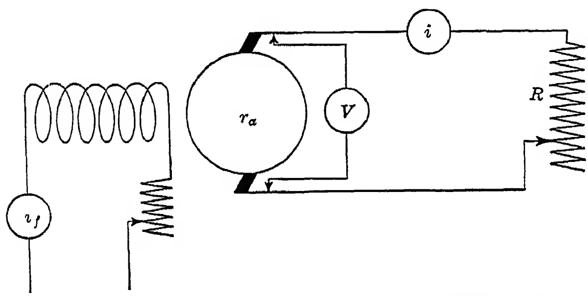


FIG. 152.—Connections for determining no-load characteristics. Separate excitation.

will vary as the external load resistance R is varied, and the terminal voltage V will be less than the generated e.m.f. E by ir_a volts, the latter being consumed in the internal resistance of the armature. That is,

$$V = E - ir_a \tag{1}$$

In this expression r_a comprises not only the resistance of the armature winding itself, but the resistance of the brushes and their connections, including the contact resistance between commutator and brushes. While the resistance of the armature is constant when the steady working temperature has been reached, the contact resistance is not constant, but varies approximately inversely as the current; that is, the total drop of potential at the contact surface between commutator and brushes is approximately constant, and is of the order of 2 volts with ordinary grades of carbon brushes, provided the current per sq. in. of contact area does not exceed 45 amperes (or 5 to 7 amperes per sq. cm.).

In the case of copper brushes, which are used only with low-voltage machines, the contact drop is of the order of 0.04 volt with current densities ranging from 65 to 160 amperes per sq. in. (10 to 25 amperes per sq. cm.).

Strictly, therefore,

$$V = E - ir_a' - \Delta e \qquad (2)$$

where r_a' is the constant part of the armature resistance, and Δe is the drop over the variable contact resistance. In what follows this correction is ignored for the sake of simplicity.

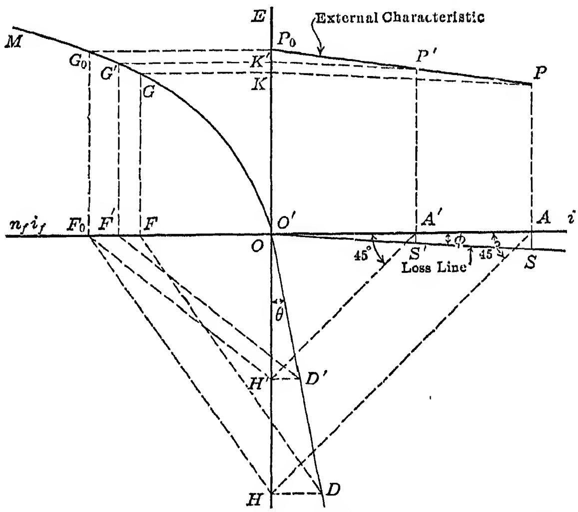


FIG. 153.—Construction of external characteristic of separately excited generator.

In Fig. 153 let $O'M$ be the magnetization curve plotted with ampere-turns per pair of poles $(n_f i_f)$ as abscissas and generated e.m.f. (E) as ordinates. If the excitation is adjusted by means of the field rheostat (Fig. 152) until it has the value represented by the abscissa OF_0, the generated e.m.f. will be $F_0G_0 = OP_0$, and this will also be the terminal voltage when the machine is running on open circuit $(i = 0, R = \infty)$.

On closing the external circuit through a load R of such resistance that a current of i amperes is drawn from the machine, the terminal voltage drops below its no-load value; (1) because the demagnetizing action of the armature reduces the useful flux Φ, and therefore also the generated e.m.f. E; (2) because of

the drop of potential, ir_a, in the armature. In Fig. 153 let OA represent to scale any value of load current (i) and draw AH at an angle of 45 deg. so that $OH = OA = i$; or what amounts to the same thing, lay off a scale of line current along OH as well as along OA. At the point H draw a horizontal line HD of length

$$HD = \frac{\alpha Zi}{180\,a} = \text{demagnetizing ampere-turns per pair of poles}$$

to the same scale previously adopted in drawing the magnetization curve M. If the straight line OD is then drawn, the intercept $H'D'$ corresponding to any other value of line current such as $OA' = OH'$ will represent to scale the demagnetizing ampere-turns per pair of poles, since that quantity is directly proportional to the armature current for a fixed setting of the brushes. Subtracting HD from the fixed field excitation OF_0, the remainder, OF, is the net excitation, and the corresponding value of generated e.m.f. is $FG = OK$; the subtraction can be effected graphically by joining points H and F_0 and drawing DF parallel to HF_0; or by transferring the length HD to F_0F by means of a pair of dividers.

Corresponding to the current $i = OA$, the drop of potential in the armature will be $ir_a = AS$, where AS is laid off to the scale of voltage previously chosen in drawing curve M. If the line OS (sometimes called the *loss line*) is then drawn, the intercept $A'S'$ corresponding to any other value of current such as OA' will represent to scale the ohmic drop due to that current.

Since $FG = OK$ is the generated e.m.f. corresponding to the current OA, the terminal voltage can be found by subtracting AS from OK; this is accomplished graphically by drawing KP through the point K parallel to OS until it intersects the ordinate through A in the point P. The latter point is then on the *external characteristic* of the machine. Additional points, such as P', are readily found by the construction indicated in the figure.

This method is subject to small errors because of the fact that it neglects a possible demagnetizing effect due to cross magnetization (Chap. V) and also due to the short-circuit currents in the coils undergoing commutation. The former may be taken into account, if necessary, by slightly increasing the angle θ.

It will be observed that the form of the external characteristic P_0P is dependent upon the form of the magnetization curve $O'M$, as well as upon the angles θ and φ. There will be a different

characteristic corresponding to each setting of the field excitation, OF_0. The student will find it very instructive to run through the construction using a value of field excitation such that the point G_0 falls on, or slightly below, the knee of the magnetization curve.

Applications of Separately Excited Generator.—The construction of Fig. 153 shows that the separately excited generator has an inherent tendency to regulate for constant voltage. The external characteristic is necessarily drooping, but the change in voltage from no load to full load can be made quite small by keeping down the demagnetizing action of the armature and by designing the armature winding to have a sufficiently low resistance to limit the ohmic drop to a small percentage of the rated voltage. For these reasons the separately excited generator is suitable for constant potential service, but is seldom used in heavy power installations for the reason that equally satisfactory results can be secured from self-excited machines of the shunt and compound types; but in laboratory and commercial testing the use of separate excitation is often very convenient and is frequently used.

107. Effect of Speed of Rotation on the External Characteristic.—For a given value of the excitation and, therefore, of the flux, the generated e.m.f. will be proportional to the speed. In case the machine is operated at a speed other than the rated speed, curve $O'M$ of Fig. 153 must be replaced by a new curve whose ordinates bear the same relation to those of the original curve that the new speed bears to the rated speed. This is shown in the three-dimensional diagram of Fig. 154. The latter figure also shows that the locus of the external characteristics for varying values of speed (but with a fixed value of field exciting current) is a wedge-shaped surface, OP_0P.

108. Load Characteristic.—By a load characteristic is meant a curve showing the relation between terminal voltage (as ordinates) and field excitation (as abscissas), subject to the condition that the current supplied to the load is constant. If this load current happens to be zero, the curve becomes the familiar no-load characteristic or magnetization curve, OM, Fig. 155. Now suppose that the resistance R (Fig. 152) of the external circuit is varied, and that the field excitation is so adjusted that the current is maintained at its normal full-load value, $i = I$. The

terminal voltage will then be $V = IR$, represented in Fig. 155 by the ordinate OA. At no load this e.m.f. would require an

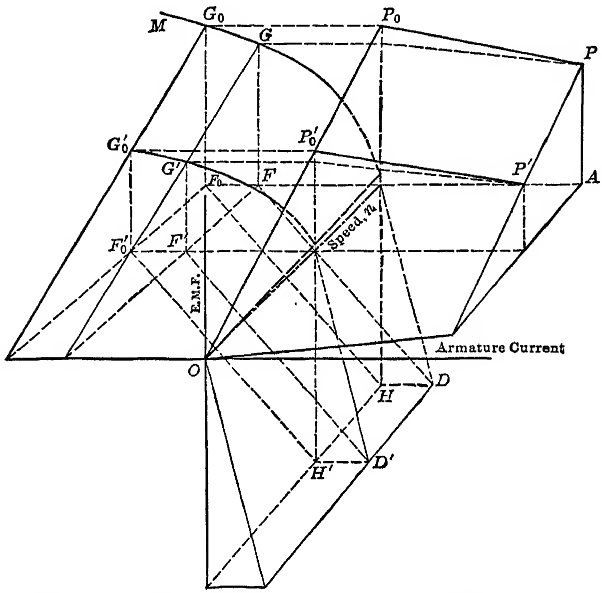


FIG. 154.—Effect of variation of speed upon external characteristic of separately excited generator.

excitation $AB = OC$, but under the assumed conditions the generated e.m.f. must be greater than OA by an amount DF, where $DF = Ir_a$, that is, the e.m.f. required to be generated to yield a terminal voltage $V = OA$ is DG, corresponding to an excitation OG. Finally, because of armature demagnetizing effect, the field excitation must be still further increased to OK, where

$$GK = (AT)_d$$

An ordinate through K then intersects AB (extended) in the point P, which point accordingly lies on the load characteristic corresponding to full-load current.

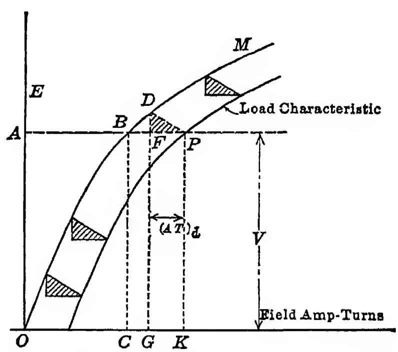


FIG. 155.—Construction of load characteristic, separately excited generator.

Since DF and $GK = FP$ remain constant in magnitude as long as $i = I$, the load characteristic is the same in shape as the no-load characteristic, but shifted downward and to the right by the constant length DP, as shown by the series of shaded triangles.

This construction is not strictly accurate, for with increasing excitation the demagnetizing component of cross-magnetization becomes greater, especially if the iron is saturated; in other words, FP should increase in magnitude as the curve rises. Further, remembering that the coefficient of dispersion, ν, is itself not constant, but that it increases with increasing excitation (X increasing, equation (34), Chap. IV), it is clear that this feature will contribute to a further increase in FP.

109. The Armature Characteristic.—It is evident from Fig. 155 that if the terminal voltage is to be maintained constant for all values of the load current, the excitation must be increased as the load increases. Thus, an increase in the current from $i = 0$ to $i = I$ requires an increase of excitation from OC to OK in order to maintain a terminal voltage equal to OA. The curve showing the relation between field excitation (as ordinate) and load current (as abscissa), under the condition of constant terminal voltage is commonly called the *armature characteristic* (Fig. 156) though a better name would be "regulation curve."

110. Characteristic Curves of the Series Generator.—*External Characteristic.*—

Let E = generated e.m.f.
V = terminal voltage
i = current in the circuit
r_a = resistance of armature winding, including brushes and brush contacts
r_f = resistance of series field winding
n_f = series field turns per pair of poles
R = resistance of external circuit
n = speed in r.p.m.

Since the same current (i) flows through the armature, the field winding, and the load circuit (Fig. 157), it follows that an increase of load causes an increase of excitation and therefore also of generated e.m.f., the speed of rotation being kept constant

at its rated value. The external characteristic will have the form of curve III, Fig. 158. If, then, to the ordinates of curve III there be added the ordinates of the loss line, curve II will result. Curve II is the *internal characteristic,* showing the relation

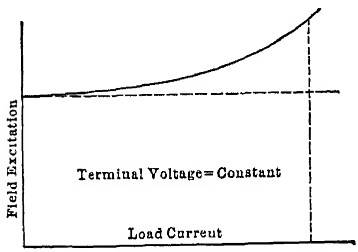


Fig. 156.—Armature characteristic or regulation curve.

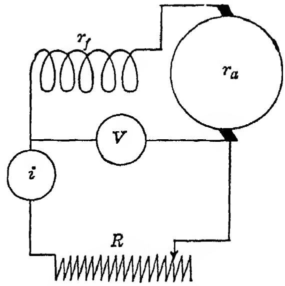


Fig. 157.—Connections of series generator, determination of external characteristic.

between the internally generated e.m.f. and the armature current. This follows because

$$E = V + i(r_a + r_f) \tag{4}$$

If there were no armature reaction, curve II would be the magnetization curve of the machine; but in the actual machine, where armature reaction exists, the magnetization curve (1) is displaced from curve II in the manner indicated in the figure. Thus, of the excitation OA required to produce terminal voltage AP and generated e.m.f. AG, a part, DG, is required to balance the demagnetizing component of armature reaction. The remainder, or OF, is then responsible for the generated e.m.f., hence D is a point on the magnetization curve.

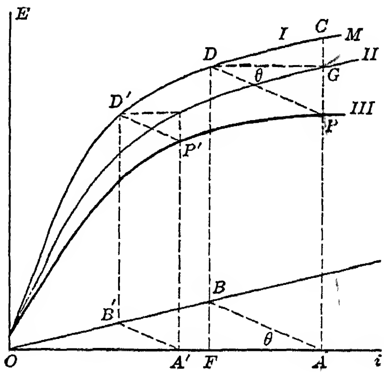


Fig. 158.—External characteristic of series generator.

Repeating this process to find other points, such as D', it will be seen that the size of the triangle PDG must be altered from point to point. But since PG and DG are both proportional to

the current, i, their ratio, and consequently the slope of DP, will remain constant. This leads to the following simple construction for obtaining the external characteristic from the given magnetization curve of the machine.[1]

The demagnetizing effect AF corresponding to any current $i = OA$ is given by

$$\frac{\alpha Z i}{180a} \cdot \frac{1}{n_f}$$

to the scale of current, and the length FB is equal to $i(r_a + r_f)$ to the scale of volts. These two lengths, when laid off as in Fig. 158,

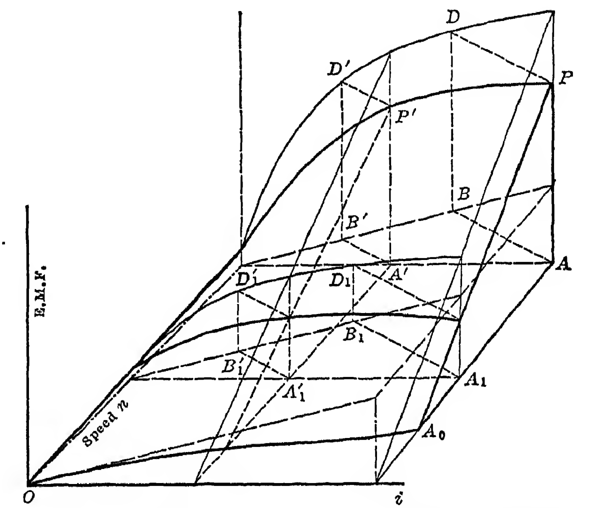


FIG. 159.—Effect of variation of speed upon external characteristic of series generator.

locate the point B, therefore also the line OB. To find a point P' on the external characteristic corresponding to current OA', draw $A'B'$ parallel to AB, through B' draw $B'D'$ vertically until it intersects curve M in D', and draw $D'P'$ parallel to $B'A'$ until it intersects the ordinate through A' in the point P'.

111. Dependence of the Form of the Characteristic upon Speed.—Variation of the speed of a series generator affects the magnetization curve in exactly the same manner illustrated in Fig. 151 for the case of the separately excited generator. Thus,

[1] This is the same construction given in Arnold's Die Gleichstrommaschine, Vol. I.

in Fig. 159, the surface bounded by $OD'D$ is the locus of the magnetization curves for various values of speed laid off along the speed axis n. Corresponding to each magnetization curve there will be an external characteristic constructed as in Fig. 158, and the locus of all such external characteristics will be a surface indicated by the heavy lines, as OA_0P. The intersection of this surface with the base (n, i) plane is a curve OA_0, which shows the relation between speed and current when the machine is short-circuited $(V = 0)$.

112. Condition for Stable Operation.—Referring to Fig. 157, it will usually be found that when the machine is driven at its rated speed there is a critical value of the load resistance R above which the machine will fail to generate, or to "build up." When the load resistance has been lowered slightly below this critical value, the terminal voltage and current will at first rise rapidly and then more slowly until a condition of equilibrium is reached, but between the initial and final condition the machine is in a state of unstable electrical equilibrium. Further reduction of R will cause the current and e.m.f. to change, but there is no further evidence of instability.

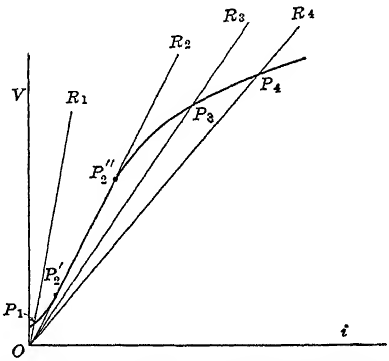


FIG. 160.—Condition for stable operation of series generator.

The reason for this behavior will be evident from a consideration of Fig. 160, in which the curve represents the external characteristic of the generator. It is evident from Fig. 157 that Ohm's law must hold for the external circuit, or

$$V = iR$$

This is the equation of a straight line through the origin, the slope of the line being proportional to R. Thus, OR_1, OR_2, OR_3, etc., correspond to successively smaller values of R, and these lines are characteristic of the external circuit. Since the points representing simultaneous values of V and i must satisfy the characteristics of both generator and external circuit, the point of equilibrium will be at their intersection. Thus, when the external

resistance is high, as R_1, the terminal voltage and current will be represented by the coordinates of point P_1. When $R = R_2$, the line OR_2 coincides for a greater or lesser range with the external characteristic, hence there is unstable equilibrium between the points P'_2 and P''_2. Values of R such as R_3 and R_4 will give stable equilibrium at points P_3 and P_4, respectively.

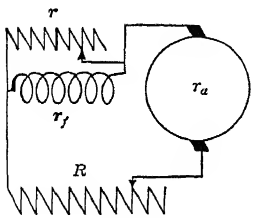


FIG. 161.—Diagram showing connections of constant current generator.

113. Regulation for Constant Current.—Inspection of the construction of Fig. 158 will show that the external characteristic of a series generator will droop more and more as the armature reaction and internal voltage drop become greater. In fact, by purposely exaggerating the magnitudes of these two quantities, the characteristic may be made to bend over to such an extent as to become nearly vertical, in which case the machine has an inherent tendency to regulate for constant current. Such a characteristic is desirable for series arc circuits, and some of the older types of Brush arc-light generators were designed on this principle.

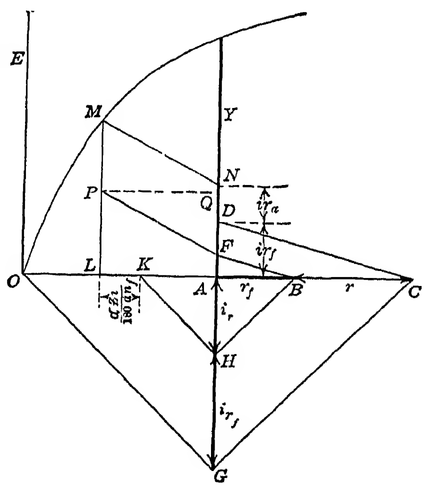


FIG. 162.—Regulation for constant current, graphical solution.

Another method of regulating a series generator to make it deliver a constant current is to provide a variable resistance shunted around the series-field winding, as in Fig. 161. It is clear that a portion of the exciting current is then by-passed around the winding, thereby cutting down the generated e.m.f. Every variation of the resistance of the load must then be accompanied by a corresponding change of the regulating shunt. The relation between the resistance of the regulating shunt and the corresponding terminal voltage of the machine is shown in a simple manner in Fig. 162.

Let OM be the magnetization curve, with abscissas equal to the current in the field winding (instead of ampere-turns), and let $OA = i$ be the constant current that the machine is required to develop; in other words, the external characteristic is to be the vertical line AY. Lay off AB to any convenient scale equal to the resistance (r_f) of the series field winding, and to the same scale lay off BC equal to any arbitrarily selected value of the resistance (r) of the regulating shunt. AB will be fixed in magnitude while BC will be variable. Vertically upward from A set off $AD = ir_f =$ constant, also $DN = ir_a =$ constant, and downward from A set off $AG = OA = i =$ constant.

The total current, i, will divide between r_f and r in such a manner that

$$i_r + i_{r_f} = i \tag{5}$$

and

$$\frac{i_r}{i_{r_f}} = \frac{r_f}{r} \tag{6}$$

Hence, if C and G are joined by a straight line and BH is drawn parallel to CG, point H will divide $AG = i$ into two parts, such that $AH = i_r$ and $HG = i_{r_f}$. Joining G and O, and drawing HK parallel to GO, OK will be the current through the series-field winding. Setting off $KL = \dfrac{\alpha Z i}{180 a n_f}$, or the equivalent demagnetizing current of the armature, OL will be the net excitation of the machine, and LM the corresponding generated e.m.f.; if MP is the ohmic drop in the machine, or $i\left(r_a + \dfrac{rr_f}{r + r_f}\right)$, the terminal voltage will be $V = LP = AQ$. To find MP graphically, proceed as follows: Connect C and D and draw BF parallel to CD; then $DF = i\dfrac{rr_f}{r + r_f}$ and $NF = i\left(r_a + \dfrac{rr_f}{r + r_f}\right)$. Therefore, point P is found by joining M and N and drawing FP parallel to MN. Finally, therefore, AQ is the terminal voltage corresponding to the shunt resistance $r = BC$. Other points may be found by exactly similar construction.

It will be evident from the above discussion that the voltage of a series generator can also be controlled by varying the position of the brushes, thereby changing angle α and affecting the length KL in Fig. 162.

Applications of Series Generator.—The characteristic curves of the series generator show that in general an increase of current drawn by the load causes a corresponding, but not strictly proportional, increase of voltage; but that it is possible to make the machine regulate for constant current output by the use of suitable auxiliary devices. Accordingly the series generator is adapted to series arc lighting service, and is still used to some extent for that purpose, but it has been practically superseded by later developments in the art; series arc circuits are now usually supplied with current from mercury arc rectifiers which are in turn supplied from alternating-current circuits through constant-current transformers. In Europe, series wound generators, regulated for constant-current service, find a limited application in the Thury system of long distance transmission of power (see Art. 121).

If a series machine is designed with an unsaturated magnetic circuit, both the magnetization curve and the external characteristic may be made to be nearly straight lines through the origin, so that the machine will develop a terminal voltage practically proportional to the current output. This feature makes the machine useful as a booster, in the manner explained in Art. 192, Chap. XI.

114. Characteristics of the Shunt Generator.—*Open-circuit Conditions.*—Let

E = generated e.m.f.
V = terminal voltage
i_a = armature current
i = external or line current
i_s = shunt-field current
r_a = armature resistance
r_s = shunt-field resistance, including regulating rheostat
R = resistance of external load circuit
n_s = field turns per pair of poles
n = speed in r.p.m.

When the load or receiver circuit of a shunt generator is disconnected, as in Fig. 163, the armature and shunt field constitute a simple series circuit identical with that of Fig. 157. It is there-

fore easily seen that variation of the shunt-field rheostat will give rise to changes in V and i_s in the manner already discussed in the case of the series generator. There is, however, this difference, that the high resistance of the shunt-field winding will limit the flow of current (i_s) to values that are small compared with the current carrying capacity of the armature, therefore the observed readings of V under the conditions assumed will not differ appreciably from E, the total generated e.m.f. Thus, with the external circuit open

$$E = V + i_s r_a \cong V \tag{7}$$

since both i_s and r_a are small. Moreover, the small flow of current in the armature means negligibly small armature reaction, hence the relation between V and i_s will be closely represented by the magnetization curve of the machine.

Since $i_s = \frac{V}{r_s}$, the effect upon the terminal voltage of changing the value of r_s will be given by a construction identical with that of Fig. 160; generally there will be a region of unstable equilibrium in the building up of the generated e.m.f.

The External Characteristic.—The form of the external characteristic showing the relation between terminal voltage V and

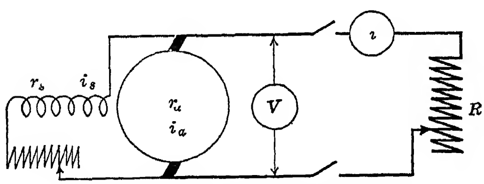


FIG. 163.—Connections of shunt generator. Determination of external characteristic.

line current i can be determined by the following method:[1] From Fig. 163 it is evident that

$$E = V + i_a r_a \tag{8}$$
$$i_a = i + i_s \tag{9}$$
$$i = \frac{V}{R} \tag{10}$$
$$i_s = \frac{V}{r_s} \tag{11}$$

The relation between the generated e.m.f., E, and the field excita-

[1] Franklin and Esty, Elements of Electrical Engineering, Vol. I.

tion, $n_s i_s$ (ampere-turns per pair of poles), is given by the magnetization curve, M, Fig. 164. If the machine is running on open circuit (R $= \infty$), let the resistance of the field rheostat be so adjusted that the excitation is represented by OF_0, the generated e.m.f. then being F_0L; this will then be nearly equal to the terminal voltage on open circuit, neglecting the small drop ($i_s r_a$) in the armature. The line ON is then the "field resistance" line, its slope being $\frac{V}{n_s i_s} = \frac{r_s}{n_s}$ that is, proportional to r_s; it corresponds to the lines OR_1, OR_2, etc., of Fig. 160. It will be clear that coordinates of all points on the line ON will represent simultaneous values of terminal voltage and field excitation.

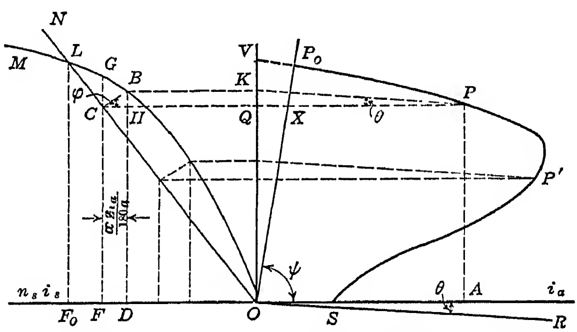


FIG. 164.—Construction of external characteristic of shunt generator.

Now let the external circuit be closed, R being so adjusted that a moderate current will flow. Then, even were the excitation to remain constant, as in the separately excited generator, the terminal voltage would fall because of the ohmic drop in the armature winding, *i.e.*, from equation (8)

$$V = E - i_a r_a \tag{12}$$

But, in the case of the shunt machine, a decrease of terminal voltage is accompanied by a proportional decrease of excitation, since $i_s = \frac{V}{r_s}$, hence, when there is an appreciable load current flowing, the flux and the generated e.m.f. are reduced, thereby causing a further reduction of V. It is clear, therefore, that the greater the load current the less will be the terminal voltage, and that the drop of terminal voltage will be greater in the shunt machine than in the separately excited machine, other things being equal.

Suppose that the load has been increased to such a value that the terminal voltage has fallen to the value OQ, Fig. 164; the problem is then to locate the point P on the horizontal line CQP such that P is a point on the external characteristic.

Through Q draw the horizontal line QC intersecting ON in C; then OF represents to scale the new value of $n_s i_s$. If there were no armature reaction, the ordinate FG would be the total generated e.m.f. corresponding to the excitation $n_s i_s = OF$, and therefore, since

$$i_a r_a = E - V$$

CG would represent to scale the value of $i_a r_a$. But since armature reaction does exist, the net excitation is less than OF by an amount FD, where

$$FD = \frac{\alpha Z i_a}{180a}$$

Actually, therefore, the net excitation is OD, and the generated e.m.f. is $E = BD$

$$\therefore i_a r_a = E - V = BD - HD = BH$$

and

$$\frac{BH}{CH} = \frac{BH}{FD} = \frac{i_a r_a}{\dfrac{\alpha Z i_a}{180a}} = \frac{180 a r_a}{\alpha Z} = \tan \varphi = \text{constant.}$$

In other words, when a point C on the field resistance line ON has been fixed, point B is found by drawing through C a line CB making the fixed angle φ with the horizontal.

Through B draw the horizontal line BK, and through K draw KP at an angle θ with the horizontal, this angle being so chosen that

$$\tan \theta = r_a$$

to the scale of the figure. It follows, then, that

$$QP = \frac{KQ}{\tan \theta} = \frac{i_a r_a}{r_a} = i_a$$

hence P is a point on a curve whose ordinates are terminal voltage (V) and whose abscissas are total armature current (i_a). Corresponding values of line current (i) can then be found by subtracting i_s; this can be done graphically by drawing the line

OP_0 at an angle ψ such that $\tan \psi = r_s$ to the scale of the figure; for it is easily seen that

$$QX = \frac{OQ}{\tan \psi} = \frac{V}{r_s} = i_s$$

Hence $AP = V$ and $XP = i$ are simultaneous values of terminal voltage and line current. Similar construction will then serve to locate additional points, such as P', as illustrated in Fig. 164.

It will be observed that the external current at first increases as the load resistance is lowered, but that eventually a critical point is reached beyond which a further lowering of the external resistance causes the current to decrease rapidly. The terminal voltage falls steadily throughout the entire process, becoming zero when the machine is dead short-circuited ($R = 0$); under this condition of complete short-circuit the external current is not zero but has a small value OS due to the fact that residual magnetism generates a small e.m.f. that is entirely consumed in driving the current through the armature resistance. It might be inferred from these facts that a shunt generator can be short-circuited without danger, but this is not the case except in very small machines; for the critical point at which the line current begins to decrease is generally far beyond the current-carrying capacity of the armature, and the winding will burn out before the current has had time to decrease to a safe value.

Applications of the Shunt Generator.—Within the range of load determined by consideration of safe heating limits, a properly designed shunt generator, when driven at rated speed, has an inherent tendency to regulate for nearly constant voltage. The drop in voltage between no load and full load, while somewhat greater than in a separately excited generator of the same design, may be kept quite small. Shunt generators are therefore suitable for constant potential circuits where the load is so close to the generator that the drop of potential in the line resistance is of no consequence; for example, a shunt generator may be used as an exciter for the field circuit of an alternator. Control of terminal voltage to meet changes of load may be made by hand adjustment of the field rheostat if the changes of load are not too rapid; and if the load is of a rapidly fluctuating character, the terminal voltage can still be kept practically constant, or can

even be made to rise with increasing load, by an automatic device such as the Tirrill regulator (see Art. 124).

A shunt generator is well adapted to such service as charging storage batteries; for as the battery approaches the condition of full charge its e.m.f. rises and so tends to reduce the amount of charging current; but because of the drooping characteristic of the shunt machine, the decrease of current is accompanied by an increase of the generator e.m.f., hence there is an automatic balance which prevents the battery from discharging back through the generator.

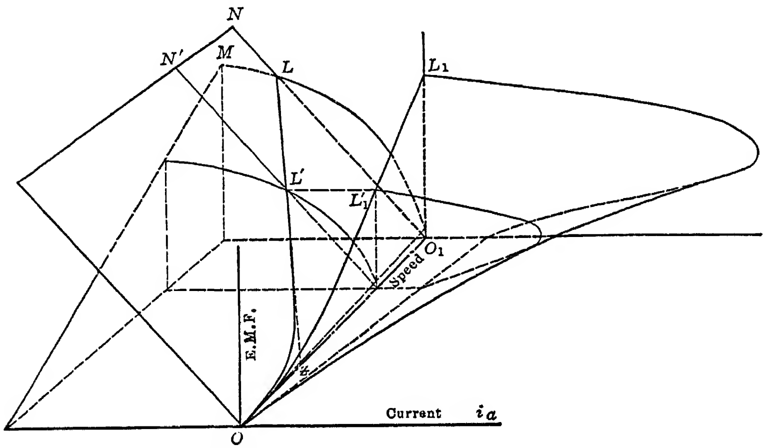


FIG. 165.—Effect of variation of speed upon external characteristic of shunt generator.

115. Dependence of the Form of the Characteristic upon Speed.—The diagram of Fig. 164 was drawn subject to the condition that both the speed and resistance of the shunt circuit remain constant. A change in speed (r_s remaining the same) will alter the form of the characteristic, and the new relations between V, i_a and n can be most easily shown by a three-dimensional diagram such as Fig. 165. In this figure the surface OO_1M, drawn to the left of the speed axis, is the locus of the magnetization curves for various values of speed, and to each magnetization curve there will correspond a characteristic L_1, L'_1, etc., the locus of which has the peculiar tubular form shown in the diagram.

If the shunt field resistance has a constant value, the locus

of the field resistance lines (ON) will be the plane OO_1N, and the intersection of this plane with the magnetization surface OO_1M will be a curve $OL'L$. The projection of this curve on the (V,n) plane will give curve OL'_1L_1, which shows the relation between terminal voltage and speed when the generator is operating on open circuit. If there were no residual magnetism, curve $OL'L$ would not pass through the origin, but would intersect the speed axis in a point Z; that is, if there were no residual magnetism, the machine would fail to build up for any speed below a critical speed, OZ.

116. Dependence of Form of Characteristic upon Resistance of Shunt Field Circuit.—If the speed of a shunt generator is kept constant and the resistance of the field circuit is varied by means

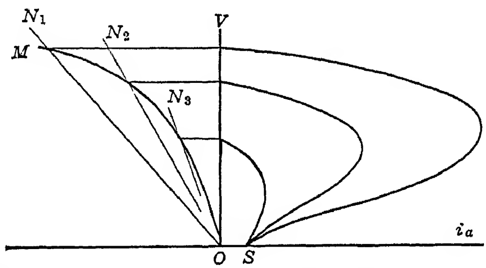


Fig. 166.—Effect of variation of shunt regulating resistance upon external characteristic of shunt generator.

of the regulating rheostat, the size and shape of the characteristic will be affected in the manner shown in Fig. 166. OM is the magnetization curve corresponding to the speed at which the machine is driven, and ON_1, ON_2, etc., are the field resistance lines corresponding to the setting of the rheostat. The construction of the several characteristics is carried out in the manner described in connection with Fig. 164.

117. Approximate Mathematical Analysis of Shunt Generator Characteristics.—It will be evident from the preceding articles that the form of the external characteristic is in all cases dependent upon that of the magnetization curve, hence an equation representing the relations between the variables V, i and n must be a function of the equation representing the magnetization curve. Since the latter would necessarily involve a relation between B and H for the iron comprising part of the magnetic cir-

cuit, and since such a relation is entirely unknown, the best that can be done is to represent the magnetization curve by an empirical equation originally due to Froelich, and which can be written in the form

$$E = \frac{ani_s}{b + i_s} \tag{13}$$

where a and b are constants, and n is the speed. If the speed is held constant, this equation represents a hyperbola, with asymptotes as shown in Fig. 167. A suitable choice of the constants a and b will make this hyperbola agree very well with the actual magnetization curve within the working range of the machine,

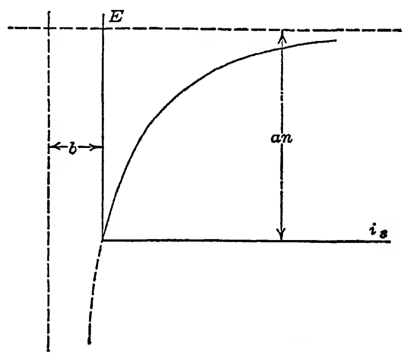


FIG. 167.—Empirical form of magnetization curve.

but it cannot be made to follow the irregularities in the actual curve at low magnetizations, and it does not take account of residual magnetism unless the equation is modified in the manner discussed in Art. 79, Chap. IV, by shifting the origin of coordinates to the right.

If both numerator and denominator of (13) are multiplied by n_s (the number of field turns per pair of poles), it becomes

$$E = \frac{an \times \text{ampere-turns per pair of poles}}{bn_s + \text{ampere-turns per pair of poles}}$$

which can also be written

$$E = \frac{An \times \text{ampere-turns per pole}}{B + \text{ampere-turns per pole}}$$

Under load conditions, the number of ampere-turns per pole (or per pair of poles) to be substituted in the above equation is the

net number obtained by subtracting the armature demagnetizing ampere-turns from the number of ampere-turns supplied by the field winding.

Using Froelich's equation in the form of equation (13), and *ignoring armature reaction,* we have the following relations (see equations 8, 9, 11):

$$E = \frac{ani_s}{b + i_s} = V + i_a r_a$$

$$i_a = i + i_s$$

$$i_s = \frac{V}{r_s}$$

whence

$$V = \frac{an\,V}{br_s + V} - \left(i + \frac{V}{r_s}\right) r_a \tag{14}$$

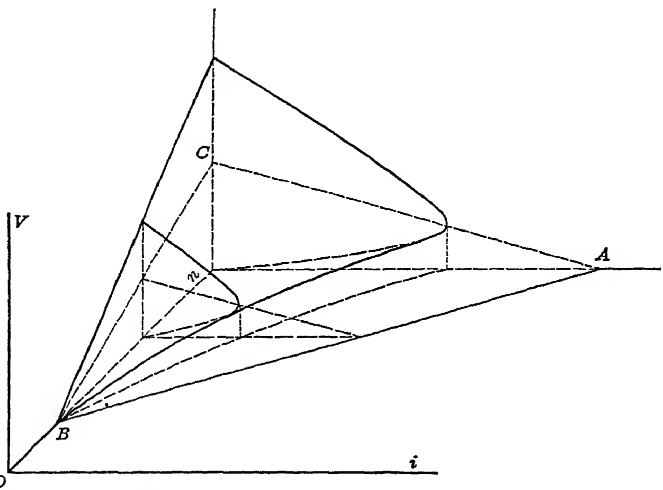


Fig. 168.—Idealized external characteristic surface of shunt generator.

Solving for V, and simplifying by assuming that r_a is small compared with r_s (*i.e.*, $r_a + r_s \cong r_s$) there is obtained

$$V = \frac{an - br_s - ir_a}{2} \pm \sqrt{\left(\frac{an - br_s - ir_a}{2}\right)^2 - ir_a r_s b}$$

$$= \frac{an - br_s - ir_a}{2} \pm$$

$$\tfrac{1}{2}\sqrt{[an - (\sqrt{ir_a} - \sqrt{br_s})^2]\,[an - (\sqrt{ir_a} + \sqrt{br_s})^2]} \tag{15}$$

This is an equation of the second degree between the three variables V, i and n, hence it represents a surface (Fig. 168) plane sections of which are conics or straight lines. Moreover, the surface is symmetrical with respect to the plane

$$V = \frac{an - br_s - ir_a}{2} \tag{16}$$

which is shown in the figure as ABC.

If in the general equation of the surface, (15), there is substituted $i = 0$, it is seen that

$$V = an - br_s \qquad \text{and} \qquad V = 0; \tag{17}$$

the first of these two equations represents open-circuit conditions, the second, short-circuit conditions. From the former it appears that $V = 0$ when $n = \frac{br_s}{a} = OB$, hence OB is the critical speed below which the machine would fail to build up if residual magnetism were not present.

Inserting in the general equation for V the condition $n =$ constant, there will result the equation of the external characteristic corresponding to the chosen value of speed. It is obvious that there are two values of the current (i) which will reduce the radical to zero, hence the characteristic intersects the plane ABC in two points; one of these values of current is $\frac{(\sqrt{an} - \sqrt{br_s})^2}{r_a}$ the other is $\frac{(\sqrt{an} + \sqrt{br_s})^2}{r_a}$. Between these values of current the radical becomes imaginary, hence the theoretical external characteristics are hyperbolas.

Example.—A 4-pole, 120-volt shunt generator, rated at 25 kw. at 900 r.p.m. has a magnetization curve represented by the equation

$$E = \frac{180 \times \text{amp.-turns per pole}}{2000 + \text{amp.-turns per pole}}$$

The field winding has 800 turns per pole, and a hot resistance, not including the field rheostat, of 20 ohms. The armature has a simplex wave winding of 194 face conductors, one turn per element, and the armature resistance is 0.0245 ohm. The angle of brush lead is equivalent to 2 segments of the commutator. If the field rheostat is adjusted to give an open-circuit voltage

of 125 volts, what will be the terminal voltage when the machine is delivering full load current?

Solution.—The field ampere-turns at no load can be found from the relation

$$125 = \frac{180 \times \text{amp.-turns per pole}}{2000 + \text{amp.-turns per pole}}$$

whence

$$\text{field amp.-turns per pole at no load} = 4550$$

$$\text{shunt field current at no load} = \frac{4550}{800} = 5.7 \text{ amp.}$$

$$\text{shunt field resistance (total)} = \frac{125}{5.7} = 22 \text{ ohms}$$

$$\text{resistance in field rheostat} = 22 - 20 = 2 \text{ ohms.}$$

Again,

$$\text{full load current} = i = \frac{25{,}000}{120} = 208 \text{ amp.}$$

$$\text{no. of commutator segments} = S = \frac{194}{2} = 97$$

$$\alpha = 2 \times \frac{360}{97} = 7.43 \text{ deg.}$$

$$\text{demag. amp.-turns per pole} = \frac{\alpha Z i_a}{360a} = \frac{7.43 \times 194}{360 \times 2}\left(208 + \frac{V}{22}\right)$$

$$= 416 + \frac{V}{11}$$

$$\text{drop in armature} = i_a r_a = \left(208 + \frac{V}{22}\right) \times 0.0245 = 5.1 \text{ (nearly)}$$

$$\text{generated e.m.f.} = V + 5.1$$

$$\therefore V + 5.1 = \frac{180\left(800\frac{V}{22} - 416 - \frac{V}{11}\right)}{2000 + 800\frac{V}{22} - 416 - \frac{V}{11}}$$

whence

$$V = 110.5 \text{ volts or } 20.7 \text{ volts.}$$

The larger value of V is the full load voltage, the smaller value that which corresponds to a line current of 208 amperes after passing around the reverse bend in the external characteristic.

118. Characteristic Curve of the Compound Generator.—*Long Shunt Connection.*—The drop in terminal voltage between no load

and full load inherent in a shunt generator can be compensated, partially or wholly, or even overcompensated, by the addition of a series field winding excited by the armature current. As has been previously pointed out, the object of over-compounding is to keep the line voltage constant at a distant point, at or near the center of distribution of the load, the increase in the voltage at the generator terminals being consumed by the resistance of the line. In a general way the compound-wound generator (Fig. 169) may be considered as combining the rising character-

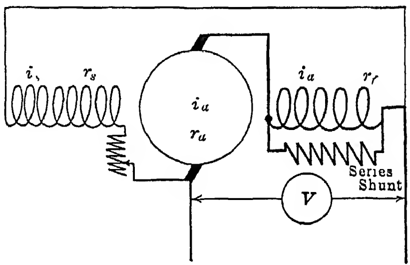


FIG. 169.—Connections of long-shunt compound generator.

istic of the series generator with the drooping characteristic of the shunt generator, the slope of the resulting curve depending upon the relative slopes of the components.

Starting as before with the magnetization curve, $O'M$, Fig. 170, the external characteristic can be constructed in a simple manner as follows:

Let ON be the shunt field resistance line, its equation being

$$V = \frac{r_s}{n_s}(n_s i_s); \tag{18}$$

the intersection of this line with the magnetization curve determines a point L whose ordinate is (very nearly) the terminal voltage at no-load. Assuming that we are dealing with an over-compounded machine, let $F_1G_1 = AP$ be the terminal voltage corresponding to a value of $i_a = OA$ (the latter being supposed to be known). The field excitation due to the shunt turns will then be given by OF_1, and the total field excitation will be OF_2, where $F_1F_2 = n_f i_a$ is the excitation supplied by the series turns. The net excitation, or OF, will be less than this by an amount $FF_2 = \dfrac{\alpha Z i_a}{180a}$ = demagnetizing ampere-turns per pair of poles,

hence the e.m.f. actually generated in the armature is FG. The difference between FG and F_1G_1, or GH, must, therefore, be the drop in the armature and series field, or $i_a(r_a + r_f)$. Summarizing,

$$F_1F_2 = G_1G_2 = n_f i_a$$
$$G_2H = \frac{\alpha Z}{180\,a} \cdot i_a$$
$$GH = (r_a + r_f)i_a$$

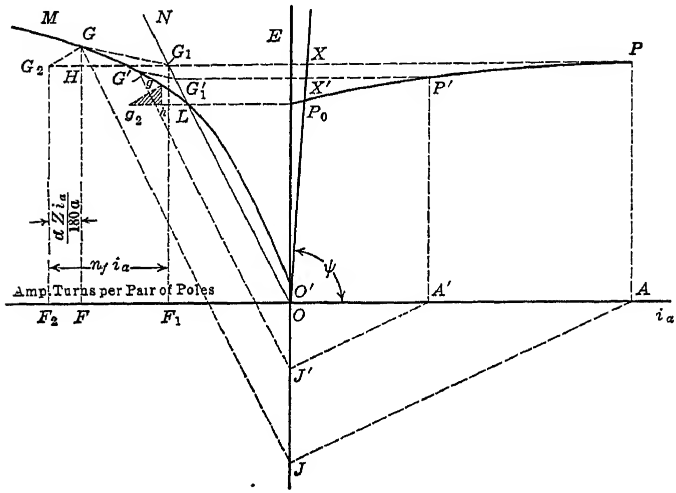


Fig. 170.—Construction of external characteristic of compound generator.

It follows, therefore, that all three sides of the triangle GG_1G_2 are proportional to i_a, hence their ratios remain fixed no matter what the value of i_a may be, and the angles at the vertices of the triangle are also constant. In particular, the slope of the side GG_1 is constant, and its length is proportional to i_a. It follows, therefore, if it is desired to find a point P' on the characteristic curve corresponding to any other current OA', it is only necessary to fit a line $G'G'_1$ between curve M and the line ON, in such manner that $G'G'_1$ is parallel to GG_1 and so that the ratio of their lengths shall be equal to the ratio of OA' to OA. This suggests the following construction:

Through the point G draw GJ parallel to ON, J being on the axis of ordinates (prolonged downward); and join J with A. Draw $A'J'$ parallel to AJ, draw $J'G'$ parallel to JG until it intersects curve M in G', and through G' draw $G'G'_1$, parallel to GG_1,

the point G'_1 being on the line ON. Draw G'_1P' horizontally until it intersects ordinate $A'P'$ in the point P', then the latter is a point on the curve showing the relation between V and i_a.

If it is desired to show the relation between V and i, it is necessary to subtract the value of i_s from each corresponding value of i_a. To this end draw the line OX at an angle ψ with the horizontal, so that $\tan \psi = r_s$ to the scale of the drawing. Then PX and $P'X'$ are the values of i corresponding to i_a equal to OA and OA', respectively.

It will be observed that this method presupposes a knowledge of the coordinates of at least one point on the characteristic. The chief value of the construction lies in the clearness with which it shows the intimate relation between the magnetization curve and the external characteristic. Thus, it becomes evident from the diagram that the external characteristic will approach a linear form more and more nearly as the magnetization curve flattens out (that is, when the figure LG_1G approaches triangular shape). If the point L is so placed that it is below the knee of the curve, the external characteristic will become more and more convex (from above), the curvature being considerable for small values of the load and less pronounced as the load increases. Considerations of this kind become important when the specifications of a machine call for a compounding that shall not depart from a linear relationship by more than a limited amount.

A study of Fig. 170 shows that a reduction in the number of series turns will shorten G_1G_2 and will make the characteristic more nearly horizontal. If it is desired to make the terminal voltage at full load equal to that at no load, that is to say, to make the machine flat-compounded, GG_1G_2 (assumed to correspond to full load conditions) will degenerate to Lgg_2, where $g_2gh = G_2GH$. It is clear from the construction that the characteristic of a flat-compounded generator, like that of an over-compounded generator, cannot be exactly a straight line because of the curvature of the magnetization curve.

Short-shunt Connection.—In this case the current through the series winding is $i = i_a - i_s$, hence the construction of Fig. 170 is not strictly applicable. But at or near full load the difference between i and i_a will be relatively small, especially in large machines, so that the above method will give a very close approximation to correct results.

Application of Compound Generators.—The compound generator is used more than any other type of direct-current generator for the reason that its characteristics adapt it to all classes of service that require constant voltage at the point of application of the load. The rise of voltage from no load to full load can be made to have any desired value, from zero up to any reasonable limit, so that the drop of potential in the transmission circuit can be compensated. Compound generators are used for supplying current to incandescent lamps, and for heavy power service such as electric railways, and in general for motor drives requiring direct-current supply at constant voltage.

119. The Series Shunt.—In practice it is quite common to design the series field windings of compound generators with a sufficient number of turns to produce the maximum per cent. of compounding that may reasonably be specified. If a lesser degree of compounding is required, the magnetizing effect of the series winding is then reduced by connecting a shunt across the terminals of the series winding, as indicated in Fig. 169. This shunt is made of German-silver strip, and serves to by-pass a portion of the main current. The total current will divide between the series winding and its shunt in the inverse ratio of their resistances, provided the load current remains steady or changes very slowly; but in cases where the load is subject to sudden fluctuations, as in street railway service, the current will not divide properly between the inductive series winding and the non-inductive series shunt while the current is changing, for the reason that the self-inductance of the series winding tends to retard any change of current therein, while the non-inductive series shunt introduces no such effect. For example, if there is a sudden increase of load current, the series winding will have less than its proportionate amount of the total current during the time the change occurs, thereby delaying the increase of excitation required to build up the voltage to the larger value demanded by the increased load. The remedy for this difficulty is to make the series shunt inductive by threading the resistor material of which it is made through a laminated iron core somewhat in the manner indicated in Fig. 31, Chap. I; to secure proper division of the total current through the series winding and its diverting shunt under all conditions, the self-inductance

of the latter must bear the same ratio to its ohmic resistance as do the corresponding quantities in the case of the series winding itself.

120. Connection of Generators for Combined Output.—When the load on a circuit exceeds the capacity of a single generator, one or more additional units must be connected to supply the excess. Thus, in a constant-current system in which the voltage varies in proportion to the load, additional generators must be connected in series when the voltage limits of the machine or machines already in service have been reached. Similarly, in constant-potential systems, additional generators must be put in parallel with those already in service when the safe current-carrying capacity of the latter has been reached.

121. The Thury System.[1]—The series system, in which series-wound generators, regulated to give constant current, are connected in series, has thus far found no application in the United States, save in those now absolete plants in which constant-current motors were supplied from high voltage arc circuits. But in Europe this system has been developed to a high state of perfection through the work of M. Thury, who has installed a number of plants operating on this principle, most of them in Switzerland, Hungary and Russia.[2]

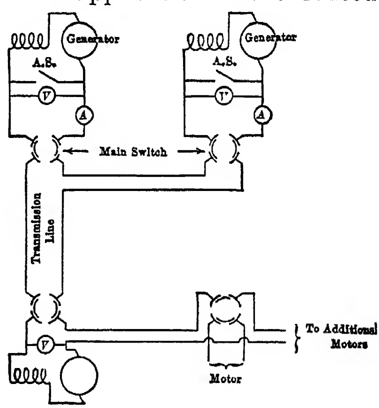


FIG. 171.—Diagram of connections, Thury system.

In the Thury system the series-wound generators are driven at constant speed and the current is kept constant by a regulating device which shifts the brushes (though it may be arranged to vary the speed). The regulating device is actuated by a solenoid through which the main line current flows. A sufficient number of generators are connected in series to develop the voltage required by the load. The load consists of series motors, also connected in series, which

[1] See also Chap. VII.

[2] Jour. Institution of Electrical Engineers, Vol. XXXVIII, p. 471. Electrical World, Vol. LXIII, No. 11, p. 583 (1914).

in turn drive generators (generally alternators) for the supply of current at the receiving or distributing end of the line. In other words, the system is generally used for the transmission of power over considerable distances, as distinguished from merely local distribution. The individual generators are grouped in pairs, each pair being driven by a water wheel (or other prime mover). In plants now operating, the maximum voltage per commutator is about 3600 volts, though it is possible to design machines of this type to give 5000 volts at 500 amperes, or 5000 kw. per pair of generators. The maximum line voltage in use at the Moutiers-Lyon plant is 57,600 volts, though a new installation projected for transmission from Trollhätten (Sweden) to Copenhagen—a distance of 200 miles—contemplates the use of a line voltage of 90,000 volts.[1]

The starting and stopping of the generators in the Thury system is very simple. Each generator is equipped with an ammeter, a voltmeter, and a switch, as in Fig. 171, the switch being so arranged that when it is in the "off" position the generator is short-circuited, and when in the "on," or running position, the machine is in series with the line. To start the machine, the switch being in the off position, the prime mover is brought up to normal speed, and the switch thrown to the running position when the ammeter reads normal current. To shut down the machine this process is reversed.

Since all of the machines are in series when under load, the potential of the circuit rises from generator to generator, hence the machines must be carefully insulated from earth to prevent breakdown of the insulation.

122. Parallel Operation of Generators.—

(*a*) *Series Generators.*—Series generators connected in parallel as in Fig. 172 will not operate satisfactorily, for the reason that if one of them suffers a momentary reduction of its output (as from a momentary drop in speed), both its voltage and current will be reduced, as may be seen from the form of the characteristic, Fig. 158. The other machine will then assume the part of the load dropped by its mate and its current and

[1] Electrical World, Vol. LXI, p. 294, 1913.

voltage will accordingly rise; the increased voltage will cause a further increase of current, hence an additional increment of load is thrown on the second machine and the load, current and voltage of the first will be still further reduced. This process will tend to continue until the first machine is driven as a motor by the second machine; moreover, the direction of rotation of the former will reverse when it becomes a motor, so that the connecting rod of its driving engine will tend to buckle. Series generators connected in parallel are, therefore, in unstable equilibrium, there being no inherent tendency to bring about a proper division of the load between the two units under consideration. This is a consequence of the rising form of the external characteristic of the series generator.

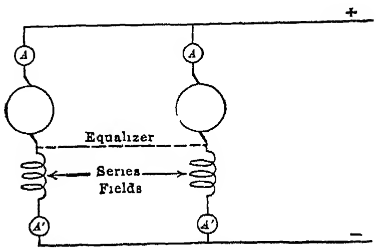


Fig. 172.—Series generators in parallel.

The natural instability of series generators in parallel can be overcome by the *equalizing connection* shown in Fig. 172 as a dashed line. The effect of this connection is to put the series field windings in parallel with each other. If then one machine assumes more than its proper proportion of the total load, the excess current will divide between the two field windings, thereby raising the excitation and voltage of the machine which has momentarily dropped its load, hence automatically readjusting the division of the load.

(*b*) *Shunt Generators.*—The drooping form of the external characteristic of the shunt generator shows that if such a machine drops its load, its voltage will automatically rise. Consequently if two shunt generators are connected in parallel, as in Fig. 173, their operation will be stable. Any tendency which causes one machine to lose its proper share of current, thereby shifting an equal amount of current to the other, will result in a rise of voltage of the first machine and a drop in the voltage of the second.

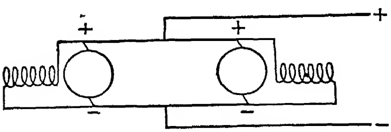


Fig. 173.—Shunt generators in parallel.

The original conditions will be restored, assuming that the prime movers are properly governed.

It is, of course, not necessary that the two (or more) generators thus connected in parallel should have the same ratings. But it is essential to good operation that the machines should divide the total load, whatever that may happen to be, in proportion to their ratings. Suppose, for instance, that two shunt

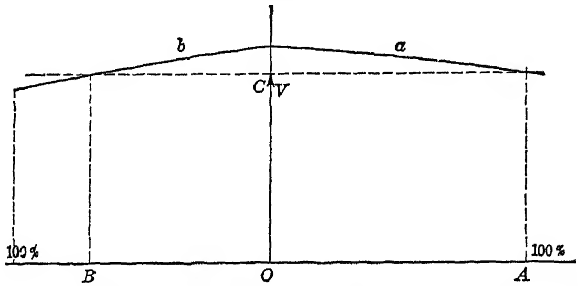


FIG. 174.—Division of load between shunt generators in parallel.

generators that are to be connected in parallel have external characteristics as shown in Fig. 174, curve (*a*) representing the characteristic of one machine, curve (*b*) that of the other. In this figure ordinates are plotted in volts and abscissas in per cent. of full-load current. Since the machines are in parallel, their terminal voltages must necessarily be equal, hence if the load is such that the terminal voltage is *OC*, machine (*a*) will deliver *OA* per cent. of its rated current and machine (*b*) *OB* per cent. The condition to be satisfied is that $OA = OB$ at all loads, therefore it follows that if the load is to divided at all times in proportion to the ratings, the characteristics when plotted in per cent. of full-load current, must be identical.

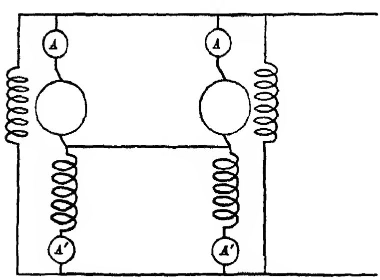


FIG. 175.—Compound generators in parallel.

(*c*) *Compound Generators.*—Inasmuch as the compound generator partakes of the characteristics of both shunt and series generators, two or more of them, if over-compounded, will operate satisfactorily in parallel only when the series fields are provided with the same equalizer

connection shown in Fig. 172. This is a consequence of the rising characteristic. But if the machines have drooping characteristics, that is, if they are under-compounded, the equalizer is not necessary. The diagrammatic scheme of connections of two compound generators in parallel is shown in Fig. 175. It is clear that if ammeters were connected as at A', they would not indicate the true current actually delivered by the machines to the external circuit, for the readings would be affected by the equalizing current. Thus, a heavily loaded generator might be

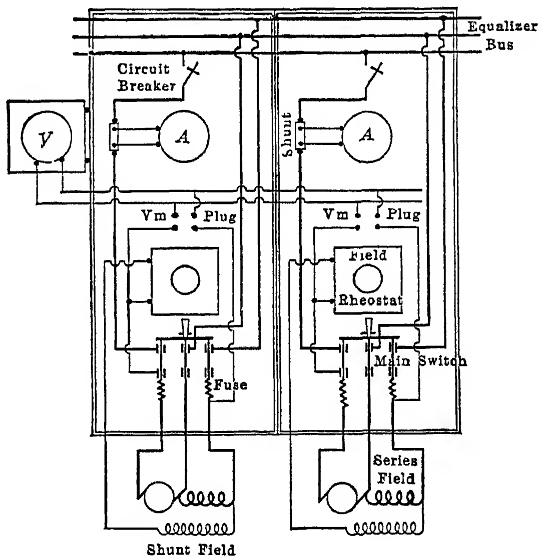


FIG. 176.—Diagram of switch-board connections, compound generators in parallel.

supplying an equalizing current of large magnitude to the other lightly loaded machines and at the same time the ammeter of the loaded machine would read low while that of the other machines would read high. For this reason the individual ammeters must be placed as at A, that is, in the lead that connects to the armature on the side *opposite* to the equalizing connection. For the same reason, if single-pole circuit-breakers are used, they should be placed in the same lead as the ammeters; thus, if two machines in parallel are each delivering full-load current and one of them should develop a momentary drop in speed, the heavy equalizing current might open its circuit-breaker, if incorrectly placed, with

the result that the entire load would be thrown on the other machine and so open its circuit-breaker also.

The complete switch-board connections of two compound generators are shown in Fig. 176. The main switch and the equalizer switch are usually combined in a triple-pole switch.

The process of paralleling a compound generator with one or more that are already running is as follows: The main switch of the incoming machine being open and its circuit-breaker closed, the prime mover is brought up to speed and the voltage of the incoming machine adjusted to equality with the bus-bar voltage by manipulation of the shunt-field rheostat; the main switch is closed, and proper division of the load is then secured, if necessary, by further adjustment of the field rheostat. To shut down a machine running in parallel with others, its load is shifted to the others by weakening its shunt-field current, and the main switch is opened when the ammeter indicates a small, or zero, current.

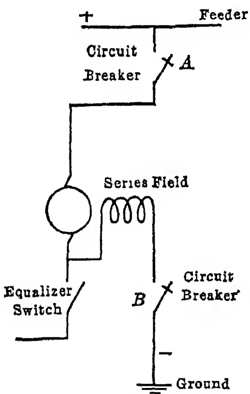


FIG. 177.—Diagram of connections of railway generator supplying grounded circuit.

If two compound generators are to divide the load in proportion to their ratings, their characteristics must obviously be identical in the manner explained in connection with shunt machines. Moreover, since the series fields are in parallel by virtue of the equalizer connection, the resistances of the series windings including the resistances of the respective equalizing leads, must be inversely proportional to the rated currents of the two machines. Neglect of this feature will result in a disproportionate division of load. For example, if the machines are at unequal distances from the switch-board, the resistance of the series field of the more remote machine will be unduly high because of the longer equalizing connection, and this machine will, therefore, not take its full share of the load.

The division of load between two over-compounded generators in parallel cannot be determined in the manner indicated in Fig. 174 for the case of shunt generators. For if the individual

characteristics of the machines are not the same, the adjustment of load between them is brought about by the equalizer connection, one machine carrying more than its normal series excitation, the other less. This has the effect of altering the external characteristics of both machines, as may be seen by referring to the construction of Fig. 170, the change being due to the modified value of the series ampere-turns. Using the construction of Fig. 174 in the case of over-compounded generators would make it appear that the machine having the more steeply rising characteristic would take the smaller load, whereas in reality the reverse is true.

The series field of a compound generator may be connected to either the positive or the negative terminal of the armature. In street-railway generators built by one well-known company the series field is connected on the negative, or grounded side; in this case it is not sufficient to use one single-pole circuit-breaker (A) on the positive or feeder terminal, but another circuit-breaker (B) must be put in the lead to the grounded bus, as shown in Fig. 177. For if circuit-breaker B were not present and the armature winding were to become grounded to the core, the short-circuit current through the armature and series field would hold up the excitation and maintain the short-circuit without the possibility of protection by circuit-breaker A.

123. Three-wire Generators.—Economy in the use of copper in distributing circuits for lighting and power dictates the selection of high voltage and moderate current, rather than low voltage and large current; but in incandescent lighting, lamps designed for 110 to 115 volts are more efficient than those operating at higher voltages. To get the benefit of the high efficiency 110–115-volt lamp and at the same time to obtain the copper economy of higher voltage, the three-wire system of distribution diagrammatically illustrated in Fig. 178 is extensively used. The individual lamps, small motors and other translating devices are connected between the outer wires and the middle or *neutral* wire, and larger motors, designed to operate on the higher voltage

of the system, are connected between the outer wires. In the early forms of three-wire systems, the splitting of the moderately high voltage between the outer wires was accomplished by using two generators in series (Fig. 178,) the neutral being tapped into their common junction.

A later arrangement, shown in Fig. 179, consisted of a main two-wire generator wound for the voltage between the outside wires, with a *balancer set* connected across the outside wires. If the load on the two sides of the system, that is, between neutral and outer wires, were exactly balanced, no current would flow in the neutral, and the neutral might then be omitted; this is sometimes done in 220-volt systems, the lamps

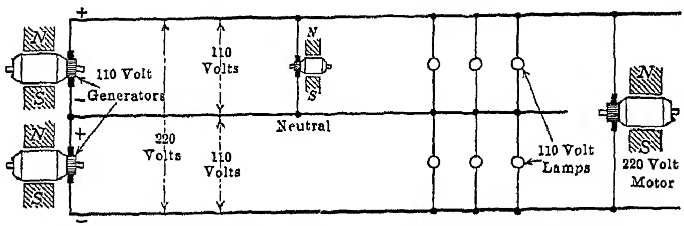


FIG. 178.—Three-wire system, two generators in series.

being connected in series in pairs and connected across the main wires. But if the load is not exactly balanced, the neutral will carry a current equal to the difference between the currents supplied to the two sides of the system. The attempt is always made to balance the system as completely as possible, but provision is usually made for an unbalancing of about 10 per cent., that is, 10 per cent. of full-load current in the neutral wire. When a system employing a balancer set becomes unbalanced, the voltage on the more lightly loaded side tends to be higher than on the more heavily loaded side; in this case, the machine on the side having the lighter load operates as a motor and drives the other as a generator; the latter then supplies current for the excess load on its side of the system, and thus automatically tends to balance the system. With perfect balance of load both machines of the balancer set operate as motors running without load.

Systems of the kind shown in Figs. 178 and 179 are open to the objection that they involve the use of more than one piece of running machinery and so require extra attendance and main-

tenance, in addition to being higher in first cost and lower in efficiency than a single machine of the same capacity. These objections are overcome by a system originally devised by Dobrowolsky, and shown diagrammatically in Fig. 180. A coil of wire, CED, wound on an iron core, is tapped into the main armature winding of the generator at the points C and D, 180 electrical degrees apart, that is, points that occupy homologous posi-

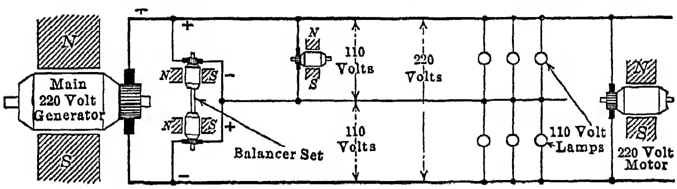


FIG. 179.—Three-wire system with balancer set.

tions with respect to poles of opposite polarity. The difference of potential between C and D is alternating, so that the coil is traversed by an alternating current which goes through one cycle (two alternations) per revolution per pair of poles; this alternating current is small because of the large self-inductance due to the iron core on which the coil is wound. The middle point

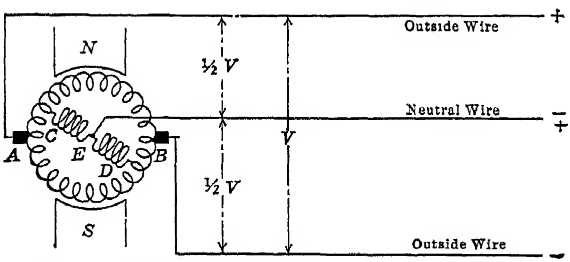


FIG. 180.—Three-wire generator with core mounted inside armature core.

of the coil, E, will have a potential midway between the potentials of C and D, and therefore also midway between the potentials of the brushes A and B, since the potentials of C and D are always symmetrically related to those of A and B, respectively. A tap brought out from the point E may then be used as the neutral wire of a three-wire system. In machines of this kind built by the General Electric Company, the coil CD is wound on a core that is mounted inside the armature core, and the connection from the middle point E to the outside circuit is made through

a single slip-ring mounted on the main shaft of the generator. The Burke Electric Company builds a three-wire generator in which the coil CD is wound in the same slots that carry the main armature winding, in the manner indicated in Fig. 181.

The balance coil CD may also be placed outside of the genera-

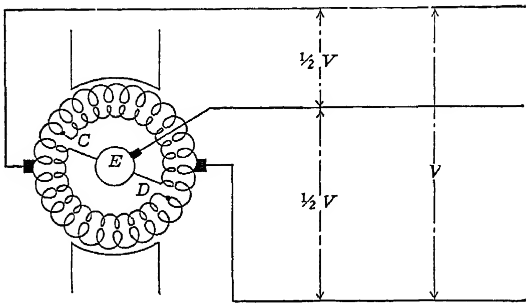


FIG. 181.—Three-wire generator with auxiliary winding in slots.

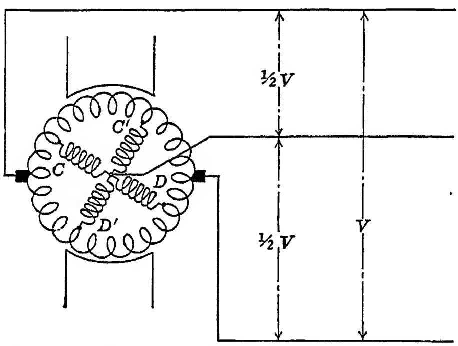


FIG. 182.—Three-wire generator with two coils tapped into armature winding.

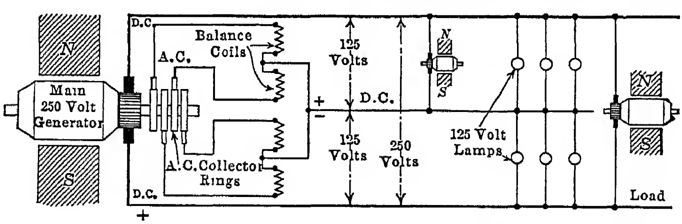


FIG. 183.—Three-wire generator with two auxiliary coils mounted externally

tor, connection to the armature winding being made in that case through two slip-rings; or two balance coils, connected to the armature winding as in Fig. 182, may be used. The alternating voltages between the points C, D and C', D' are 90 electrical

degrees apart, that is, one of them is a maximum when the other is zero, and *vice versa*. Fig. 183 shows the connections when two balance coils, mounted externally to the generator, are used; this is the standard construction used by the Westinghouse Electric and Manufacturing Company.

If three-wire generators are to be compounded, the series field winding must be in two equal parts, half of the turns being in series with one of the outer wires, the other half in series with the other outer wire, as in Fig. 184. If two or more three-wire generators are to be operated in parallel, two equalizer connections must be used, hence the main switch of a three-wire generator is usually constructed with four blades.

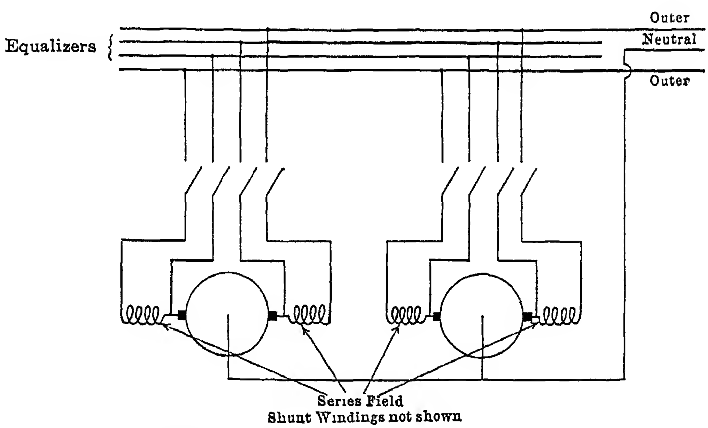


FIG. 184.—Diagram of connections of compound three-wire generators in parallel.

124. Tirrill Regulator.—It has been shown in preceding articles how the voltage of shunt and compound generators may be regulated either by manual adjustment of the rheostat in the shunt field circuit or by the automatic compounding effect of the series field winding. In lighting circuits where steady voltage is of the greatest importance, accurate and automatic regulation of voltage may be obtained by the use of the Tirrill regulator; this device makes it possible to maintain a steady voltage at the generator terminals irrespective of changes in the load or of fluctuations of speed, and also to compensate for line drop by increasing the generator voltage as the load increases.

The regulator maintains the desired voltage by rapidly opening and closing a shunt circuit connected across the terminals of the exciter field rheostat. The field rheostat is so adjusted that when the regulator is disconnected the generator voltage is about 35 per cent. below normal; on closing the regulator circuit the rheostat is short-circuited and the generator voltage rises. When the voltage reaches a predetermined value, the short-circuit around the rheostat is opened and the voltage again falls. The opening and closing of the short-circuit around the rheostat is so rapid that the voltage does not actually follow the changes of the field circuit resistance, but merely tends to do so, with the result that incipient changes of voltage are immediately checked.

An elementary diagram of connections of the regulator is shown in Fig. 185. The opening and closing of the by-pass around the exciter rheostat is accomplished by means of contacts on the armature of a differentially wound relay magnet of U

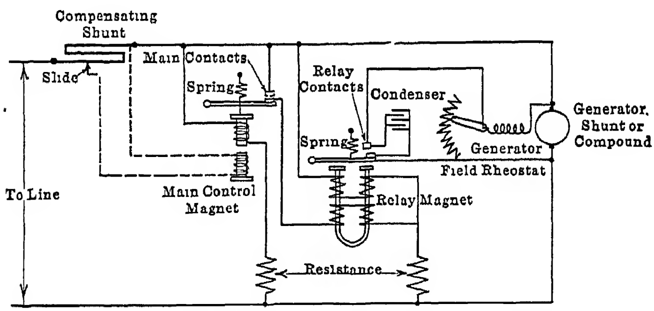


FIG. 185.—Diagram of connections of Tirrill regulator.

shape. One winding of the relay magnet is connected directly across the main bus-bars, in series with a current-limiting resistor; the other winding is also connected across the bus-bars, but through a pair of main contacts actuated by the main control magnet. The latter is wound with a potential coil connected directly across the bus-bars, and a current coil (which may or may not be used) whose magnetizing action opposes that of the potential coil.

The operation of the regulator is as follows: If the generator voltage falls, the current through the potential coil of the main control magnet is weakened and the spring closes the main contacts. Current then flows through both windings of the relay

magnet which is then demagnetized and the spring closes the relay contacts, thereby short-circuiting the field rheostat. As the voltage rises the armature of the main control magnet is again pulled down, the main contacts are opened, and the relay magnet is again energized, thus again inserting the rheostat in the field circuit. If the current coil of the main control magnet is used, its differential action will cause the voltage to rise higher before the main contacts are opened than would otherwise be the case, thus giving a compounding action. The degree of compounding

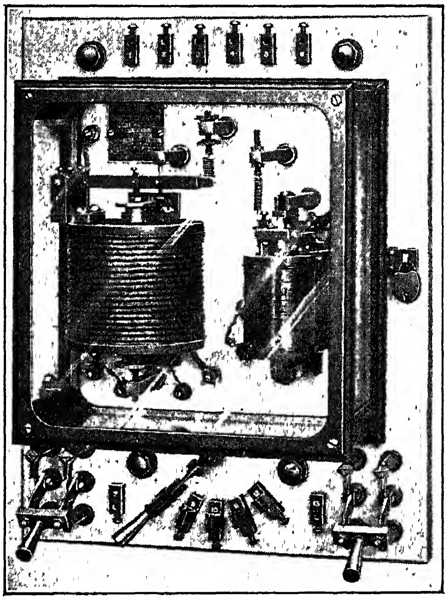


FIG. 186.—Voltage regulator, General Electric Co.

may be varied by means of the sliding contact on the compensating shunt with which the current coil is in parallel. The condenser shown in the figure is for the purpose of reducing sparking at the relay contacts. A perspective view of a simple regulator built by the General Electric Company is shown in Fig. 186.

When several compound generators of moderate capacity are worked in parallel, a simple regulator may be connected to one of them and the others allowed to "trail." The generator

provided with the regulator will take the fluctuations in load, and the load on the others will be equalized through the compound windings. Regulators are also built for controlling the voltages of two or more generators operating in parallel; instead of using a single relay magnet, from two to ten are employed, part of them serving to short-circuit sections of the field rheostat of one generator, the others performing a like function for the other machines. Regulators of this kind are suitable for two-wire or three-wire generators with shunt or compound windings, and will compensate for line drops up to 15 per cent.

In the case of very large machines, it is advisable to use separate excitation and to connect the regulators so that they act upon the exciter fields. Of course, in such a case the main control magnet would be actuated by the main bus-bar voltage and line current.

PROBLEMS

1. A 10-pole, 220-volt generator rated at 400 kw. at 200 r.p.m. has a magnetization curve represented by

$$E = \frac{540 \times \text{amp.-turns per pole}}{15{,}400 + \text{amp.-turns per pole}}$$

The armature has a simplex lap winding of 820 conductors, each element having one turn, and the brushes are advanced two commutator segments beyond the geometrical neutral axis. The armature resistance is 0.003 ohm. The shunt field winding has 575 turns per pole, and the resistance of the entire shunt circuit, not including the regulating rheostat, is 10.5 ohms.

If the field circuit is separately excited from 250-volt mains, how much resistance must be placed in series with the field winding to develop an open-circuit voltage of 250 volts at a speed of 220 r.p.m.?

2. The machine of Problem 1 is operated as a separately excited generator at 200 r.p.m. and with a field excitation sufficient to develop an open-circuit voltage of 240 volts. (*a*) What will be the terminal voltage when it is delivering 2000 amp. to the load? (*b*) If the excitation is now adjusted until the terminal voltage is 250 volts, the external circuit remaining unchanged in effective resistance, what will be the terminal voltage on opening the load circuit?

3. The separately excited generator of Problem 1 is run at a speed of 220 r.p.m. Find the field current required to produce a terminal voltage of 230 volts when the armature current is (*a*) zero; (*b*) 1000 amp.; (*c*) 2000 amp. Plot a curve showing the relation between field current and armature current.

4. A series generator has a resistance of 0.35 ohm and the armature demagnetizing amp.-turns per pole at full load amount to 6 per cent. of

the field amp.-turns per pole. The magnetization curve of the machine may be expressed by Froelich's equation, such that at a speed of 1170 r.p.m. an exciting current of 22 amp. develops an e.m.f. of 84 volts, while an exciting current of 11 amp. develops an e.m.f. of 56 volts. Find the terminal voltage when the machine is operated as a series generator at 1150 r.p.m. and is delivering a current of 25 amp.

5. The machine of Problem 1 is operated as a shunt generator at a speed of 220 r.p.m. and with a field resistance such that it gives an open-circuit voltage of 230 volts. Find its terminal voltage when the armature carries the rated full-load current of the machine.

6. The machine of Problem 1 is provided with a series field winding having $2\frac{1}{2}$ turns per pole, the total resistance of the entire series winding being 0.001 ohm. A shunt around the series winding reduces the series field current to 1000 amp. at full load. If the connections are such that the machine is a long-shunt compound generator, and the open-circuit voltage is 220 volts, what will be the terminal voltage when the rated full-load current of the machine flows through the armature? What is the resistance of the series shunt?

7. Two shunt generators, rated at 75 kw. and 150 kw., respectively, are adjusted until each has its rated open circuit voltage of 220 volts. The smaller machine has a 5 per cent. voltage regulation, while that of the larger is $2\frac{3}{4}$ per cent. The relation between terminal voltage and output is linear in both machines. If the machines are connected in parallel and the load amounts to 200 kw., what is the bus-bar voltage and how much load is carried by each machine?

8. A 6-pole armature has a simplex lap winding, not provided with equalizing connections. The measured resistance of the armature is 0.006 ohm. Due to irregular setting of the pole pieces and a slightly eccentric adjustment of the bearings, the e.m.fs. generated in the six armature circuits are, respectively, 220, 220.3, 220.7, 221.1, 220.5 and 220.2 volts. If the armature is delivering a current of 400 amperes to the load, what is the terminal voltage and how much current flows in each armature circuit?

*

CHAPTER VII

MOTORS

125. Service Requirements.—In the industrial application of the motor drive, there are three principal classes of service, characterized by *constant speed*, *adjustable speed*, and *variable speed*. Constant-speed motors, of which the shunt motor is an example, maintain an approximately constant speed at all loads when supplied from constant potential mains, and are used for such purposes as driving line shafting, fans, etc. In the case of adjustable speed motors, the speed can be fixed at any one of a large number of values between a minimum and maximum value, and when so set will remain substantially constant for all loads within the limits of the machine's capacity, the impressed voltage remaining constant throughout; motors of this kind are used, for example, in individual drives for machine tools. Variable-speed motors include those types in which the speed is inherently variable, changing as the load changes, with constant impressed voltage; examples of this class are the series motor and the cumulative compound-wound motor; their speed characteristics make them especially suitable for that class of service in which it is desirable to reduce the speed as the load increases, as in street railway and in hoisting service.

Intelligent operation of motors involves a knowledge of the relations between speed, torque (or turning moment), load (or output), and the electrical and magnetic quantities involved. These relations determine the operating or *mechanical characteristics*, which will be discussed for the different types of motors.

126. Counter E.M.F., Torque and Power.—It has been shown in Chap. II that when a current is sent into the winding of an armature which is under the influence of a magnetic field the individual conductors of the winding are subjected to a lateral thrust and that motion ensues. The immediate effect of this motion is to generate in the conductors an e.m.f. whose direction is

opposite to that of the current. This *counter-generated e.m.f.* is called the "*back e.m.f.*" or the "*counter e.m.f.*," and its magnitude is given by equation (7), Chap. II.

The effective development of torque in the case of a motor is dependent upon a proper space relation between the field flux and the armature current. If, for instance, the brushes are so set that the axis of the armature current coincides with the axis of field flux, as in Fig. 187*a*, there is no resultant tendency to rotation; but if the axes of armature current and field flux are at right angles to each other, as in Fig. 187*b*, the torque will be a maximum for a given current in the winding.

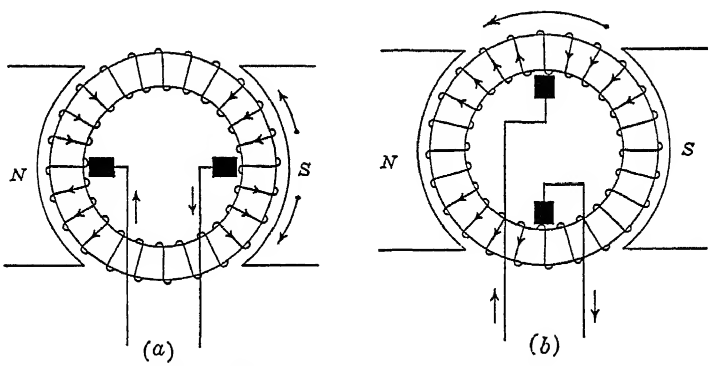


FIG. 187.—Effect of brush position on the torque.

Counter E.M.F.—In the case of a separately excited or of a shunt motor, the voltage impressed upon the armature terminals must be consumed in overcoming the back e.m.f. plus the ohmic drop due to the resistance of the armature winding and the brush contacts.

$$\therefore V = E_a + i_a r_a \tag{1}$$

where

$$E_a = \frac{p}{a}\frac{\Phi Z n}{60 \times 10^8}$$

In the series and long-shunt compound motor, there is an additional drop due to the resistance of the series field winding, hence

$$V = E_a + i_a(r_a + r_f) \tag{2}$$

In the case of the short-shunt compound motor the relation is

$$V = E_a + i_a r_a + i r_f \tag{3}$$

In general

$$V = E_a + i_a r' = \frac{p}{a} \frac{\Phi Z n}{60 \times 10^8} + i_a r' \tag{4}$$

or

$$n = \frac{V - i_a r'}{\Phi Z'} \tag{5}$$

where

r' = resistance of armature and circuits in series therewith

and

$$Z' = \frac{p}{a} \frac{Z}{60 \times 10^8} \tag{6}$$

It is also seen that

$$i_a = \frac{V - E_a}{r'} \tag{7}$$

an equation which is of importance in connection with the starting of motors, as explained later.

In equations (1), (2), (3), and (4), the term or terms representing the ohmic drop in the armature and any part of the field winding in series with it are quite small in comparison with the impressed voltage, within the safe limits of load; for if they were not, the loss of power represented by the product of the drop of potential and the current would injuriously lower the efficiency of the motor and would at the same time result in undue heating, thereby reducing the safe load of the motor. The ohmic drop in the armature, expressed in per cent. of the impressed voltage, is smaller the larger the machine. It follows, therefore, that to a first degree of approximation *the counter e.m.f. is practically equal and opposite to the impressed voltage.* This is a fundamental principle of all types of motors, including alternating-current motors at well as direct-current motors. Since most motors operate with constant impressed voltage, it follows that the counter e.m.f. is nearly constant within the working range for which the motor is designed; but it will be observed that for a given design the counter e.m.f. is proportional to the product of the flux per pole

and the speed, hence if the flux is constant (or nearly so) under running conditions, the speed of the motor will likewise be substantially constant; on the other hand, if the flux is variable under running conditions the speed will also be variable in nearly inverse ratio.

These conclusions should be compared with those readily deducible from the generator characteristics already discussed. Thus, in generators, the terminal voltage will be substantially constant if the flux is constant, provided the speed is held fixed, as in the case of separately excited, shunt and flat-compound machines; but where the flux varies, as in series generators, the terminal voltage also varies. It will be seen, therefore, that there is a reciprocal relation between generators and motors, voltage in the case of generators being the inverse of speed in motors.

Torque and Power.—Multiplying equation (4) by i_a, and transposing, there results

$$Vi_a - i_a^2 r' = E_a i_a \tag{8}$$

The term Vi_a represents the power supplied to the armature, and $i_a^2 r'$ is the power dissipated as heat in the ohmic resistance of the armature circuit. The difference between these two terms, or $E_a i_a$, must therefore be the amount of *mechanical power developed by the armature.* Not all of this developed power is available at the shaft or pulley, for some of it is lost in overcoming the friction of the bearings and brushes, windage (air resistance), and hysteresis and eddy currents in the armature core and pole faces.

If P = total mechanical power, in watts, developed in the armature,

T = torque in dyne-centimeters $\div 10^7$

$$\therefore P = E_a i_a = 2\pi \frac{n}{60} T \text{ watts} \tag{9}$$

or

$$T = \frac{60}{2\pi n} E_a i_a = \frac{60}{2\pi n} \frac{p}{a} \frac{\Phi Z n}{60 \times 10^8} i_a$$

$$= \frac{60}{2\pi} Z' \Phi i_a \tag{10}$$

where Z' has the meaning shown in equation (6).

The above unit of torque is inconvenient for practical application; expressing torque in kilogram-meters, pound-feet, and pound-inches, respectively,

$$\left.\begin{aligned} T &= \frac{60}{2\pi} \times \frac{10^7}{980 \times 10^3 \times 10^2} Z'\Phi i_a = 0.975 Z'\Phi i_a \text{kg.-m.} \\ &= \frac{60}{2\pi} \frac{10^7}{980 \times 453.6 \times 30.48} Z'\Phi i_a = 7.05 Z'\Phi i_a \text{ lb.-ft.} \\ &= 84.6 Z'\Phi i_a \text{ lb.-in.} \end{aligned}\right\} \quad (11)$$

It is clear from these equations that the torque is dependent only upon the flux and the armature current, and is independent of speed. It is to be understood, of course, that the brushes are properly set, in such a way that the axis of commutation is perpendicular to the axis of the flux, otherwise the above equations will not hold true

The four equations numbered (4), (5), (7), and (11), may be said to summarize in analytical form the physical facts involved in the operation of any motor of the usual types. These facts may be stated as follows:

Assume that the motor has been started in some suitable manner, and that it is running with only a small load, the source of supply being a circuit which maintains constant voltage at the motor terminals. Since the load, or mechanical output, is small by hypothesis, the electrical input need only be sufficient to supply the small power demanded by the load plus the losses in the motor itself; and since the losses must necessarily be only a small per cent. of the rated capacity of the motor, the input will be small under the conditions stated, and the armature current will be correspondingly small. The current is kept small by reason of the fact that the counter e.m.f. developed by the rotation of the armature through the field flux opposes the impressed voltage. The speed of the motor, which with the flux determines the counter e.m.f., automatically adjusts itself to such a value that the difference between the impressed voltage and the counter e.m.f. permits the flow of just enough current (see equation (7)) to develop the torque required to carry the load (see equation (11)). If now the mechanical load on the motor is

increased, the electrical power input to the motor must also increase, hence more current must be supplied from the line; but the current can increase only in case the counter e.m.f. is decreased, and this demands a decrease of the product of flux and speed. If the motor is a constant flux machine, the speed will therefore fall, a conclusion which seems entirely logical as a mere result of the increased load; the speed will again become constant at the lower value when the adjustment of the magnitude of the counter e.m.f. permits the flow of just enough current to develop the increased torque required by the increased load. The sequence of reactions in this case is very similar to what happens in a steam engine controlled by a fly-ball or inertia governor; an increase of load momentarily checks the speed, and the automatic response of the governor admits more steam until a condition of equilibrium is again established.

It is quite possible, however, to design a motor in such a way that an increase of load brings about an increase of speed. As was explained in the paragraph above, the increased mechanical load demands an increase in the amount of current supplied by the line, and this in turn requires a decrease in the counter e.m.f.; the latter can be decreased by decreasing the flux as well as by lowering the speed, consequently if the flux is caused to be decreased in a porportion greater than the necessary decrease in counter e.m.f., the speed will actually increase as a result of the increased load. This is the case that arises in the differentially compounded motor discussed more in detail in Art. 131.

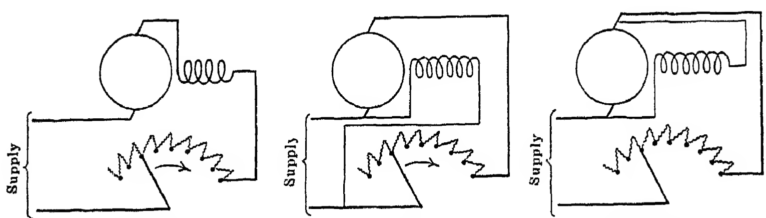
127. The Starting of Motors.—If a motor is called upon to start a heavy load from rest, the starting torque may be as large as, or even larger than, the full-load running torque. If the flux at starting has its normal full-load value, the starting current, by equation (11), will then have to be equal to, or perhaps somewhat greater than, its full-load value. Other things equal, the starting current may be smaller the greater the flux. But since $E_a = 0$ when the armature is stationary, it is clear from equation (7) that at the moment of starting $i_a = \frac{V}{r'}$, and, therefore, that the normal small running resistance of the armature circuit (r_a or $r_a + r_f$) must be increased during the starting period by the insertion of a starting rheostat in order to limit the flow of current to a reason-

able value. Thus, a 10-h.p. 220-volt shunt motor would take an armature current of approximately 40 amperes when carrying its rated load, and would have an armature resistance of about 0.5 ohm. If the full voltage were impressed directly upon the armature, the initial current would be 440 amperes, or more than ten times normal full-load current. To limit the starting current to the full-load value, the resistance that must be put in series with the armature should be

$$\frac{220}{40} - 0.5 = 5 \text{ ohms}$$

The resistance of the starting rheostat is usually so adjusted that the initial current is somewhat greater than that giving full-load torque.

Fig. 188*a* shows diagrammatically the connections of the starting rheostat in the case of a series motor, and Fig. 188*b* those of a shunt motor. It should be carefully noted that in Fig. 188*b* the rheostat is in series with the armature only, so the shunt field



(*a*) Series (*b*) Shunt Correct (*c*) Shunt Incorrect

FIG. 188.—Connections of starting rheostats.

winding receives the full line voltage at all times, including the starting period. Fig. 188*c* shows an incorrect set of connections, since here the shunt field current is seriously reduced at the start, thereby reducing the flux and also the torque, and, if the motor is unloaded, causing the speed to rise dangerously high.

If an ordinary rheostat of the kind illustrated in Fig. 188 were used in commercial installations, there would be danger of burning out the armature if, after an interruption of the service and the consequent stopping of the motor, the voltage should again be applied to the supply line; for in that case the full line voltage

would be thrown directly across the low resistance of the armature (or armature and series field), resulting in a very heavy current. For this reason most motor starting rheostats are provided with a "no-voltage release" which automatically restores the starting lever of the rheostat to the starting position when the line voltage is removed; quite frequently there is also an "overload release," which opens the circuit and automatically cuts in the starting resistance if the current becomes excessive for any reason. The connections of such a rheostat are shown in Fig. 189, and Fig. 190 illustrates a starting rheostat of this type made by the Ward Leonard Company.

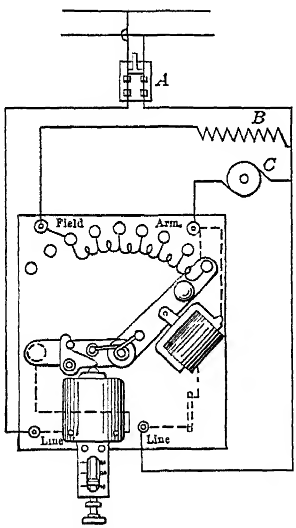


FIG. 189.—Diagram of connections of starting rheostat having no-voltage and overload release.

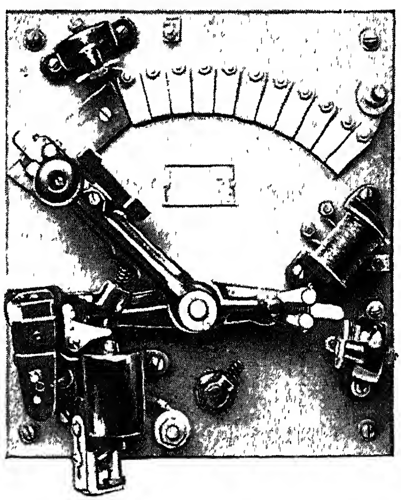


FIG. 190.—Motor starting rheostat with no-voltage and overload release.

It is seen from equation (5) that if the flux Φ is reduced to a small value while the e.m.f. impressed upon the armature remains constant, the speed will rise to a dangerously high value. In other words, the motor will "run away" and may wreck itself. This contingency may arise in the case of a shunt motor if the field circuit is opened, as by a broken wire or loose connection;

and in the series motor by an accidental short circuiting of the terminals of the series winding. This behavior is due to the tendency of any motor to run at such a speed that the back e.m.f. shall be nearly equal to the impressed e.m.f., the lowering of the flux demanding an increased speed.

128. Characteristics of the Separately Excited Motor.—

(a) *Speed Characteristics.*—Let it be assumed that both the impressed voltage, V, and the field exciting current are constant. It follows from the speed equation

$$n = \frac{V - i_a r_a}{\Phi Z'}$$

that were it not for the demagnetizing action of the armature current, the denominator of the fraction would be constant and the speed would decrease slightly and uniformly with increasing values of i_a, as in Fig. 191. This assumes that r_a is constant; in other words, that the temperature of the armature is maintained at its normal running value. The separately excited motor with constant excitation is, therefore, inherently self-regulating as regards speed. Both equation (5) and Fig. 191 indicate that if $i_a = 0$, $n = \frac{V}{\Phi Z'}$. Actually, if $i_a = 0$, there is no torque and no rotation. When the motor is "running free" (that is, unloaded), there is still some current through its armature since sufficient power must be supplied by the line to overcome internal losses due to windage, friction, hysteresis and eddy currents. The minimum value of armature current is indicated by the point A in the figure. The speed $n = \frac{V}{\Phi Z'}$ may be called the ideal zero-load speed; it is the speed that would be reached if there were no losses, in which case also, $E_a = V$ and $i_a = 0$.

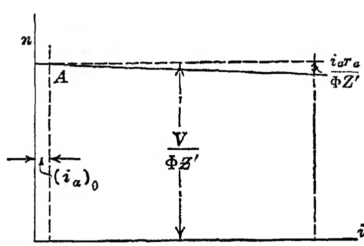


Fig. 191.—Approximate speeed characteristic of separately excited motor.

It is obvious that the speed may be varied through wide limits by varying either Φ or V, or both of them. Thus, the speed can be raised by reducing Φ or by increasing V. However, the possible range of speed due to the adjustment of the excita-

tion is rather restricted, unless special devices are used, because there are limits to the field strength above or below which there are serious commutation difficulties. Variation of V gives little or no trouble so far as commutation is concerned, provided the flux is originally adjusted to about its normal value unless, indeed, V is raised to too great an extent. The fact that the field excitation and the armature impressed voltage are independently variable in the separately excited motor gives to this type its chief advantage.

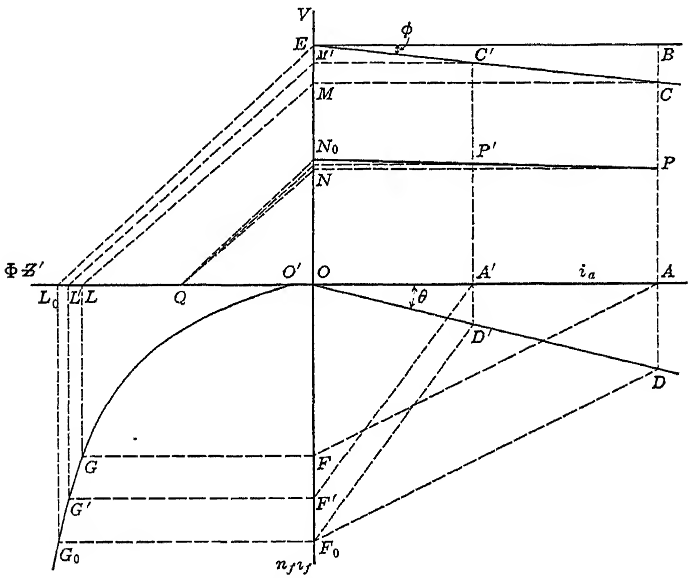


FIG. 192.—Construction of speed characteristic of separately excited motor.

(b) *Effect of Armature Reaction.*—The form of the speed equation shows at a glance that the effect of armature reaction, since it reduces Φ, will be to raise the speed, thereby partially neutralizing the slowing-down effect of armature resistance and improving the speed regulation. The armature of a motor may therefore be designed magnetically more powerful than the armature of an otherwise identical machine intended for use as a generator.

The curve showing the relation between speed and armature current can be constructed in the following manner:

Let $O'G$, Fig. 192, be the magnetization curve of the machine, abscissas (drawn downward from O) representing ampere-turns per pair of poles ($= n_f i_f$) and ordinates (drawn to the left of O) representing values of $\Phi Z'$. Select any convenient scale of armature current along OA, and a scale to represent the impressed voltage along OV. Assume that the field excitation is constant and equal to OF_0, and that the voltage impressed upon the armature is likewise constant and equal to $OE = V$.

Draw a straight line EC through the point E so that $\tan \varphi = r_a$, to the scale of the figure; then for any value of i_a, such as OA, the intercept on the ordinate at A between EC and EB will be $BC = i_a r_a =$ ohmic drop in the armature. The back e.m.f. is then

$$E_a = V - i_a r_a = AB - BC = AC$$

Similarly, draw OD making an angle θ with OA, such that the intercept AD, corresponding to $OA = i_a$, is equal to $\dfrac{\alpha Z i_a}{180a} =$ demagnetizing ampere-turns per pair of poles, to the scale previously adopted along OF_0.

If the armature were currentless ($i_a = 0$) as in the ideal no-load condition, the value of $\Phi Z'$ would be $F_0G_0 = OL_0$, and the ideal no-load speed is then

$$n_0 = \frac{V}{\Phi Z'} = \frac{OE}{OL_0}$$

At any other load, as when $i_a = OA$, the demagnetizing effect is given by AD; joining D with F_0, and drawing AF parallel to DF_0, the net excitation is reduced to OF and $\Phi Z'$ becomes $FG = OL$; at the same time the back e.m.f. is $AC = OM$, hence the speed is

$$n = \frac{V - i_a r_a}{\Phi Z'} = \frac{OM}{OL}$$

In this way values of speed can be computed for various values of i_a and the results plotted to obtain the desired curve, N_0P. But the diagram lends itself readily to a complete graphical solution, as follows:

Select any convenient point Q on the $\Phi Z'$ axis, and draw QN_0 parallel to L_0E; then

$$n_0 = \frac{OE}{OL_0} = \frac{ON_0}{OQ}$$

Similarly, join L and M and draw QN parallel to LM.

$$\therefore n = \frac{OM}{OL} = \frac{ON}{OQ}$$

Since OQ is constant, it follows that ON_0 and ON are respectively proportional to n_0 and n, and may be made *equal* to the speed by a suitable choice of scale. Projecting N across to P, the latter being on the ordinate through A, P will be a point on the required

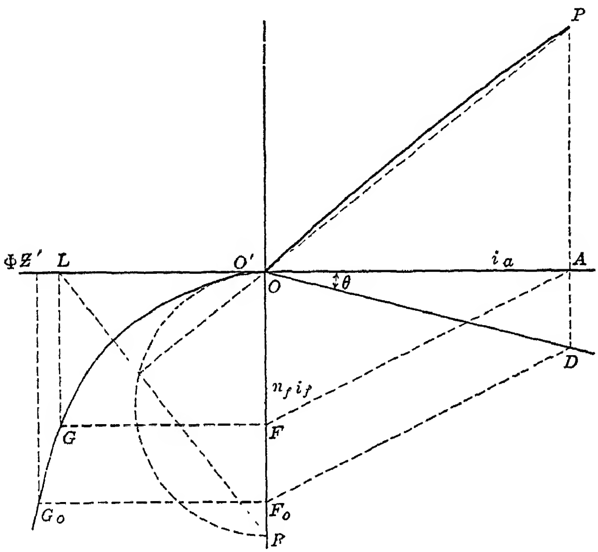


FIG. 193.—Construction of torque characteristic of separately excited motor.

curve. In precisely the same manner, P' is a point corresponding to $i_a = OA'$. It is clear that the speed curve cannot be exactly straight because of the curvature of the magnetization curve.

(c) *The Torque Curve.*—From equation (11), the torque is

$$T = 7.05\ \Phi Z' i_a \text{ pound-feet}$$

Referring to Fig. 192, this becomes

$$T = 7.05\,FG.OA = 7.05\,OL.OA$$

when $i_a = OA$. This may be written

$$\frac{T}{OA} = \frac{OL}{\text{constant}}$$

which equation suggests the following construction for the curve showing the relation between the armature current and the torque:

In Fig. 193 draw the axes of coordinates, the $\Phi Z'$ curve, and the line OD just as in Fig. 192. Proceed as before to locate points G and L. Select any convenient constant length OR, draw LR, and then draw OP perpendicular to LR (using the semicircle on OR as a construction line) until it intersects the ordinate through A in the point P. By construction

$$\frac{AP}{OA} = \frac{OL}{OR} = \frac{OL}{\text{constant}}$$

hence AP is proportional to the torque, or may be made equal to the torque by a suitable choice of the length OR, and P is then a point on the desired torque curve. The torque curve is slightly concave downward, but to all intents and purposes it is a straight line through the origin, since Φ is almost constant for all values of i_a within the working range.

129. Characteristics of the Shunt Motor.—

Speed and Torque Characteristics.—A plain shunt motor operated on constant potential mains, and having fixed field resistance, differs in no way from a separately excited machine with constant impressed voltage and constant excitation. The determination of its operating characteristics is, therefore, to be carried out in exactly the same manner as described in the preceding article.

The chief point of difference between the shunt and the separately excited motor is that in the former the field excitation and the impressed voltage are not independently variable, as in the latter. The possible range of speed variation is therefore less in the shunt motor than it is in the separately excited motor.

An interesting and important fact in industrial applications of the shunt motor is that the speed rises perceptibly as the temperature of the motor is increased. This is due to the fact that the higher temperature of the field winding increases its resistance and so reduces the exciting current and the flux; at the same time the increased resistance of the armature winding tends to decrease the speed, but the influence of the field preponderates. Shunt motors for industrial service are not provided with regulating field rheostats as in the case of generators, hence the field

winding must be designed to have the proper resistance to give the required speed at the operating temperature. In machines of standard design, the difference between full-load speed, running cold, and full-load speed running hot, may be 10 per cent. of the higher value.

130. Characteristics of the Series Motor.—

(*a*) *Speed Characteristic.*—Inspection of the general equation for the speed

$$n = \frac{V - i_a(r_a + r_f)}{\Phi Z'}$$

shows that the speed of the series motor must decrease quite rapidly with increasing load for the reason that Φ increases with increasing i_a. In other words, while the numerator of the fraction decreases, the denominator increases. Theoretically, if $i_a = 0$, $\Phi = 0$, hence at no load the speed would be infinite; practically, while the flux does not become zero because of residual magnetism, it still becomes so small that the speed reaches a dangerously high value, assuming that V remains constant. For this reason, a series motor must always be so installed as to be positively connected to its load, by gearing or direct connection, never by belting, and the minimum load must be great enough to keep the speed within safe limits; such is the case, for instance, in railway motors, hoists, rolling mills, etc.

Assuming that the motor is to be operated on constant potential mains, its speed characteristic can be determined by a modification of the methods described in the case of the separately excited motor, as follows:

Let $O'G$, Fig. 194, be the curve which gives the relation between $\Phi Z'$ and the exciting current (i_a). Let OE represent to scale the constant impressed voltage, V, and draw EC so that $\tan \varphi = (r_a + r_f)$, to the scale of the figure; then BC will represent to the same scale the internal ohmic drop corresponding to $i_a = OA$, and AC will be the back e.m.f. Also, draw OD so that $\tan \theta = \frac{\alpha Z}{180a} \cdot \frac{1}{n_f}$ where n_f is the number of field turns per pair of poles; then AD will be the demagnetizing effect expressed in equivalent amperes instead of in ampere-turns per pair of poles.

When the armature current is $i_a = OA$, the field excitation (in

amperes) is $OF = OA$, the point F being found by drawing AF at an angle of 45 deg. with the horizontal. The demagnetizing effect is $AD = FH$, hence the net excitation is OH and the corresponding value of $\Phi Z'$ is $HK = OL$.

$$\therefore n = \frac{V - i_a(r_a + r_f)}{\Phi Z'} = \frac{AC}{HK} = \frac{OM}{OL}$$

Selecting a point Q on the $\Phi Z'$ axis such that OQ is constant, and drawing QN parallel to LM,

$$n = \frac{OM}{OL} = \frac{ON}{OQ}$$

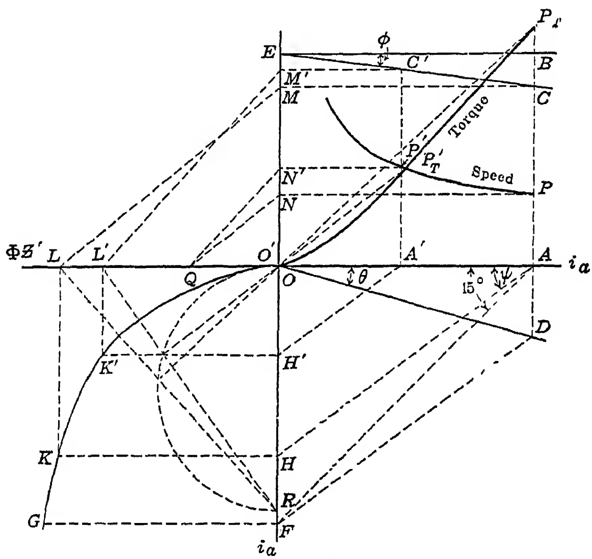


FIG. 194.—Construction of speed and torque characteristics of series motor.

hence ON is proportional to the speed. Projecting N upon the ordinate at A, the resulting point P is a point on the speed-current curve.

Since

$$FH = AD = \frac{\alpha Z}{180 a n_f} i_a = i_a \tan\theta$$

$$OH = OF - FH = i_a - i_a \tan\theta$$

and

$$\tan\psi = \frac{OH}{OA} = 1 - \tan\theta = \text{constant}$$

it follows that for any other current, as $i_a = OA'$, it is only necessary to draw $A'H'$ parallel to AH in the process of locating P'.

(*b*) *The Torque Characteristic.*—As before, the torque is

$$T = 7.05\ \Phi Z' i_a = 7.05\ OL.OA \text{ pound-feet}$$

when $i_a = OA$.
Selecting a point R such that OR = constant, and drawing OP perpendicular to LR,

$$\frac{AP_T}{OA} = \frac{OL}{OR}$$

whence AP_T is proportional to the torque, and P_T is a point on the torque-current curve.

It will be observed that the torque curve of a series motor deviates considerably from the linear form due to the fact that the flux varies with the current. If the magnetization curve were a straight line, that is, Φ proportional to i_a, the torque would be proportional to $(i_a)^2$, and the curve would be a parabola; actually it is a curve of higher order, lying between the linear and parabolic curves.

Lines such as OP_T can readily be drawn perpendicular to LR by constructing a semicircle on OR as a diameter and drawing a line through O and the point where LR cuts the circle.

The torque curves discussed in connection with the separately excited, shunt and series motors refer to the total developed torque, as given by equation (11). The actual torque at the pulley that would be measured by a brake test is less than the total torque by an amount which corresponds to the torque required to overcome internal friction and iron losses. The curve of useful torque may be obtained from that of total torque by subtracting from the ordinates of the latter the "lost torque"; the useful torque passes through zero value when i_a has an appreciable value (see Fig. 210).

131. Characteristics of the Compound Wound Motor.—

(*a*) *General.*—If the shunt and series windings of a compound wound (long shunt) machine are so connected that their magnetizing effects cooperate, or are *cumulative* when the machine is used as a generator, then, if the machine is used as a motor, the two windings will oppose each other, resulting in a *differential* effect. This is illustrated diagrammatically in Fig. 195. If the machine

17

is designed to over-compound as a generator, the differential motor action will be considerable, resulting in a decided decrease of flux under load conditions, and hence a speed higher than would obtain without the series winding. In general, the case is similar to that of a shunt motor with exaggerated armature demagnetizing effect.

In the same way a differentially wound generator, having a drooping e.m.f. characteristic when driven at constant speed, becomes a cumulative-compound motor with a drooping speed characteristic when supplied with constant terminal voltage.

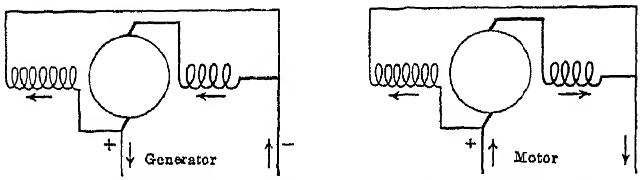


FIG. 195.—Relative directions of shunt and series exciting current in compound machine.

(*b*) *Construction of Speed Characteristic.*—

(I) *Differential Compounding.*

The only difference between this case and the one discussed in connection with Fig. 192, is that now

$$\tan \varphi = r_a + r_f$$

and

$$\tan \theta = \frac{\alpha Z}{180a} + n_f$$

assuming that the resistance of the shunt field winding is constant. The construction has been carried out in Fig. 196, from which it appears that if n_f is sufficiently large, the speed rises with increasing load. It is clear that there is a particular value of θ for which the speed will be the same at full load as at no load, but that it cannot be made absolutely constant at all loads (in the absence of special regulating devices) because of the curvature of the magnetization curve $O'G$.

The torque curve is also shown in Fig. 196, the fixed point R being used in its construction. The curve is concave downward, due to the fact that the flux decreases with increasing current.

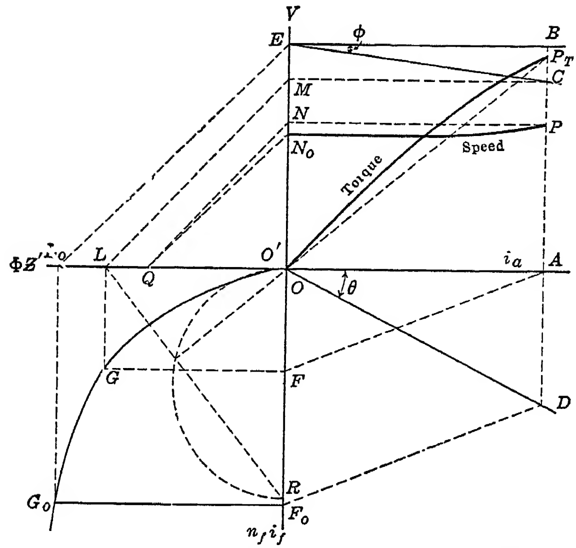


FIG. 196.—Construction of speed and torque curves of differentially compound motor.

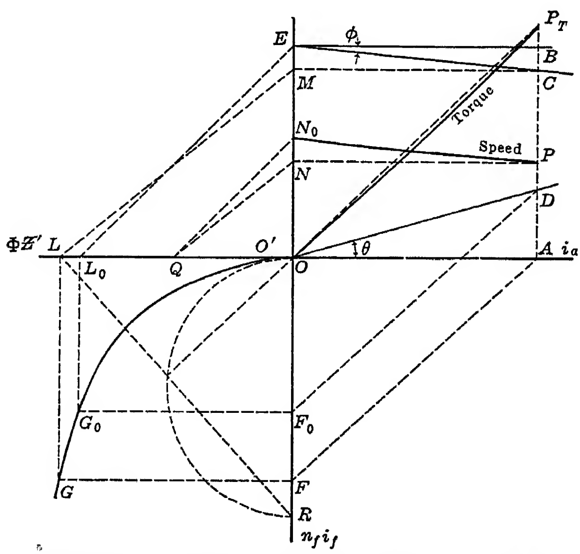


FIG. 197.—Construction of speed and torque curves of cumulative compound motor.

(II) *Cumulative Compounding*
Here,

$$\tan \varphi = r_a + r_f$$

and

$$\tan \theta = \frac{\alpha Z}{180a} - n_f$$

therefore, the line OD must be drawn *above* OA instead of below it; otherwise the construction shown in Fig. 197 is the same as in the previous cases for both the speed and torque curves. The speed now falls considerably with increasing load, and the torque curve is concave upward. The cumulative compound motor has characteristics which are intermediate between those of the shunt and series motors. It differs from the latter especially in this, that its speed rises to a definite limit when full load is suddenly thrown off, instead of running away.

132. Counter E.M.F.—The Reversing Motor.—The existence of the counter-generated or back e.m.f. can be shown in a striking

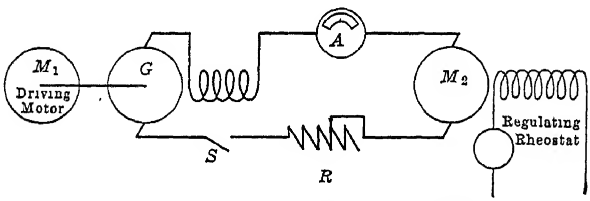


FIG. 198.—Connections for the experiment of the reversing motor.

manner by the arrangement shown diagrammatically in Fig. 198. A series generator, G, is driven at constant speed by a suitable motor, M_1, and the generator is electrically connected through the switch S and the regulating resistance R to the armature of an unloaded and separately excited motor, M_2. The field of M_2 is excited by a constant current supplied from any convenient source.

On closing the switch S, the series generator will build up both e.m.f. and current, provided the resistance of the circuit is below the critical value. At the same time, the speed of M_2 will rise and its back e.m.f. will rise nearly proportionally, so that the active e.m.f. in the circuit, available for producing current, is the difference between the e.m.fs. of generator G and of motor M_2.

The current therefore falls after rising to a certain value and with it the generated e.m.f. of G decreases. Meanwhile, the speed of M_2 tends to be maintained because of its acquired momentum, its back e.m.f. overpowers the generated e.m.f. of G, the current in the circuit reverses (as may be shown by the two-way ammeter A), and M_2 momentarily becomes a generator tending to drive G as a motor in opposition to motor M_1. But as M_2 has no driving power other than its energy of rotation, it very quickly comes to rest. Since the current through the circuit has been reversed, the residual magnetism of G also reverses, consequently as soon as M_2 has come to rest G begins to build up again, but with polarity opposite to that in the first instance. Motor M_2 then speeds up again in the reverse direction until its back e.m.f. overpowers the generator, it again stops, and the entire cycle of changes is repeated, over and over again.

The function of the resistance R is simply to prevent the current from reaching excessive values, and its magnitude will depend upon the machines used in the experiment. The rate at which motor M_2 will build up in speed depends upon the moment of inertia of its armature and upon the torque, the latter in turn depending upon the magnitude of the exciting current of M_2. The larger the excitation of M_2, the greater will be the torque for a given armature current, and the more rapid will be the process of picking up speed; moreover, the greater the excitation, the less will be the speed to produce a given back e.m.f. Finally, therefore, it will be seen that the reversals of M_2 will be more and more rapid, the greater the excitation of M_2.

The above reasoning will serve to explain why a series generator cannot be used to charge a storage battery. For as the charging proceeds the counter e.m.f. of the battery rises, hence reducing the effective e.m.f. in the circuit; the current, therefore, falls and as it decreases, the e.m.f. of the generator also decreases. The current will therefore continue to fall off until it becomes zero, and then the battery discharges through the generator, tending to make it run backward as a motor.

133. Starting of Differentially Wound Motors.—Differentially wound compound motors are seldom used in practice for the reason that in most cases the slightly drooping speed characteristic of the plain shunt motor meets the requirements of constant

speed to a sufficient extent. Moreover, the differentially wound motor is subject to "racing" in case of heavy overload, due to the considerable reduction of field flux caused by the large current in the series field winding. Such motors are also liable to start up in the wrong direction on throwing the handle of the starting rheostat to the first notch; for the high inductance of the shunt winding, due to the large number of turns, may so impede the rise of the shunt field current that the current in the series winding, which builds up much more rapidly because of the small inductance of that circuit, may overpower the magnetizing effect of the shunt winding, and so reverse the flux and the direction of rotation. In that case the motor, if unloaded, will rapidly speed up in the wrong direction, thereby developing a considerable counter e.m.f. and so reducing the current flow and the torque; the acquired momentum of the armature may even for a brief interval cause the machine to become a generator and send current back to the line. In the meantime the shunt field current has been building up, and when the armature finally stops because of reduced torque or the dissipation of its energy of rotation, a heavy flow of current through the armature and series winding will result because there is now no counter e.m.f. The machine will then start up in the right direction, but if the initial current flow is sufficiently great the series excitation may overpower that of the shunt winding, and so bring about another reversal of rotation. This process may go on indefinitely unless the design constants of the machine are such that the successive impulses are damped out, that is, do not synchronize with the natural period of oscillation of the armature.

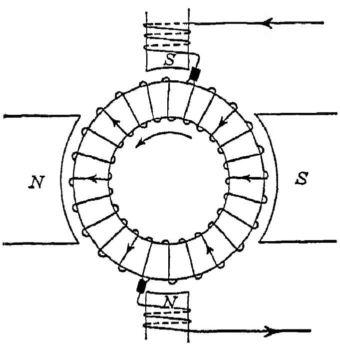


FIG. 199.—Differential effect in interpole motor due to backward shift of brushes.

A similar state of affairs may arise in the case of shunt motors provided with interpoles if the brushes are not properly placed. Normally the axis of commutation coincides with the axis of the interpoles, but if the brushes are accidently shifted backward,

against the direction of rotation, as in Fig. 199, the interpoles will produce a component of flux in opposition to that of the main poles, and so convert the machine into one having the characteristics of a differentially wound motor.

134. Control of Speed of Shunt Motors.—Inspection of the fundamental equation for the speed of a motor

$$n = \frac{V - i_a r'}{\Phi Z'}$$

reveals the fact that there are three principal methods for regulating the speed, namely, *rheostatic control,* by varying the resistance r', which includes the armature resistance r_a; *voltage control,* by varying the impressed voltage V; and *field control* by varying Φ. A fourth method occasionally used involves changing Z' by using an armature having two windings and two commutators which may be connected either in series or in parallel.

(*a*) *Rheostatic Control.*—In this method the effective resistance of the armature is increased by connecting in series with it (but

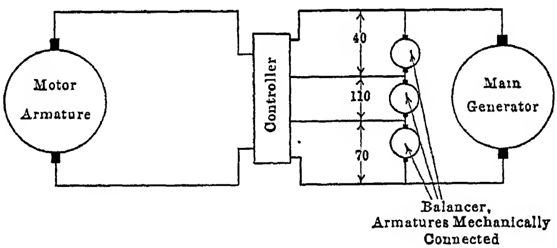


Fig. 200.—Speed regulation of motor by means of voltage control.

not in the main line or field circuit) a variable resistance. This has the effect of imparting a pronounced droop to the speed characteristic (Figs. 191 and 192), the downward slope of the characteristic being proportional to the combined resistance of the armature winding and external resistor. A motor used in this way has poor speed regulation, that is, the speed will fluctuate between rather wide limits as the load changes; moreover, the method is inefficient because of the loss of power due to the flow of the armature current through the external resistor. It is not to be recommended in industrial installations, but is frequently convenient in laboratory investigations and in special tests.

(*b*) *Voltage Control.*—Subdividing the voltage of the main generator or bus-bars by means of a balancer set, as in Fig. 200, makes it possible to impress upon the armature of the motor a number of different voltages, to each of which there will correspond a definite speed characteristic such as is illustrated in Fig. 192. For any given impressed voltage the speed will be substantially constant and will be approximately proportional to the impressed voltage. The variation in speed between full-load and no-load with normal voltage will usually be between 2 and 10 per cent., the smaller limit holding for large motors, the larger limit for small motors. It should be understood that the motor connections are such that the voltage impressed on the shunt field winding is not changed when the armature is switched from one circuit to another, in order that the field flux may remain sub-

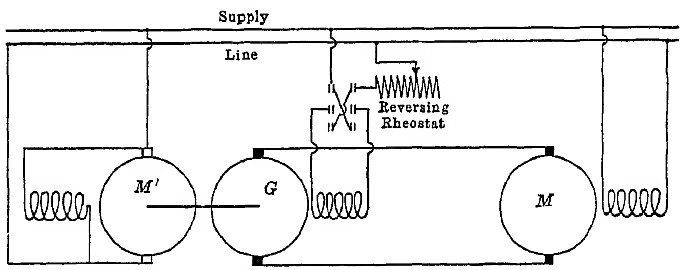


FIG. 201.—Diagram of connections of Ward Leonard system of speed control.

stantially constant. The armature connections are changed by means of a special controller, somewhat resembling an ordinary railway motor controller.

With the arrangement indicated in Fig. 200 it is possible to impress six different voltages upon the motor, namely, 40, 70, 110, 150, 180 or 220 volts, giving six different speeds. Intermediate speeds may then be secured by adjusting the flux by means of a rheostat in series with the shunt field winding. This method is extensively used for driving machine tools, such as lathes, boring mills, etc. It has the disadvantage of requiring a considerable investment in copper due to the extra wires of the distributing circuits.

Where uniform gradation of speed in either direction is required, as in the operation of the turrets of battleships or in steering by

electrically controlled rudders, the Ward Leonard system may be used. The motor M, Fig. 201, whose speed is to be regulated, is separately excited from the main supply lines and its armature is supplied from an auxiliary generator G, the latter being driven at constant speed by a shunt motor M' which takes its power from the line; instead of driving the generator G by a motor, any other form of prime mover may be used. The field of the generator is excited from the constant voltage supply line, and may be adjusted from zero to a maximum, in either direction, by means of a reversing field rheostat; in this way it is possible to obtain a

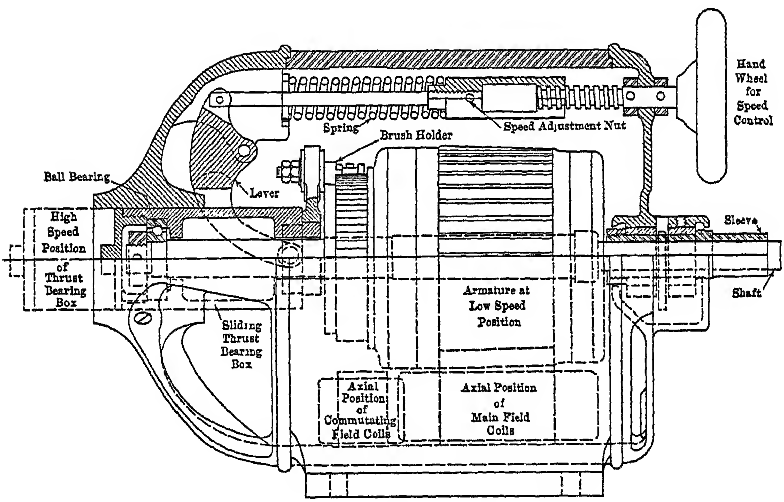


FIG. 202.—Sectional view of Lincoln adjustable speed motor.

smooth variation of the voltage impressed upon the motor. This method is very effective, but is naturally expensive because of the auxiliary motor-generator set.

(c) *Field Control.*—The simplest and cheapest method of regulating the speed of a shunt motor is that in which the flux is varied by means of a rheostat in the shunt field circuit. If the machine operates normally with a nearly saturated magnetic circuit, all resistance of the rheostat being cut out, the speed may be approximately doubled by weakening the field current; beyond this point the field intensity at the pole tips becomes so weakened by armature reaction, especially under load conditions,

that commutation is seriously interfered with. Consequently this method is limited to those cases in which a very moderate range of speed will suffice.

The interpole motor affords means whereby a wide range of speed is made possible, a ratio of maximum to minimum speed of 5 or 6 to 1 being fairly common. The principle of the interpole motor involves the neutralization of the armature reaction of the motor by placing auxiliary poles in the axis of commutation and exciting them by the same current that flows through the arma-

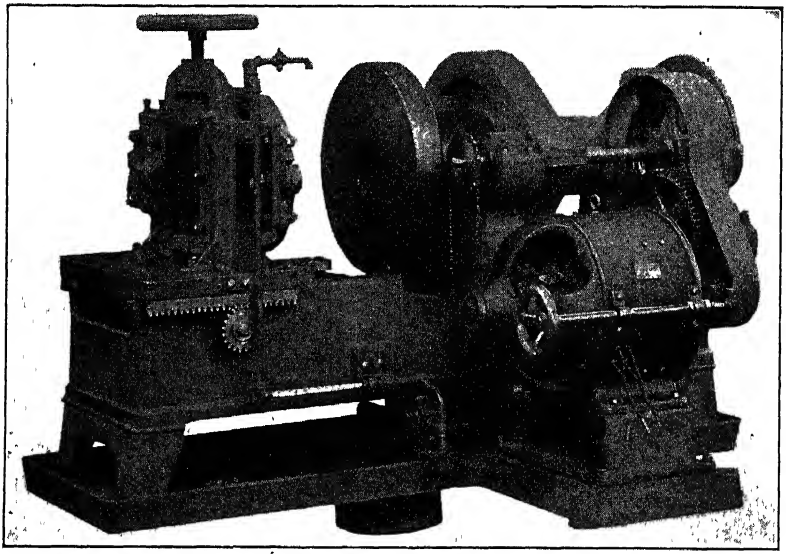


FIG. 203 —Lincoln adjustable speed motor driving pipe cutting machine.

ture, the winding of the auxiliary poles being so designed that the m.m.f. of the armature is either exactly balanced or else slightly overcompensated. In this way the main field may be varied through a wide range without producing sparking, the interpoles always producing a field of the proper strength to reverse the current in the coils undergoing commutation. Interpole motors are used to a very large extent where variable speed is a necessity, as in machine tool operation. They are generally provided with a controller which serves not only to start the motor, and to reverse its direction, but also to vary its speed as desired.

The methods thus far described effect the variation of speed by adjustment of the electrical circuits of the machine. But the flux and, therefore, the speed can be varied by mechanical devices which change the length of the air-gap. In the Lincoln adjustable speed motor, shown in section in Fig. 202, the armature core is conical, so that as the armature is moved sideways by means of the handwheel the effective length of air-gap may be increased or decreased at will. A range of speed of 10 to 1 is readily obtained in the smaller sizes. Commutation difficulties at high speeds (and weak field) are avoided by using interpoles. Fig. 203 shows a similar machine made by the Reliance Electric

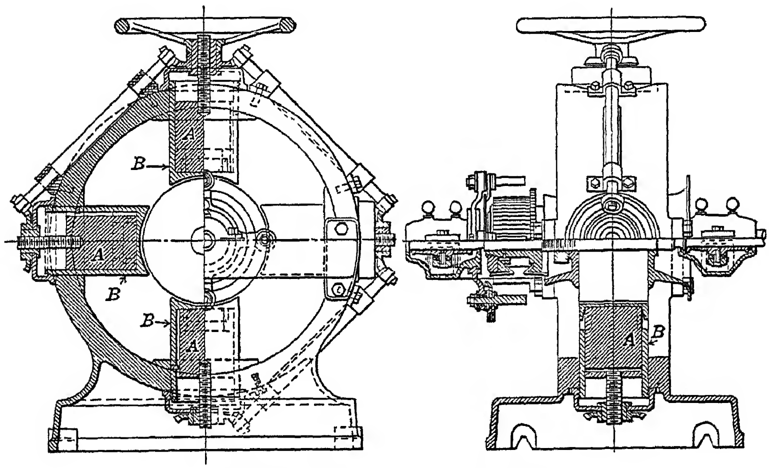


Fig. 204.—Stow multi-speed motor.

and Engineering Co., the motor here being geared to a pipe cutting machine.

Speed variation is obtained in the Stow multi-speed motor by plungers which are moved in and out of the hollow pole cores by means of a handwheel and bevel gears, as shown in Fig. 204. The speed is increased by drawing the plungers away from the armature, thereby weakening the field; the thin shell of iron thus left at the pole tips becomes saturated, and the commutating field is therefore sufficiently intense to prevent sparking at the upper limit of the speed.

135. Applications of the Series Motor.—The rapid drop in speed of the series motor as its load is increased makes this type

of machine especially valuable for traction purposes, as in street railways and hoisting service, and in rolling mills. In railway work, for example, motors having a constant-speed character-

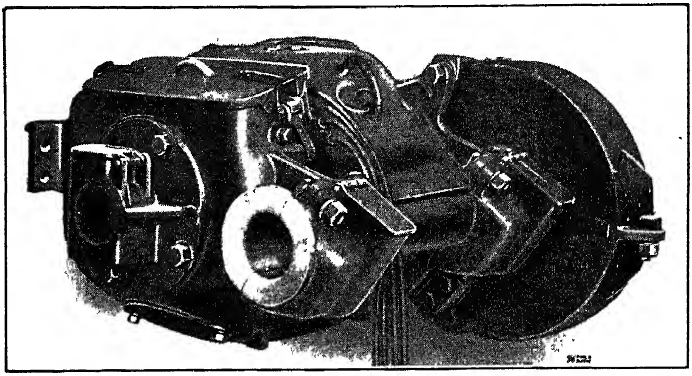


FIG. 205.—Box frame railway motor, forced ventilation. (General Electric Co.)

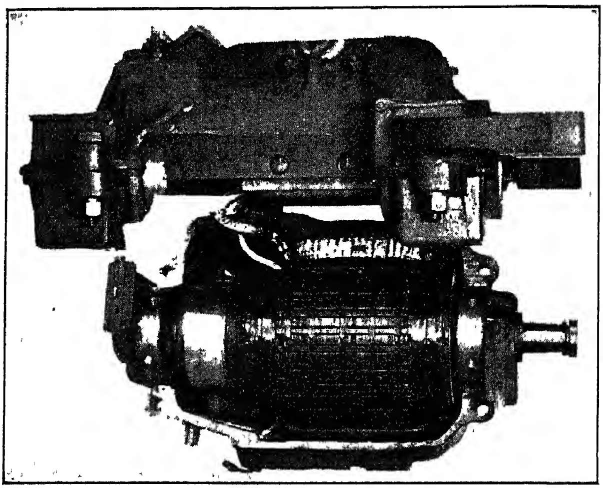


FIG. 206.—Split frame commutating pole railway motor, Westinghouse Elec. & Mfg. Co.

istic like the shunt motor are seldom used for the reason that the current taken by such a motor in going up a steep grade is excessive; for since the speed of such a motor will remain substantially constant if the impressed voltage is constant, the additional power

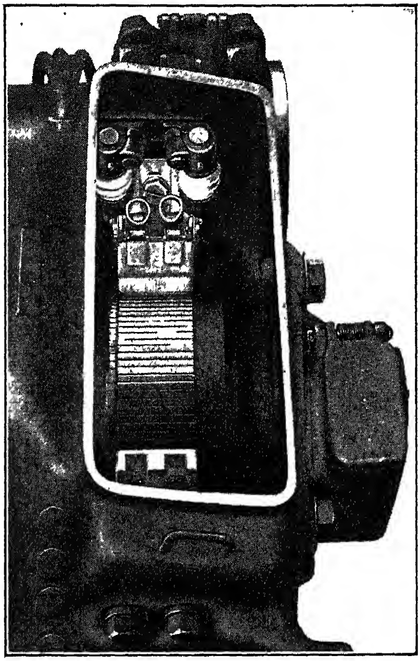


Fig. 207.—Commutator and brushes of motor of Fig. 204.

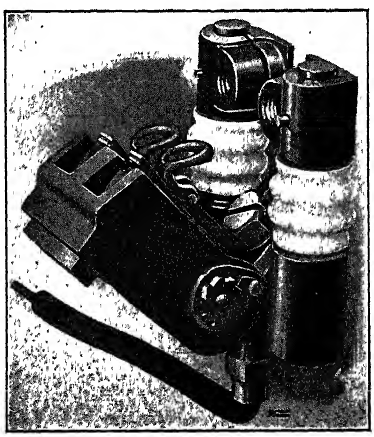


Fig. 208.—Brush holders of motor of Fig. 204.

required to climb the grade demands a proportionally increased current. The series motor, on the other hand, will slow down as the load increases, automatically preventing an excessive load, and, to a certain extent, tending to maintain a constant load on the system; at the same time it develops a torque more than proportional to the current, while in the shunt motor the torque increases less than proportionately to the current.

Series motors for railway, automobile, hoisting and rolling mill service are generally of the totally enclosed type. In rail-

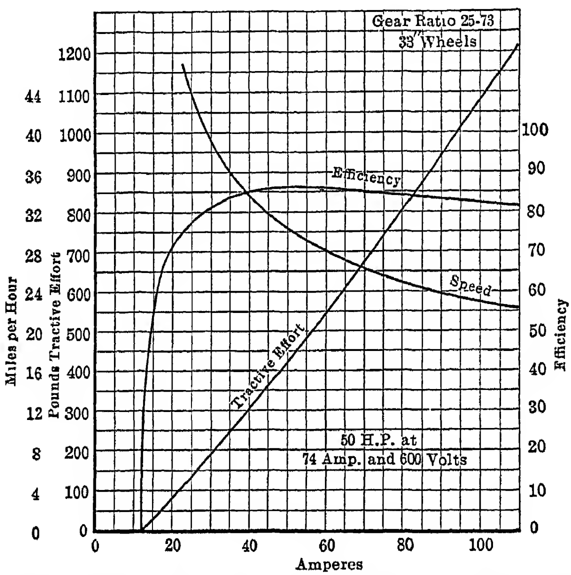


FIG. 209.—Characteristic curves of motor of Fig. 205. (General Electric Co.)

way and automobile service, in particular, the motors must be waterproof and of rugged construction to withstand the rough usage to which they are subjected by reason of poor roadbed and improper handling of the starting controller. A too rapid cutting out of the starting resistance results in very heavy current, excessive torque, and a wracking of the armature winding.

Fig. 205 illustrates a recent type of railway motor made by the General Electric Company. It is of the box frame, commutating pole type with forced ventilation, the inlet and outlet for

the cooling air being at the pinion end of the frame. In the box frame type the armature may be removed from the frame through the opening at the commutator end. Fig. 206 shows a split frame commutating pole motor made by the Westinghouse Electric and Manufacturing Company. Figs. 207 and 208 show the commutator and brush rigging of the motor illustrated

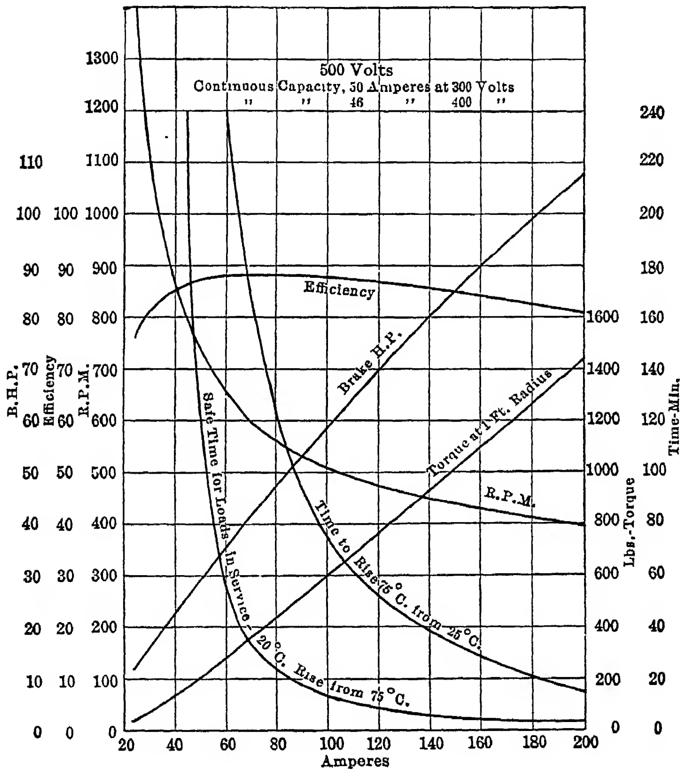


FIG. 210.—Characteristic curves of motor of Fig. 206. (Westinghouse Elec. & Mfg. Co.)

in Fig. 205. The characteristic curves of the motor of Fig. 205 are shown in Fig. 209; those of Fig. 206 in Fig. 210.

Equations of Characteristic Curves of the Series Motor.[1]—In Art. 117, Chap. VI, there was presented a mathematical discus-

[1] Empirical Equations of the Speed and Torque Characteristics of the Series Motor, by A. S. Langsdorf, Washington University Studies, Vol. 6, No. 1 (1918).

sion of the characteristic curves of the shunt generator, based on the use of Froelich's equation for representing the relation between flux and excitation. An analogous study of the characteristic curves of the series motor, leading to empirical equations for the speed and torque characteristics, is given here partly because such equations may at times be useful, but largely because it furnishes a good example of some of the methods of developing equations to fit experimentally determined graphs.

For example, let it be required to determine the equations of the speed and tractive effort curves of Fig. 211 (tractive effort being proportional to torque).

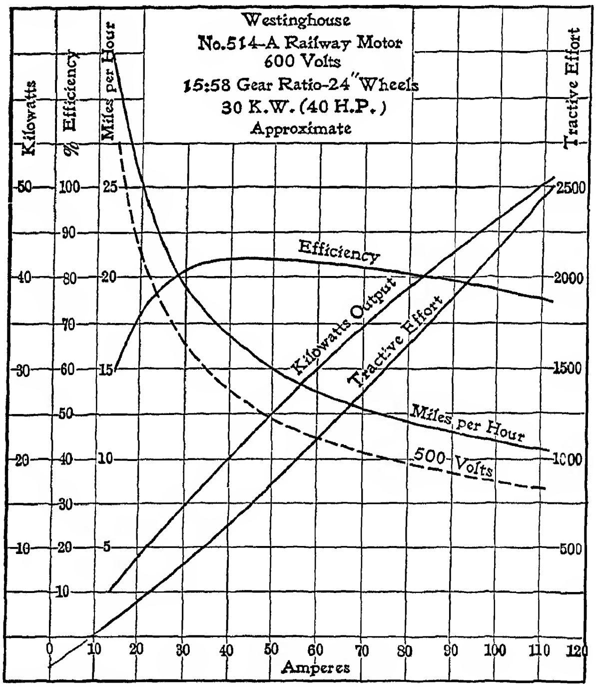


FIG. 211.—Characteristic curves of railway motors.

Assuming that the relation between flux and excitation is given by

$$\Phi = \frac{ai}{b + i} \tag{12}$$

it follows that the effect of the demagnetizing ampere-turns of the armature is in this case accounted for; for the net excitation

is expressed by $(1 - k)i$, where k is a constant equal to the ratio of armature demagnetizing turns per pole to the field turns per pole; strictly, therefore, the form of Froelich's equation should be

$$\Phi = \frac{a(1-k)i}{b + (1-k)i},$$

but on dividing numerator and denominator of this expression by the term $(1 - k)$, it reduces to the same type form as (12).

From equation (4), page 244

$$V = \Phi Z'n + ir$$

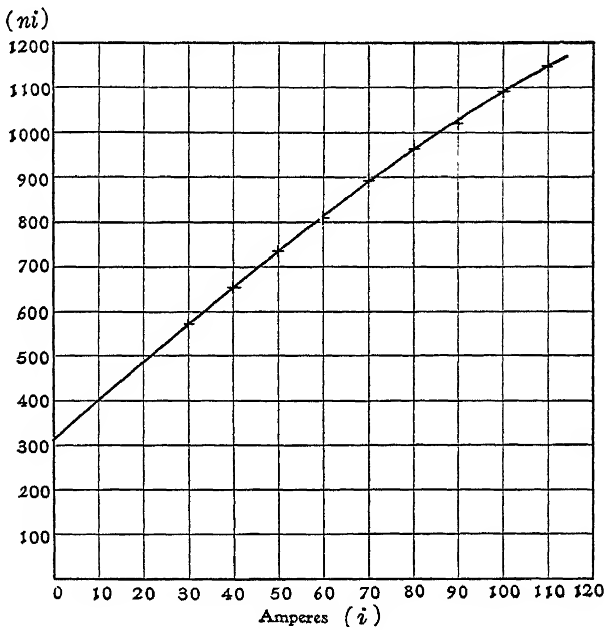


FIG. 212.—Plot of ni and i.

where r is the total resistance of the motor, and on substituting for Φ from (12) it follows that

$$V = \frac{aZ'ni}{b + i} + ir$$

whence

$$ni = \frac{Vb}{aZ'} + \frac{V - br}{aZ'}i - \frac{r}{aZ'}i^2 \tag{13}$$

If in equation (13) we regard the product ni as a new variable, say y, with the variable i as argument, the equation will represent a parabola with its axis parallel to the y axis. If, therefore, a table of simultaneous values of n and i is prepared by reading the coordinates of a series of points from the speed characteristic of Fig. 211, as has been done in Table I, and if there is then computed from these data the products ni having as factors the corresponding pairs of values of n and i, it may reasonably be anticipated that a curve plotted between ni and i will have a parabolic form. The curve of Fig. 212 has been constructed in this manner, and it shows clearly that the method is thus far justifiable.

It will be observed, however, that if i is made zero in equation (13), the equation becomes,

$$(ni)_{i=0} = \frac{Vb}{aZ'}$$

which means that if the curve of Fig. 212 is produced backward until it intersects the ni axis, the intercept on that axis will give to scale the numerical value of the constant term $\frac{Vb}{aZ'}$. In the particular case considered, this turns out to be 320. If the curve between ni and i turns out to be somewhat flat, and if the points determinable from the original data, or from Fig. 211, lie considerably to the right of the axis of ordinates of

TABLE I

1	2	3	4	5
i amperes	n speed, miles/hour	ni	$\frac{ni-320}{i}$	n computed
30	19.2	576	8.53	19 22
40	16.4	656	8 40	16.43
50	14.7	735	8.30	14.71
60	13.5	810	8.17	13.52
70	12.65	886	8 08	12.64
80	12.0	960	8.00	11.96
90	11.4	1026	7 84	11.39
100	10.9	1090	7 70	10.92
110	10 5	1155	7.59	10.50

Fig. 212, there may be some uncertainty in the value of $\frac{Vb}{aZ'}$ so determined. In such a case it may be necessary to make several trials before a satisfactory value is found, but in any case the trial value must be tested in the manner indicated in the remaining part of the analysis, as follows:

Using the value of $\frac{Vb}{aZ'}$ as found from Fig. 212, equation (13) can be written in the form:

$$\frac{ni - \frac{Vb}{aZ'}}{i} = \frac{V - br}{aZ'} - \frac{r}{aZ'}i \tag{14}$$

and the value of the quantity $\frac{ni - \frac{Vb}{aZ'}}{i}$ can then be computed

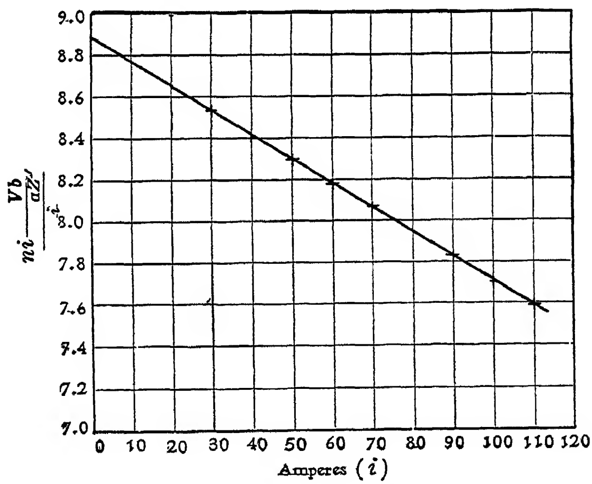


Fig. 213.—Plot of $\frac{ni - \frac{Vb}{aZ'}}{i}$ and i.

from the data of Table I. Inspection of equation (14) reveals the fact that this new quantity bears a linear relation to i, and when these two quantities are plotted there should result a straight line. Such a line is shown in Fig. 213, which is plotted from the data in columns 1 and 4 of Table I. The fact that a

straight line is obtained proves that the theory of the method is sound, and incidentally that the value of $\frac{Vb}{aZ'}$ was correctly obtained from Fig. 212; if the resultant line is found to be not straight, either the value of the constant was not correctly determined, in which case another trial should be made; or else the method is at fault.

Continuing the analysis of equation (14) and its graphical representation in Fig. 213, it will be seen that the intercept of the line on the axis of ordinates is $\frac{V - br}{aZ'}$, and that its slope is $\frac{-r}{aZ'}$. It is therefore easy to find the actual numerical values of these quantities, which are found to be:

$$\frac{V - br}{aZ'} = 8.9$$

$$\frac{r}{aZ'} = 0.0118$$

Substituting these values in equation (14), and solving for n, there results,

$$n = 8.9 + \frac{320}{i} - 0.0118i \tag{15}$$

which is the desired equation (n being given in miles per hour).

By assigning various values to i, say the values in column 1 of Table I, corresponding values of n may now be computed and compared with the original data. Computed values of n are given in column 5 of Table I, and it will be noted that the agreement is practically absolute, because the ordinates of the curve of Fig. 211 cannot be read with accuracy beyond the third significant figure.

The torque, and therefore the tractive effort, developed by the motor is proportional to Φi, but because of friction the net tractive effort available at the draw-bar is less than that developed by a nearly constant amount. It is therefore legitimate to write,

$$T = k\Phi i - T_f \tag{16}$$

where T is the net tractive effort, and T_f is that part of the total tractive effort which is lost in friction. Substituting (12) in (16) and transposing,

$$T = \frac{kai^2}{b + i} - T_f = \frac{ci^2}{b + i} - T_f \tag{17}$$

where c is a new constant.

Equation (17) can be written,

$$c\frac{i^2}{T + T_f} = b + i \tag{18}$$

The form of equation (18) then suggests the following procedure: In Fig. 211 produce the curve of tractive effort backward until it intersects the axis of ordinates; the intercept, which is found to be 150 pounds (in the negative direction) in the case

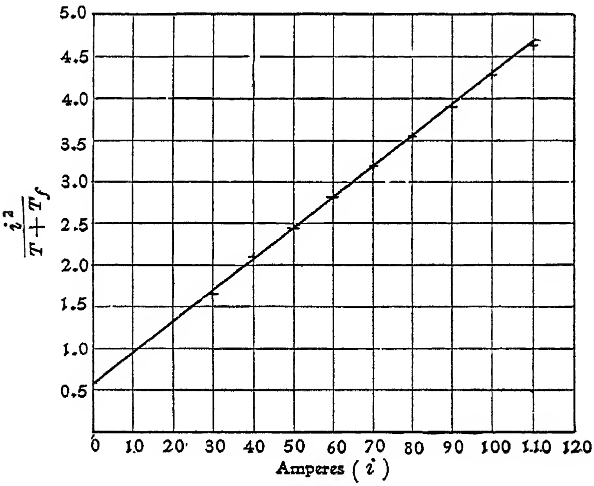


FIG. 214.—Plot of $\frac{i^2}{T + T_f}$ and i.

under consideration, may then fairly be taken as the value of T_f. Prepare Table II by reading simultaneous values of T and i from Fig. 211, and compute column 4 of the table by forming the ratio $\frac{i^2}{T + T_f} = \frac{i^2}{T + 150}$. Equation (18) then indicates

that this quantity, when plotted with i, should yield a straight line. If it does not do so, a new trial value of T_f should be used. In the case here considered, the plot of $\frac{i^2}{T + T_f}$ and i, where T_f is 150, yields the straight line of Fig. 214. Selecting two points on this line, and substituting their coördinates in equation (18), and solving to find the constants, it is found that

$$T = \frac{i^2}{0.68 + 0.0361i} - 150 \qquad (19)$$

TABLE II

1	2	3	4	5
i amperes	T tractive effort, lbs.	$T + T_f$	$\frac{i^2}{T + T_f}$	T computed
30	380	530	1 698	360
40	600	750	2.13	605
50	860	1010	2 48	855
60	1120	1270	2 84	1114
70	1370	1520	3 22	1380
80	1640	1790	3 58	1640
90	1900	2050	3.95	1910
100	2150	2300	4.35	2180
110	2440	2590	4.68	2450

Assuming the values of i given in Table II, the values of T computed from equation (19) are as shown in column 5. The discrepancies are again very small, thus verifying the correctness of the formula for T.

136. Cycle of Operation of Railway Motors.—The horse-power rating of a railway motor has little significance in determining its suitability for a particular equipment; the nominal rating is defined as the mechanical output at the car or locomotive axle measured in kilowatts, which the motor will carry for one hour without exceeding a temperature rise, measured by thermometer, of 90° C. at the commutator and 75° C. at any other normally

accessible part, the motor being tested on a stand with an impressed e.m.f. of rated value, and the motor covers being arranged to secure maximum ventilation without external blower.[1] In any given case the motors must be so selected that they will not overheat and the heating depends in part upon the average value of the square of the current taken throughout the whole of the working period, including stops. The current has its largest

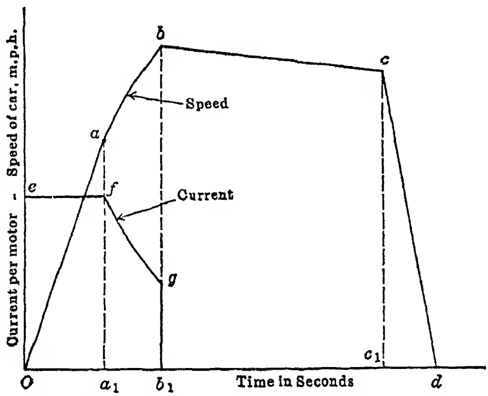


FIG. 215.—Speed-time and current-time curves of railway motor.

value during the starting or acceleration period, hence the heating is largely dependent upon the number of stops in any given schedule.

When the car—or train—is started, the resistance in series with the motors should be cut out step by step in such a manner that the current through each motor remains practically constant until all the resistance is out of the circuit. The torque per motor will then be constant, hence the draw-bar pull and the resulting acceleration will also be constant, and the speed of the car will increase uniformly, as indicated by the line Oa, Fig. 215. When the resistance is all out, the speed will continue to increase, but at a steadily decreasing rate, as represented by the curved line ab, and during this interval the current will decrease from the initial constant value Oe in the manner indicated by curve fg. The cause of the decreasing current is the increasing counter e.m.f. due to the rising speed. After the time Ob_1, the current is

[1] See Standardization Rules, A.I.E.E.

shut off, and the car allowed to coast, the speed accordingly falling in the manner shown by line *bc*. The brakes are then applied and the speed rapidly falls from cc_1 to zero. The broken line *Oabcd* is called a *speed-time* curve, and its area is proportional to the distance traveled by the car in the time *Od*.

The slope of the line *Oa* is the acceleration of the car; the value ordinarily used varies from 1 to 2 miles per hour per second, and this in turn determines the draw-bar pull, torque and current when the weight of the car, gear ratio and type of motor are known.

137. Series-parallel Control.—In cars having a two-motor equipment the motors and starting resistance are at first all connected in series, and after the resistance has been cut out, the connections are quickly changed so that the motors themselves are in parallel with a resistance between them and the line; this resistance is then cut out, so that finally the motors are in parallel directly across the full voltage of the line. The elementary diagram of connections is shown in Fig. 216. In four-motor equipments, the motors are usually connected in parallel in pairs, and the two pairs are then connected in series-parallel just as though each pair were a single machine.

The series-parallel control is a much more economical method than if each motor had its own starting rheostat, or than if the motors were permanently in parallel with a single resistance for starting purposes. For example, assume a two-motor equipment with the following data:

V = line or trolley voltage
I = current per motor during acceleration period
r = resistance of each motor
t = duration of acceleration period, in seconds.

At the moment of starting, the motors being in series (Fig. 216*a*), the starting rheostat must have a resistance of R_1 ohms such that

$$I = \frac{V}{R_1 + 2r}$$

or

$$R_1 = \frac{V}{I} - 2r$$

The loss in the rheostat at the first instant is then at the rate of I^2R_1 watts; but as the motor speeds up at a uniform rate under the assumption of constant current, the counter e.m.f. also increases uniformly, and in order to keep the current constant the resistance must be cut out at a uniform rate. All of the resistance should be out of circuit in a time $\frac{t}{2}$ seconds, and the motors then switched to the parallel position (Fig. 216*b*). During the first half of the acceleration period the energy lost in the rheostat is then

$$W_{R_1} = \tfrac{1}{2}I^2R_1\frac{t}{2} = \tfrac{1}{4}I^2R_1t \text{ watt-seconds.}$$

At the instant when all of the resistance R_1 is out of circuit, each motor receives half of the line voltage and this condition may be maintained efficiently if it is desired to continue running at reduced speed. But if the speed is to be increased at the original rate, as in Fig. 215, the motors must be put in parallel and a new resistance R_2 inserted between them and the line. In order that there may be no break in the smoothness of the acceleration, each motor must continue to take I amperes, and at the first instant after the transition has been made the resistance R_2 must consume $V/2$ volts since the remaining $V/2$ volts are taken up by the motors. The resistance R_2 must then have such a value that

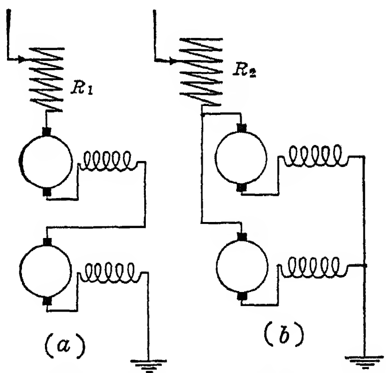


FIG. 216.—Elementary diagram of connections, series-parallel control.

$$R_2 = \frac{\tfrac{1}{2}V}{2I} = \frac{V}{4I} \text{ ohms}$$

and the energy lost in the rheostat during the second half of the acceleration period will be

$$W_{R_2} = \tfrac{1}{2}(2I)^2R_2\frac{t}{2} = I^2R_2t \text{ watt-seconds.}$$

The total loss in the rheostat is then

$$W = W_{R_1} + W_{R_2} = I^2t\left(\frac{R_1}{4} + R_2\right) = \tfrac{1}{2}I^2t\left(\frac{V}{I} - r\right) \text{ watt-seconds.}$$

If, now, the motors had been originally in parallel, as in Fig. 216*b*, with a resistance of R_3 ohms between them and the line, the value of R_3 would have to be

$$R_3 = \frac{V}{2I} - \frac{r}{2} \text{ ohms}$$

in order to allow a current of I amperes to flow through each motor. The loss in the rheostat would then be

$$W_{R_3} = \tfrac{1}{2}(2I)^2R_3t = 2I^2R_3t = I^2t\left(\frac{V}{I} - r\right) \text{ watt-seconds}$$

or exactly twice as great as in the case of series-parallel control.

138. Railway Controllers.—The successive changes in the starting resistance and the change from series to parallel connection are accomplished by means of a *controller.* As the controller is changed from notch to notch the resistance is varied by finite amounts, so that the current does not remain absolutely constant throughout the acceleration period, as represented by the line *ef*, Fig. 215, but in reality this curve assumes a saw-tooth form lying partly above and partly below the desired constant value. The two positions of the controller in which the motors are in full series and in full parallel, respectively, are called *running points* because in these positions there is no loss in the rheostat; all other positions are called *resistance points* except in the interval in which the transition from series to parallel connection takes place.

Controllers are commonly designated by characteristic letters which indicate the type to which they belong. Thus, type R controllers are those in which rheostatic control is used, without the customary series-parallel arrangement; they are used for single motor railway equipments, mining locomotives with one or two motors, and for cranes and hoists. Type K controllers are designed for series-parallel operation of two or more series motors, and have the characteristic feature of not breaking the power circuit during the transition from series to parallel

connection. Type *L* controllers are also designed for series-parallel control of series motors, and include the feature of opening the power circuit during the transition period; this type is now seldom used. Type *B* controllers have the usual power circuit connections, and in addition allow the motors to run as generators for energizing magnetic brakes of the axle or track type.

Fig. 217 represents a *K*-10 controller made by the Westinghouse Electric and Manufacturing Co. and Fig. 218 shows the succes-

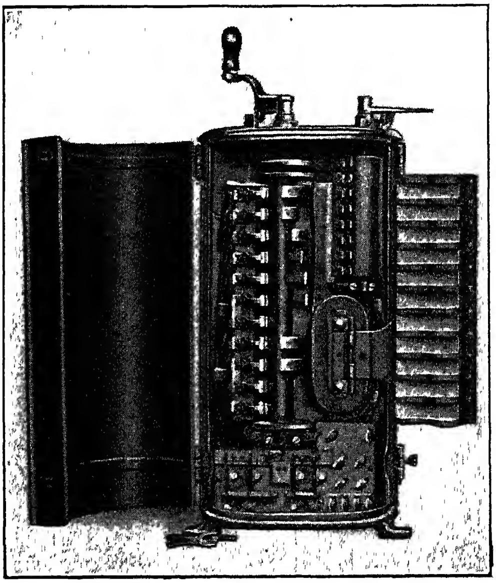


Fig. 217.—K-10 controller, Westinghouse Elec. & Mfg. Co.

sive stages of the connections. The oval shaped part near the middle of Fig. 217 is a solenoid connected in the main power circuit; its function is to create a powerful magnetic field at the contacts between the stationary contact fingers and the segments on the controller spindle. This field is so directed as to blow out the arcs that form on breaking the circuit. Fig. 219 represents a more recent type of *K* controller made by the General Electric Co.; instead of a single magnetic blow-out coil, there are individual blow-out coils for each contact. The diagram of

connections of this controller are shown in Fig. 220. It will be observed that this diagram differs from that of Fig. 218 in that during the transition period the latter involves short-circuiting and immediately thereafter open-circuiting one of the motors (or pair of motors) while in the latter both motors are continu-

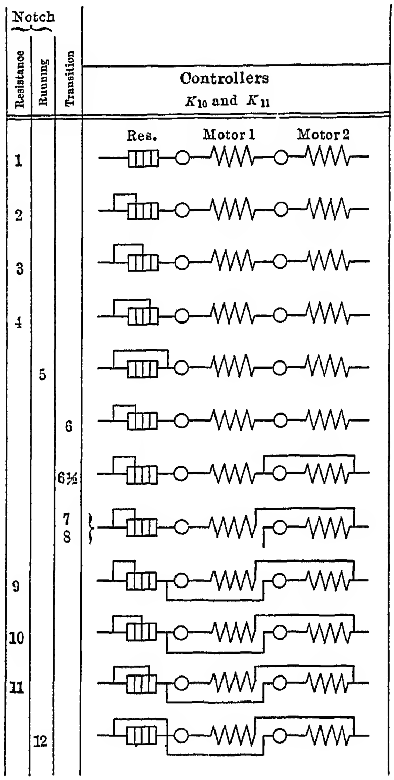


FIG. 218.—Successive stages of connections, K-10 controller.

ously in circuit. The system of transitional connections shown in Fig. 220 is called the *bridge* control.

All controllers, with the exception of certain *R* types, have two handles, one for the usual operation of accelerating the car, the other for the reversal of the direction of its motion. These two handles are mechanically interlocked in such a manner that the reversing handle cannot be moved unless the main handle is in the "off" position, and the main handle cannot be moved

unless the reversing handle is in either the forward or reverse position. The reversing handle changes the direction of rotation of the motors by interchanging the connections of the field windings with respect to the armature terminals.

If the car is running and it is desired to reduce speed, the controller handle should be turned quickly to the off position and then brought back again to the proper notch before the speed

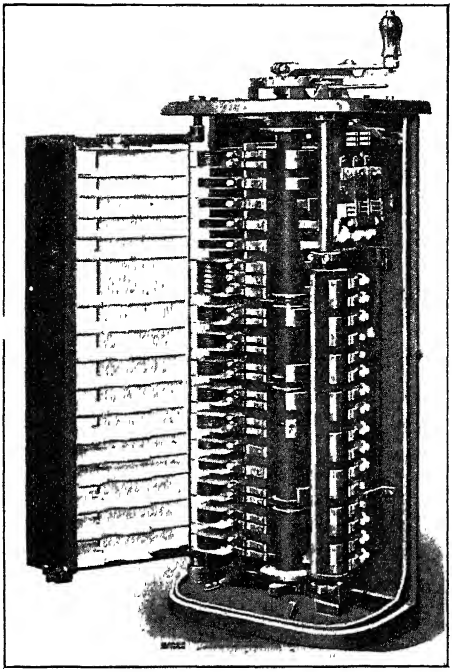


FIG. 219.—Railway motor controller with individual blow-out coils, General Electric Co.

has fallen too low. A slow turning off is apt to draw destructive arcs at the contact fingers.

A characteristic feature of all of the controllers described above is that the main current passes directly through them. This is perfectly feasible in the case of a single car, or motor car and trailer, but where several motor cars and trailers are to be operated as a train, the multiple-unit type of control, called type *M*, must be used. The controller for this service carries only a

small auxiliary current supplied by the line, and this current actuates electromagnets which operate *contactors* that control the main current. The contactors are usually mounted in waterproof iron cases under the car bodies. In this system a

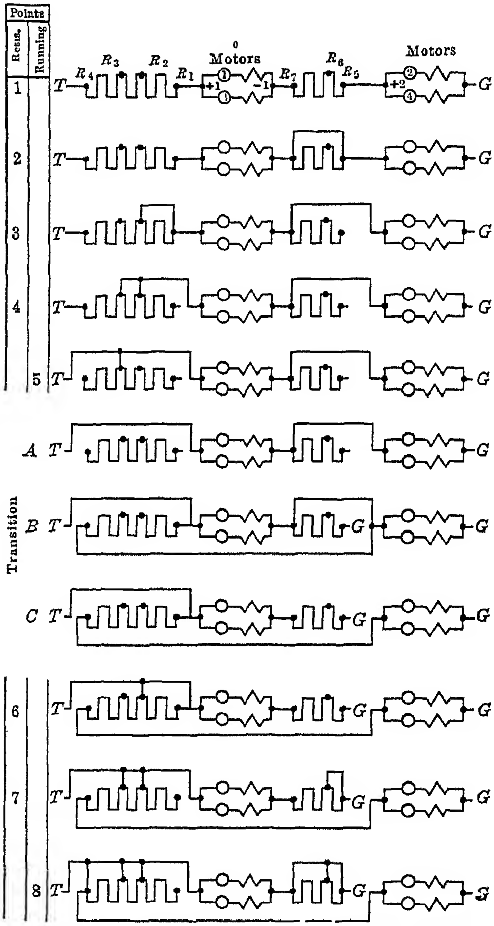


FIG. 220.—Bridge control, series-parallel system.

single controller, called a master controller, serves to operate the contactors of all the motor cars in the train, the auxiliary circuit being extended from end to end of the train.

A system similar to the type *M* is also used in large single cars,

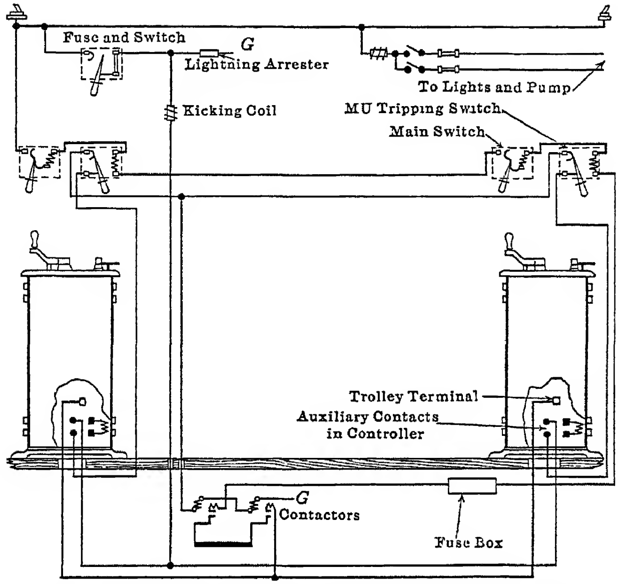


FIG. 221.—Railway motor controller with auxiliary circuits.

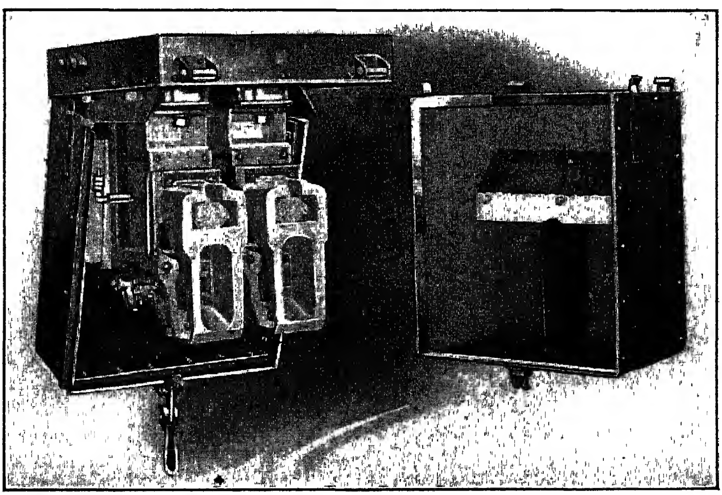


FIG. 222.—Contactor with cover removed.

where very heavy current through the controller itself might be objectionable. An elementary diagram of connections of such a controller is shown in Fig. 221, and the contactors are shown in Fig. 222. This type of control is not suitable on a system in which the trolley voltage is apt to be low, as at the end of a long feeder at times of heavy load; for in such a case it is possible that the current passing through the auxiliary circuit may be insufficient to operate the contactors.

139. Division of Load between Motors.—Two or more shunt motors designed for the same voltage, when connected in parallel to the same supply circuit, and with their shafts rigidly coupled, will divide the load in proportion to their capacities provided their speed-current curves (Fig. 192) are identical in the manner discussed in connection with Fig. 174; that is, if the speed curves, plotted in terms of per cent. of full-load current, are identical. The same thing is true of series motors operating in parallel.

Series wound motors when connected in series in a constant-current circuit, will develop approximately constant torque if the brushes are kept in a fixed position; but if the torque were constant the speed would have to vary in direct proportion to the load. To obviate this variation of speed, and in particular to keep the speed constant, the series motors in the Thury constant-current system (see Chap. VI) are provided with regulators which change the position of the brushes, thereby affecting the torque instead of the speed.

An interesting case of unequal division of load between series wound motors is afforded by the case of a car starting on an up-grade on slippery rails. Assume for example that the rear end of the car is more heavily loaded than the forward end; on turning the controller handle to the first notch, the same current will flow through both motors (or both pairs of motors) since they are in series with each other; therefore each will develop the same torque. If the weight on the forward trucks is fairly light, the adhesion between the wheels and the rail may not be sufficient to prevent slipping, in which case the forward motor will speed up and spin the wheels. The counter e.m.f. of the forward motor will increase as its speed rises, so that its impressed voltage must also increase; but any increase of the voltage on the forward

motor will be at the expense of that impressed on the already overworked rear motor, so that the result will be to stall the car unless the front wheels can be prevented from slipping.

PROBLEMS

1. A 230-volt, 25-h.p. shunt motor has an armature resistance of 0.134 ohm and a field resistance of 76.7 ohms. At full load the current input to the machine is 93.5 amp. Find the power loss in the armature and field windings, the counter e.m.f. and the total mechanical power developed by the motor at full load.

2. The motor of Problem 1 is provided with a series field winding whose resistance is 0.025 ohm. If the load is such that the total current input is 93.5 amp., solve for the same quantities called for in Problem 1 assuming (*a*) long-shunt connection; (*b*) short-shunt connection.

3. The shunt motor of Problem 1 takes a line current of 6.1 amp. at no load, and runs at a speed of 1175 r.p.m. What is the ideal zero load speed?

4. The shunt motor of Problem 1 has a magnetization curve such that an exciting current of 1.5 amp. produces two-thirds as much flux as that produced by a field current of 3.0 amp. What will be the (ideal) no-load speed if the field rheostat is adjusted to give a total shunt resistance of 85 ohms?

5. When the shunt motor of Problem 1 is carrying its full-load current, the armature demagnetizing amp.-turns per pole amount to 7 per cent. of the field amp.-turns per pole. Find the speed at (*a*) full load; (*b*) at a load such that the armature current is 150 per cent. of full-load current.

6. What resistance must be put in series with the armature of the shunt motor specified in the above problems to make it develop (*a*) full-load torque at the moment of starting; (*b*) a starting torque 50 per cent. greater than full-load torque?

7. Using the shunt motor specified in the above problems, and impressing successively voltages of 200, 250, 275 and 300 volts upon the machine terminals, what will be the corresponding (ideal) no-load speeds?

8. The series field winding specified in Problem 2, when connected long shunt, produces 20 per cent. as much excitation as the shunt winding when the armature carries full-load current. Find the speed of the motor when the series winding is connected (*a*) differentially; (*b*) cumulatively, the armature current being 90 amp.

9. Bearing in mind that a straight line through the origin drawn tangent to the magnetization curve (specified in Problem 4) represents the relation between flux and the excitation consumed by the air-gap, what will be the (ideal) no-load speed of the shunt motor of the preceding problems if the pole faces are bored out to such an extent that the effective length of air-gap is increased by 10 per cent.?

10. The shunt motor specified in Problems 1 to 8 is driven as a generator at a speed of 1200 r.p.m., the field rheostat being adjusted until the shunt

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exciting current is 2.8 amp. Find the terminal voltage when the armature current is 50 amp.; (*a*) when the direction of rotation results in a forward lead of the brushes; (*b*) when the direction of rotation is reversed.

11. A street car equipped with four motors, grouped in two pairs, the motors of each pair being permanently in parallel, is running under power. The controller is thrown to the off position and the reversing handle is turned to the reverse position. Specify what will happen and explain fully.

12. From the curves of Fig. 209 construct a new set of curves showing (*a*) armature speed in r.p.m.; (*b*) motor torque in lb.-ft., in terms of current input.

13. A car is equipped with four motors which have the characteristics shown in Fig. 209. Each motor is mounted on its own axle, the gear ratio being 25:76. The driving wheels of the forward pair of axles are 33 in. in diameter, while those of the rear pair are 32 in. in diameter. If the motors are in parallel and the speed of the car is 24 miles per hour, what is the total current taken by the car and what is the total tractive effort?

14. Determine the equations of the speed and tractive effort curves shown in Fig. 209.

CHAPTER VIII

COMMUTATION

140. Fundamental Considerations.—Each of the a parallel paths comprising the entire armature winding consists of $Z/2a$ turns in series in each of which the current is i_a/a amperes. As the commutator segments to which the terminals of the individual winding elements are connected pass under the brushes, the elements are successively switched from a path or circuit in which the current has one direction to an adjoining circuit in which the current has an opposite direction. During this transition period, or period of commutation, the current must be reduced from its original value to zero and then built up again to an equal value in the opposite direction. The period of commutation is of very brief duration, of the order of 0.0005 to 0.002 second, and it may easily happen that the reversal of current is either retarded or unduly accelerated; in either case, the current at the end of the period will tend to have a value which differs from that of the circuit to which the commutated coil is about to be connected, and the result of the final equalization is a spark between the brush and the commutator segment. The study of the commutation process therefore has for its object the determination of the conditions which will result in sparkless operation.

The time variation of the current in a winding element may be represented by a diagram such as Fig. 223, in which ordinates represent values of current and abscissas the time. Immediately before the beginning of the commutation period AB, the current in the coil under consideration has the value $+ i_0 = i_a/a$; after the completion of the commutation it must have a value $- i_0$, assuming that the winding is symmetrical and, therefore, that the currents in all of the armature circuits are the same. In the time interval $AB = T$, the current may vary in the manner shown by such typical curves as a, b, c, d, e, f,

each of which corresponds to a definite set of physical conditions. These curves are called the *short-circuit current curves.*

Curve a shows that the current has been reversed too rapidly, overreaching its final value, such a condition being characterized as overcommutation. Here the current may reach its proper final value without a spark, but it may involve such large localized current densities at the contact surface between commutator segment and brush as to lead to sparking and, perhaps, to glowing (incandescence) of the brush and certainly to excessive loss and heating and deterioration of the brush.

Curve b represents a case in which the current comes to its

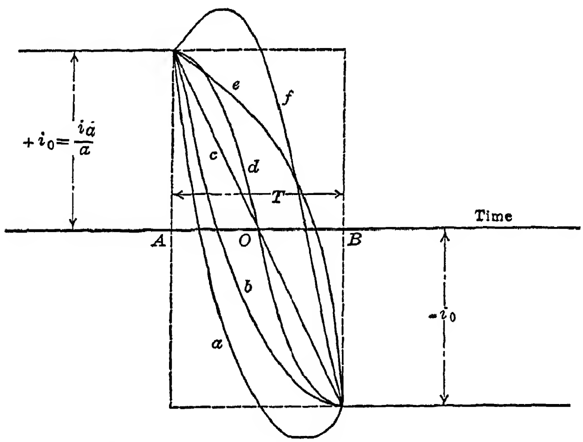


FIG. 223.—Types of short-circuit current curves.

final value smoothly, with a zero rate of change at the end of the commutation period. This will generally result in satisfactory commutation.

Curve c indicates a uniform transition of the current from its initial to its final value. When this occurs the commutation is said to be linear. Linear commutation is very desirable, for, as will appear later, it gives rise to uniform current density at the brush contact surface and the loss of power at the contact surface is a minimum.

Curve d represents the so-called "sinusoidal" commutation; the curve is one-half of a sine curve. Such a short-circuit current curve would generally result in satisfactory commutation.

Curve *e* represents a limiting case in which the final rate of change of current is infinite; that is, the curve is tangent to the vertical line drawn through *B*. Under such conditions sparking would invariably result.

Curve *f* shows "undercommutation," that is, the current is not reversed with sufficient rapidity. Even though the final value of current may be correct, this condition may involve excessive current density under the brushes and hence possible glowing, just as in the case of overcommutation.

It should be understood that these curves represent only the more important cases. In practice, the short-circuit current curves may assume an infinite variety of forms, subject always to the condition that the initial and final values of current must be equal in magnitude and opposite in sign, the armature winding being assumed to be symmetrical.

No account has here been taken of the effect of mechanical irregularities such as vibration of the brushes, unevenness of the commutator surface, etc. Such mechanical defects will invariably produce sparking even though the magnetic and electrical conditions are otherwise perfect. Vibration of the brushes causes the short-circuit current curves to take on a saw-tooth form.

141. Physical Basis of the Theory of Commutation.—The theory of commutation is much less advanced than that of other parts of the theory of direct-current machines; that is to say, the commutation characteristics cannot be predetermined with anything like the degree of accuracy that is possible in the calculation of the general performance characteristics. Notwithstanding this fact, practice based upon more or less empirical rules has so far outstripped theory that manufacturers commonly guarantee sparkless operation between no-load and 50 per cent. overload with a fixed setting of the brushes.

The elementary theory of commutation is relatively simple and has been extensively discussed by numerous writers. It involves the fact that the coil undergoing commutation has induced in it an e.m.f. of self-induction due to the changing current in the coil, the self-induced e.m.f. acting always in such a direction as to oppose the change of current; and in case the short-circuited coil is in inductive relation to one or

more coils which are simultaneously undergoing commutation there will also be induced in it an e.m.f of mutual induction. For these reasons the theory may be designated the "inductance" theory. There is also to be considered the fact that the short-circuited coil may be situated in a magnetic field—the fringing field near a pole tip, or the reversing field due to a commutating pole—and that the rotation of the coil through this field produces a generated e.m.f. in the coil. As has been previously pointed out, this generated e.m.f. should in general be so directed as to neutralize the retarding effect of self-induction, in which case the process is referred to as *voltage commutation.* If, however, the commutated coil is not acted upon by any extraneous field, that is, if there is no generated e.m.f. acting in it, the process is called *resistance commutation* inasmuch as the self-induced e.m.f. is controlled only by the ohmic drops in the coil and at the brush contact surface. Resistance commutation is largely relied upon in machines having a fixed brush position and no special commutating devices such as interpoles; to this end carbon brushes are employed, the high contact resistance serving to keep the short-circuit current within reasonable limits. Examples of this type of machines are afforded by railway and hoisting motors in which the brushes are permanently set at the geometrical neutral because of the frequent reversal of direction of rotation.

The ohmic drops at the transition surface between the commutator segments and the brushes, and in the short-circuited coils and their connecting leads are almost as important as the e.m.fs. due to self- and mutual-induction and to rotation through the magnetic field. The contact resistance between commutator and brushes is very much more complex in its nature than that of ordinary metallic conductors; it resembles that of the electric arc in many of its properties, for it is dependent upon such factors as the current density, the direction of the current, the temperature, material and chemical structure of the contact surfaces, and upon the nature of the current itself (continuous, alternating, or pulsating); it varies, moreover, with the contact pressure and the relative velocity of the surfaces.[1]

[1] A very complete presentation of the effects of all of these factors will be found in Arnold's Die Gleichstrommaschine, Vol. I.

It is probable that the passage of the current across the transition surface between commutator and brush causes an ionization of the gaseous layer between them and that this sets up a counter e.m.f. similar to that encountered in the arc stream. From this standpoint the drop of potential across the contact surface is the sum of the counter e.m.f. and the true ohmic drop; the quotient obtained by dividing the observed drop by the current is then not a true resistance, but what may be called an effective resistance, made up of the true resistance plus a fictitious resistance equivalent in its effects to the counter e.m.f. The transition layer between commutator and brush is the seat of an energy storage, and breakdown in the form of sparking may be expected when the amount of the stored energy exceeds a critical value. On this basis, neither current density nor transition drop taken separately is a sufficient criterion of the sparking limit; this is confirmed by an experiment of Professor Arnold's in which the current density passing from a carbon brush to a metal surface was raised until the brush glowed, but without producing sparking.

THE INDUCTANCE THEORY

142. General Equation, Case of Simple Ring Winding.—For the sake of simplicity there will first be considered a simple ring winding in which the brush width b is equal to the width β of a commutator segment. Under this condition only one winding element will be short-circuited at a time, as shown in Fig. 224, and the effect of the mutual induction of other coils is eliminated. It has been explained in Chap. VI that the axis of commutation must be slightly displaced from the neutral axis (in the direction of rotation in the case of generators), in order that the fringing field at the leading pole tip may generate in the short-circuited element an e.m.f. of sufficient magnitude to balance the retarding effect of the self-induced e.m.f. But during the commutation period the short-circuited coil moves through the fringing field from a position in which the generated e.m.f. has a certain value to another position in which the generated e.m.f. is appreciably larger, so that the reversing e.m.f. is not constant. If the distribution of the flux in the air-gap is determined experimentally, as by the pilot brush method de-

scribed in Chap. V, it will be found that, as a rule, the curve of flux distribution is approximately linear for short distances between pole tips; therefore, the commutating e.m.f. may be closely represented by the function

$$E_c = e + ht \tag{1}$$

where

t = time counted from the beginning of the commutation period

h = constant

e = commutating e.m.f. at the beginning of the period, when $t = 0$.

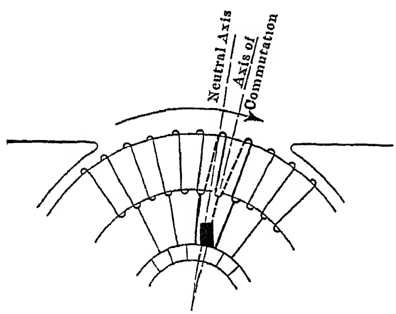


FIG. 224.—Short-circuited element, simple ring winding.

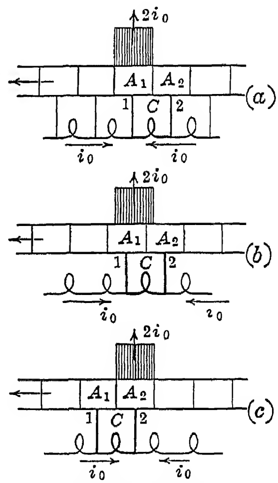


FIG. 225.—Successive phases of short-circuit of coil.

Parts a, b, and c of Fig. 225 represent, respectively, the initial, intermediate, and final stages of the commutation of a coil C. In position a the coil is a part of the right-hand branch of the winding and carries the current $i_0 = i_a/a$. All of the current reaching the brush from the two adjoining paths must then pass through lead 1. Similarly, in the final position c, coil C has become an integral part of the left-hand branch, its current has been fully reversed, and the combined current of the two paths must reach the brush by way of lead 2. The b position, in which coil C is short-circuited, shows that immediately after segment A_2 has reached the brush, the current from the right-

hand branch may reach the brush by way of both leads 1 and 2, and coil C therefore carries less current than before; as the contact area of segment A_1 diminishes and that of A_2 increases, the original current through C is diverted more and more from lead 1 to lead 2. At the same time that the right-hand branch current is being throttled in this way out of coil C, the left-hand branch current finds its way more and more readily through coil C and the increasing contact area A_2, and less and less readily through the diminishing contact area A_1.

If the transfer of the brush current, $2i_0$, from lead 1 to lead 2 occurs uniformly during the commutation period, *linear commutation* results. In that case the current in C will have zero value when the insulation between segments A_1 and A_2 is directly under the middle of the brush; and leads 1 and 2 will then each be carrying current i_0, from the left- and right-hand circuits, respectively. But if the axis of commutation is too near the leading pole tip (in the case of a generator) the e.m.f. generated in C by its motion through the field will act to accelerate the transfer of current from lead 1 to lead 2, therefore giving rise to abnormal current densities at the contact area A_2; this is the case of overcommutation, Fig. 223*a*. On the other hand, a commutating field that is too weak will delay the transfer of current from lead 1 to lead 2, so that the current density may become excessive at contact area A_1; this corresponds to the curve of undercommutation, Fig. 223*f*.

143. Elementary Mathematical Relations.—Fig. 226 is the same as Fig. 225*b* except that the currents in the various paths are indicated. It may be assumed that the currents from the left-hand and right-hand paths pass to the commutator by way of leads 1 and 2, respectively, and that the current in the coil C has a value which at any instant is i amperes, its path being completed through the brush. In the figure the current i is represented as flowing in a clockwise direction through the short circuit but at a later instant during the commutation period it will have reversed. From the figure it follows that the currents in leads 1 and 2 are, respectively,

$$\left.\begin{aligned} i_1 &= i_0 + i \\ i_2 &= i_0 - i \end{aligned}\right\} \tag{2}$$

$$\therefore i_1 + i_2 = 2i_0 = \text{total current from the brush.} \tag{3}$$

Now let

R_c = resistance of coil C
R_l = resistance of each commutator lead
R_b = resistance of contact area of the entire brush.

Counting the time t from the beginning of the short-circuit of coil C, the contact resistances of areas A_1 and A_2 are, respectively,

$$R_1 = R_b \frac{T}{T - t}, \quad R_2 = R_b \frac{T}{t}$$

In the closed circuit consisting of coil C, commutator leads 1 and 2, contact areas A_1 and A_2, the two commutator segments and the brush, the sum of all the potential drops (with due regard to sign) must equal zero, in accordance with Kirchhoff's law. The sign of any e.m.f. is to be taken positive or negative if it acts in, or in opposition to, some arbitrarily assumed positive

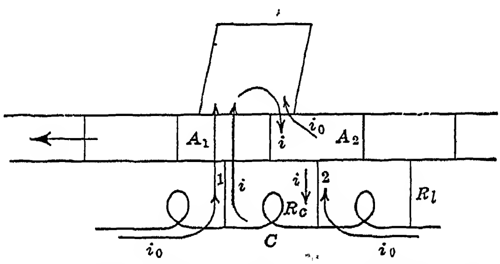


FIG. 226.—Current paths in short-circuited coil.

direction, respectively; similarly, a drop of potential due to a current flowing in the positive direction is to be taken with the negative sign, and *vice versa*. Taking the counter-clockwise direction as positive, we have

$-L\frac{di}{dt}$ = e.m.f. of self-induction (negative)
iR_c = ohmic drop in coil C (positive)
i_1R_l = ohmic drop in lead 1 (positive)
i_1R_1 = ohmic drop in contact area A_1 (positive)
i_2R_2 = ohmic drop in contact area A_2 (negative)
i_2R_l = ohmic drop in lead 2 (negative)
E_c = commutating e.m.f. (positive).

$$\therefore L\frac{di}{dt} + iR_c + (i_0 + i)R_l + (i_0 + i)R_b\frac{T}{T - t} - (i_0 - i)R_b\frac{T}{t} - (i_0 - i)R_l + E_c = 0 \tag{4}$$

which may be written

$$L\frac{di}{dt} + iR + \frac{R_bT}{T - t}(i_0 + i) - \frac{R_bT}{t}(i_0 - i) + E_c = 0 \quad (5)$$

where

$$R = R_c + 2R_l$$

This equation involves the justifiable assumption that the resistances of the commutator segments and of the brush are negligible.

The complete integration of this differential equation has been worked out subject to certain conditions[1] and results in an equation of the form

$$i = F(t)$$

subject to the terminal conditions that when $t = 0$, $i = i_0$, and when $t = T$, $i = -i_0$. The complete results of the integration are not essential in most cases, since it is generally the end of the commutation period that is most important so far as sparking is concerned.

144. Discussion of the General Equation.—At the last moment during the commutation process, when $t = T$, and $i = -i_0$

$$E_c = e + hT = E_T$$

$$\frac{R_bT}{t}(i_0 - i) = 2R_bi_0$$

$$\frac{R_bT}{T - t}(i_0 + i) = \frac{0}{0}, \text{ or indeterminate.}$$

The value of the expression $\left(\frac{i_0 + i}{T - t}\right)$ can, however, be evaluated by differentiating numerator and denominator separately with respect to the independent variable t, giving

$$\left.\frac{i_0 + i}{T - t}\right|_{t = T} = \frac{\frac{di}{dt}}{-1} = -\frac{di}{dt}$$

The general equation then reduces to

$$L\frac{di}{dt} - i_0R - R_bT\frac{di}{dt} - 2R_bi_0 + E_T = 0$$

or

$$\left(\frac{di}{dt}\right)_{t = T} = -\frac{i_0(R + 2R_b) - E_T}{R_bT - L} \quad (6)$$

[1] Über den Kurzschluss der Spulen und die Vorgänge bei der Kommutation des Stromes eines Gleichstromankers, by Paul Riebesell. Kiel, 1905.

From the last equation (6) there may be deduced several important conclusions as follows:

1. If $\frac{R_bT}{L} = 1$, the final value of $\frac{di}{dt}$ will be infinite, *i.e.*, $\left(\frac{di}{dt}\right)_{t=T} = \infty$, provided $i_0(R + 2R_b)$ differs from E_T. If this were the case, the e.m.f. of self-induction, $-L\frac{di}{dt}$, would also be infinite and sparking would result at the trailing edge of the brush. Therefore, $\frac{R_bT}{L}$ *must in general differ from unity.*

2. Inspection of Fig 223*a* shows that in case of overcommutation the final rate of change of current in the coil is positive in sign; since this condition of overreaching is to be avoided, the rate of change of current should always be negative, hence the numerator and denominator of equation (6) must be of the same sign. If, then,

$$\frac{R_bT}{L} > 1, \tag{7}$$

it follows that

$$E_T < i_0(R + 2R_b) \tag{7a}$$

and if

$$\frac{R_bT}{L} < 1 \tag{8}$$

it must follow that

$$E_T > i_0(R + 2R_b) \tag{8a}$$

In the expressons (7*a*) and (8*a*) the term $i_0(R + 2R_b)$ is only slightly greater than $2i_0R_b$, which represents the drop at the brush contact, and this is usually of the order of 1 volt with average carbon brushes. E_T can only be less than this value when the brushes are set very close to the neutral axis; but since many machines are required to operate with the brushes in a fixed position close to the neutral axis, it follows that in such cases the relation $\frac{R_bT}{L} > 1$ is the condition to be satisfied to insure good commutation, and it is accordingly this relation which is most commonly given as the all-important criterion

for sparkless operation. It is quite clear, however, from the pair of relations given by (8) and (8*a*) that $\frac{R_b T}{L}$ may be less than 1 provided E_T is sufficiently greater than the brush contact drop, and this is the case that actually occurs in high speed machines of large capacity such as turbo-generators; for in machines of this type the duration of the commutation period, T, is short and the inductance L is comparatively high because of the relatively long armature core. In such cases it is necessary to make E_T sufficiently large by the use of commutating poles, thereby definitely controlling the flux density in the neutral zone and consequently the e.m.f. generated in the short-circuited element. To sum up, the pair of relations (7) and (7*a*) holds

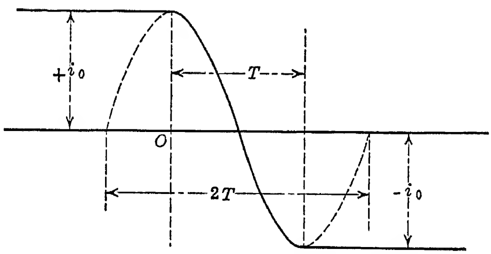


FIG. 227.—Sinusoidal commutation.

for the case of resistance commutation, where there are no special auxiliary devices; while (8) and (8*a*) relate to voltage commutation where special devices are used.

3. The final rate of change of current will be zero, *i.e.*,

$$\left(\frac{di}{dt}\right)_{t=T} = 0,$$

if

$$E_T = i_o(R + 2R_b) \tag{9}$$

in which case the conditions for good commutation will be favorable. This state of affairs is shown in curve *b*, Fig. 223.

145. Modified Form of Sparking Criterion.—The condition $\frac{R_b T}{L} > 1$ can be put into another form which has a simple physical

interpretation; multiplying both sides of the inequality by $2i_0L/T$, we have

$$2i_0R_b > \frac{2i_0}{T} L = e_r \tag{10}$$

The term $2i_0R_b$ is the drop of potential at the brush contact surface and is usually of the order of 1 volt with carbon brushes. The term $2i_0/T$ is the average rate of change of current during commutation, hence $e_r = 2i_0L/T$ is the average reactance voltage, or average e.m.f. of self-induction. It follows, therefore, that $e_r < 1$ is the condition to be satisfied.

The criterion $R_bT/L > 1$ shows that the brush contact resistance R_b and the time of commutation T must be large, and that L must be kept small. This accounts for the fact that carbon brushes are ordinarily superior to metal brushes, since the former have the larger contact resistance.

The self-inductance L can be kept within limits by designing the elements with a small number of turns, the inductance being proportional to the square of the number of turns; in large machines the elements are designed with only a single turn; ordinarily the number of turns per element should not exceed 2 or 3, though in railway motors the number is frequently 4 or 5. Furthermore, since the value of L is determined by the number of flux linkages per ampere of current in the coil, L can be kept down by limiting the axial length of the armature; this means, for a given capacity of machine, a relatively large diameter, hence the possibility of using a large commutator, a correspondingly large number of segments, and a small number of turns per segment and per element.

It would appear at first glance that an increase in brush width would give good results because of the increased value of T. But this apparent advantage is offset because of the fact that the wide brush simultaneously short-circuits several additional coils whose mutual inductance is equivalent to an increase in the self-inductance of the original coil (see Art. 154).

The criterion $R_bT/L > 1$, or $e_r = 2i_0L/T < 1$ is frequently expressed in a still different form. H. M. Hobart has proposed a method which assumes that the commutation curve (Fig. 223d) is one-half of a sine curve of period $2T$, as indicated in Fig. 227.

the maximum ordinate being i_0, whose equation, referred to origin O, is

$$i = i_0 \cos \frac{2\pi t}{2T}$$

The instantaneous value of the self-induced e.m.f. is then

$$e = -L\frac{di}{dt} = \frac{\pi}{T} L i_0 \sin \frac{2\pi t}{2T}$$

and its maximum value, which Hobart calls the reactance voltage, is

$$e_{max} = \frac{\pi}{T} L i_0 = \frac{2 i_0}{T} L . \frac{\pi}{2} = \frac{\pi}{2} e_r \tag{11}$$

Hence, if $e_r < 1$, $e_{max} < 1.57$. As usually stated, however, $e_{max} < 2$. It follows from the previous discussion that if $e_r > 1$, or $e_{max} > 2$, the commutating e.m.f. must be so adjusted that $E_T > i_0(R + 2R_b)$.

146. Linear Commutation.—Equation (5) can be utilized to determine the conditions necessary for a uniform transition of the current from its initial to its final value. Thus, in case of linear commutation, curve c, Fig. 223, the current i at any instant t is given by

$$i = i_0 - \frac{2 i_0}{T} t = i_0 \frac{T - 2t}{T}$$

and

$$\frac{di}{dt} = -\frac{2 i_0}{T}$$

Substituting these values in (5), there results after some transformation

$$E_c = i_0 \left[\frac{2L}{T} - \frac{R}{T}(T - 2t) \right] \tag{12}$$

It follows from (12) that when

$$t = 0, \; E_c = e = i_0\left(\frac{2L}{T} - R\right)$$

$$t = T, \; E_c = E_T = i_0\left(\frac{2L}{T} + R\right)$$

In other words, the commutating e.m.f. must not only vary as a linear function of the time, as shown by equation (12), but it must also change with the load since it is directly proportional to i_0. It follows, therefore, that if conditions for perfect linear commutation were satisfied for one particular load they would

not be satisfied for other loads, without special corrective devices.

An interesting consequence of linear commutation is that the current density at the brush contact is constant. Thus, referring to Fig. 226,

$$i_1 = i_0 + i$$
$$i_2 = i_0 - i$$

and if

$$i = i_0 \frac{T - 2t}{T}$$

it follows that

$$i_1 = 2i_0 \frac{T - t}{T}$$
$$i_2 = 2i_0 \frac{t}{T}$$

But the contact areas A_1 and A_2 are given by

$$A_1 = A \frac{T - t}{T}$$
$$A_2 = A \frac{t}{T}$$

where A is the total brush area; hence the current density is

$$\frac{i_1}{A_1} = \frac{i_2}{A_2} = \frac{2i_0}{A} = \text{constant.} \tag{13}$$

It can also be shown that the ohmic loss due to the resistance of the brush contact is a minimum in the case of linear commutation, that is, when the current density is uniform. For let it be assumed that the short-circuit current in coil C, Fig. 226, is not linear; in this case the actual non-linear current can be thought of as made up of a linear current, i_l, and an extra current, i_x, where the latter may have any general form. It follows then that

$$i = i_l + i_x$$

and

$$i_1 = i_0 + i = i_0 + i_l + i_x$$
$$i_2 = i_0 - i = i_0 - i_l - i_x$$

The contact resistances at the areas A_1 and A_2 are, respectively,

$$R_1 = R_b \frac{T}{T - t}$$

and

$$R_2 = R_b \frac{T}{t}$$

and the ohmic loss at the contact areas is

$$W_c = i_1^2 R_1 + i_2^2 R_2$$

Substituting for i_1, i_2, R_1 and R_2, and remembering that $i_l = i_0 \frac{T - 2t}{T}$, we find that

$$\begin{aligned} W_c &= 4 i_0^2 R_b + i_x^2 R_b \left(\frac{T}{T - t} + \frac{T}{t}\right) \\ &= 4 i_0^2 R_b + i_x^2 R_b \frac{T}{t(1 - t/T)} \end{aligned} \qquad (14)$$

from which it follows that the loss is a minimum if $i_x = 0$, *i.e.*, if the commutation is linear.

147. The Current Density at a Commutator Segment.—General Case.—The uniform current density over the brush width that is characteristic of linear commutation means that the drop of potential across the contact surface is everywhere the same and, therefore, that there are no differences of potential along the brush contact (in the peripheral direction) to be equalized by a flow of current. It follows, then, that if such inequalities of potential do exist, or tend to exist, extra currents will flow along the brush and complete their paths through the short-circuited coil or coils, thereby giving rise to non-linear short-circuit current curves and a non-uniform current density. Of course, the potential differences which produce the extra currents are due to the fact that the e.m.f. generated in the short-circuited coils differs in form, as a time function, from that which would produce linear short-circuit current. It is consequently important to determine in what manner the distribution of current density is affected by a non-linear short-circuit current. For this purpose the following graphical method, due to Professor Arnold,[1] may be used.

Consider, for example, a case where the brush width is 3.5

[1] Die Gleichstrommaschine, Vol. I, p. 438, 2nd ed.

times the width of a commutator segment, and let it be assumed that the current density at any given instant is the same over the entire area of that part of the segment covered by the brush. Assume also that all the coils successively undergoing commutation have identical short-circuit current curves. The current density at a particular segment S will then change from instant to instant as it passes under the brush. Three distinct phases of its motion may be recognized:

1st, the segment approaches the brush,
2d, the segment is covered by the brush,
3d, the segment emerges from the brush,

as shown in Fig. 228, parts a, b, and c, respectively.

Phase 1.—The current crossing from segment S to the brush is

$$i_2 - i_1 = i_0 - i_1$$

the upward direction of current through the segment being taken as positive.

Phase 2.—The current crossing from segment S to the brush is

$$i_2 - i_1$$

Phase 3.—The current crossing from segment S to the brush is

$$i_2 - i_1 = i_2 + i_0,$$

since in this position coil A is carrying current from the left-hand branch circuit, *i.e.*, $i_1 = -i_0$.

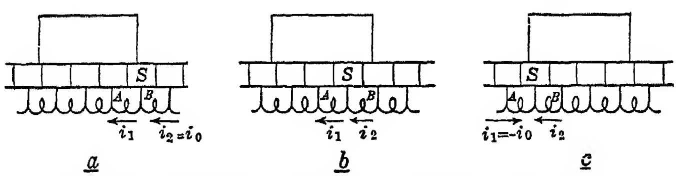


FIG. 228.—Successive phases of short-circuit, wide brush.

In Fig. 229, curves C_A and C_B represent the short-circuit current curves of coils A and B, respectively. They are drawn in their correct time positions with respect to the edge 1 of the brush, the line OO' being the axis of their ordinates; in the position shown in the figure the first phase of the motion of segment S is just beginning.

When the commutator has moved to the left a distance x_1 (during phase 1), the current in coil A is ab, and the current across segment S is

$$i_2 - i_1 = i_0 - i_1 = ac - ab = bc$$

The current density is proportional to

$$\frac{i_0 - i_1}{x_1} = \frac{bc}{Mc}$$

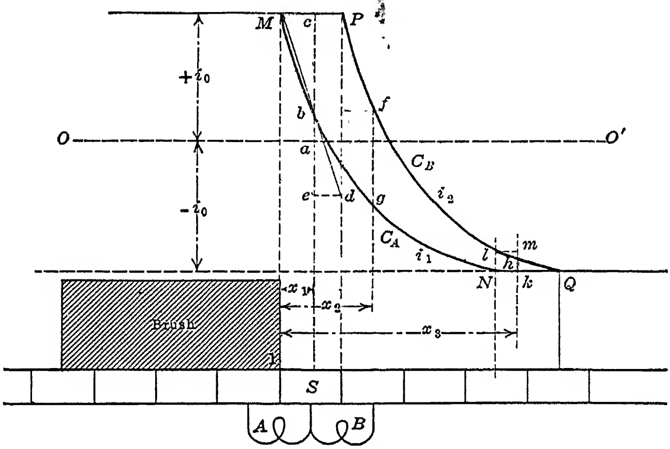


FIG. 229.—Current density at a segment.

Draw the straight line Mb and produce it until it cuts the vertical through P in the point d, then

$$\frac{bc}{Mc} = \frac{Pd}{MP}$$

and since MP is a constant length, Pd is proportional to the current density at segment S. Projecting the point d across to e, ce is the current density corresponding to the abscissa x_1. It is readily apparent from this construction that a too rapid initial reversal of the current in a coil (overcommutation) may result in excessive current density at a segment as it passes under the brush.

During the second phase, or after a travel of the commutator represented by x_2, the current across segment S, *i.e.*, $i_2 - i_1$, is given directly by the intercept fg between curves C_A and C_B. This intercept is also proportional to the current density to the same scale as ce.

During the third phase of the motion, or after a travel indicated by x_3, the current across S is

$$i_2 - i_1 = i_2 + i_0 = hk$$

At the same time the length of the segment still in contact with the brush is kQ, hence the current density is proportional to $\frac{hk}{kQ}$, or, to the scale previously adopted, it is represented by $lN = mk$. The point l is found by drawing the straight line Ql.

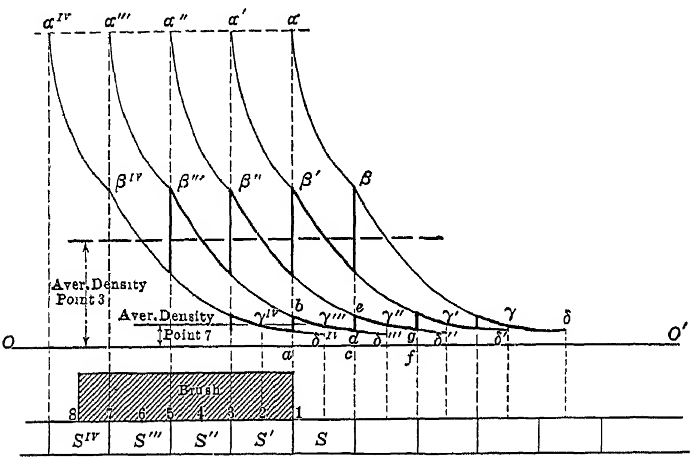


FIG. 230.—Curves of current density at segments.

Continuing this method for several points in each of the three phases of the process, the current density curve $\alpha\beta\gamma\delta$ of Fig. 230 is obtained. For convenience, the ordinates representing current density have been drawn upward with respect to the axis OO'. Similar curves, $\alpha'\beta'\gamma'\delta'$ and $\alpha''\beta''\gamma''\delta''$, etc., show the variation of current density of segments S' and S'', etc., respectively, and to the same time scale as that of curve $\alpha\beta\gamma\delta$.

148. Variation of Local Current Density at the Brush.—Except in the case of linear commutation, the current density is not the same at the same instant all along the arc of contact of the brush, nor does it remain constant at any given point. For instance, in Fig. 230 consider the point 7 of the brush, which

is just about to make contact with segment S'''. At this instant the current density of segment S''' is ab, as read from curve α''', and thereafter, until S''' has moved from under the point, the current density changes in accordance with curve bd; segment S'' then comes under point 7, the density rises suddenly from dc to ec, the point e being on curve α'', and again falls off to the value gf. The curve eg is, of course, the same as bd; in other words, the current density at a given point in the brush varies periodically. In precisely the same way the saw-tooth line with cusps at β, β', β'', etc., represents the variation of current density at point 3 of the brush.

If the average current density is found for various points along the brush, the results when plotted give the curve $1'2'3'\ldots8'$ of Fig. 231. This is the curve of average local current density that corresponds to the curve of commutation MN or PQ of Fig. 229. It is readily apparent from Fig. 231 that the local current densities may differ considerably from the average current density of the brush as a whole.

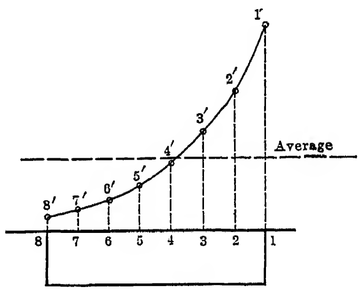
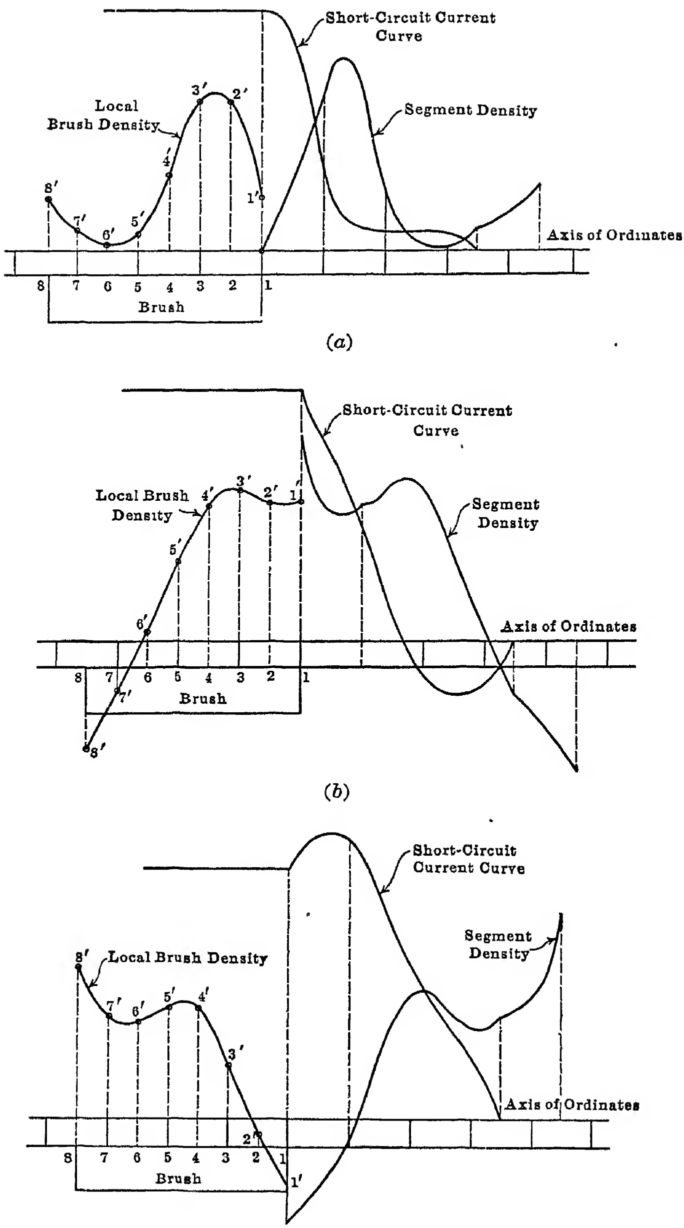


FIG. 231.—Average local current density at points along brush.

149. Further Examples.—Using the above methods, curves of current density have been constructed for several types of short-circuit current curves. They are shown in Figs. 232, a, b, and c. The case of linear commutation is not shown, since it is obvious from the previous analytical discussion, as well as from the geometry of the construction, that the current density is the same at all times both at the segments and at each point of the brush.

Figs. 232a and b represent cases in which the rate of change of current is too great during the initial stages of the commutation period. This results in correspondingly great current densities at the commutator segments as they come under the brush, and high average brush current density near the heel. In Fig. 232b the overreaching of the current in the coils causes a reversal of direction of current at the receding segments. Fig. 232c shows a case of undercommutation, with consequent reversal of direc-



(c)

FIG. 232.—Curves of current density at segments and brushes.

tion at the heel of the brush and excessive densities near the end of the commutation period.

150. Simultaneous Commutation of Adjacent Coils.—Inasmuch as the process of commutation in a coil is affected by the mutual induction of neighboring short-circuited coils, it is important to be able to predetermine the number and relative positions of those coils in the same neutral zone which are simultaneously short-circuited. In the case of the simple ring winding heretofore considered, all coils in the same zone are short-circuited by a single brush; if b is the brush width and β the width of a commutator segment, the ratio b/β fixes the number of coils short-circuited at the same time. This ratio is generally

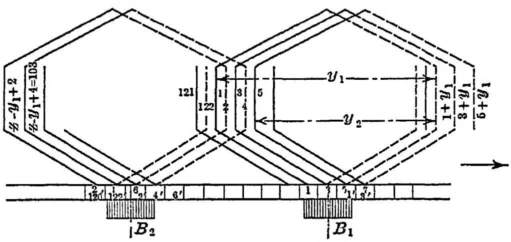


FIG. 233.—Simultaneous short-circuit of elements of lap winding.

a mixed number, and the actual number of coils short-circuited will vary alternately between the two integers lying on either side of it. In lap and wave windings, however, the conditions are as a rule not so simple, since in a given neutral zone some of the conductors are short-circuited by a brush of one polarity, others by a neighboring brush of opposite polarity, as illustrated in Fig. 233. The diagram represents a duplex lap winding having the following constants:

$$Z = 122 \qquad S = 61 \qquad p = 6 \qquad a = 12$$

$$y = m = \frac{a}{p} = 2 \qquad y_1 = 23 \qquad y_2 = -19 \qquad \frac{b}{\beta} = 2.5$$

It is clear from the figure that in the position shown, conductors 1 and 4 are simultaneously short-circuited; a moment earlier conductors 1, 3 and 4 were short-circuited. The successive combinations of short-circuited coils can be conveniently studied by means of the following graphical method, due to Professor Arnold.[1]

[1] Die Gleichstrommaschine, Vol. I, p. 354, 2nd ed.

1. *Lap Windings.*

It will be observed that in the winding here selected brushes B_1 and B_2 are not identically situated with respect to the segments of the commutator in contact with them. This is a consequence of the fact that S/p is not an integer.

Coil edges 1, 3, 5, etc., drawn in full lines to indicate that they occupy the tops of the slots, are connected to commutator segments which are correspondingly numbered in the top row of figures. The other sides of the same coils, whose numbers are $1 + y_1$, $3 + y_1$, $5 + y_1$, etc., are connected to segments which are numbered 1′, 3′, 5′, etc. (*i.e.*, dropping the term y_1 and priming the numeral) in the bottom row of figures. A coil will then be short-circuited when the brush B_1 is in contact with any pair of segments which bear the same numbers. A similar arrangement is indicated in the case of coil edges 2, 4, etc.

Now, coil edge 2 is connected to one on the left which is separated by a pitch y_1 from 2, and by $y_1 - 1$ from 1. Segment 2 is therefore separated from 1 by ½ $(y_1 - 1)$ segments. But brushes B_2 and B_1 are separated by S/p segments, hence the relative shift of segments in the vicinity of B_2 with respect to those at B_1 is

$$\triangle = \frac{S}{p} - \tfrac{1}{2}(y_1 - 1) \tag{15}$$

and is toward the left when $\triangle$ is negative, toward the right when it is positive. In the case considered in Fig. 233, $\triangle = -\tfrac{5}{6}$.

The simultaneous action of the two brushes can now be studied

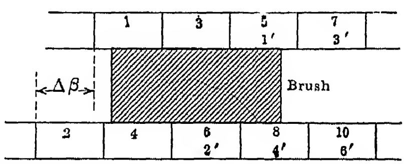


FIG. 234.—Diagram showing coils simultaneously short-circuited—lap winding.

by means of a diagram like Fig. 234; take a strip of paper cut to the width of the hatched area to represent the brush and slide it between the two commutators; when it touches segments similarly numbered, the corresponding coils will be simultaneously short-circuited.

2. *Wave Windings.*

Fig. 235 represents a portion of a duplex wave winding having the following constants:

$$Z = 122 \qquad S = 61 \qquad p = 6 \qquad a = 4$$

$$y = y_1 = y_2 = 21 \qquad \frac{b}{\beta} = 2.5$$

In the position shown, coil edges 122, 1, 2, 3, and 4 are short-circuited.

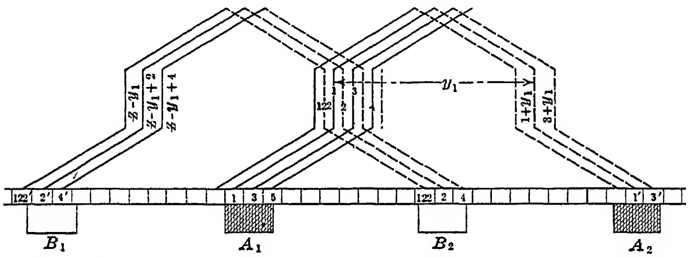


FIG. 235.—Simultaneous short-circuit of elements of wave winding.

Number the segments connected to coil edges 1, 3, 5, etc., with corresponding numbers, and the segments connected to the other sides of the same coils 1′, 3′, 5′, etc.; similarly with respect to the other coils occupying the same neutral zone, as 2, 4, 6.

Brushes A_1 and A_2, which are of the same polarity, are not similarly placed with respect to the segments in contact with them. The brushes are separated by $2S/p$ segments, while the ends of an element are separated by $y = \dfrac{2S}{p} \pm m$ segments, where $m = a/p$; segments 1′, 3′, 5′ are therefore shifted with respect to A_2 by an amount $m\beta$, as compared with the relative positions of segments 1, 3, 5 with respect to brush A_1. The shift is to the right if m is positive (as in the case illustrated), to the left if m is negative. The short-circuiting of these elements can then be shown by drawing two commutators one above the other, as in the upper part of Fig. 236.

Obviously, brushes B_1 and B_2, and the segments in contact with them, are related to each other in the same way as are A_1 and A_2. Now B_1 is separated from A_1 by S/p segments, while segment 2′ is separated from segment 1 by $\frac{1}{2}(y_1 - 1)$ segments,

The displacement of segments 2′, 4′, etc., with respect to B_1, as compared with that of segments 1, 3, 5 to A_1, is then

$$\Delta = \frac{S}{p} - \tfrac{1}{2}(y_1 - 1)$$

and is to the right if Δ is positive (in the winding considered $\Delta = +\tfrac{1}{6}$), to the left if it is negative. The complete relations are shown in Fig. 236.

If a strip of paper whose width is equal to that of the brush is moved across the fictitious commutators represented by A_1, A_2, B_1 and B_2, it will touch a series of similarly numbered segments and the corresponding coils will be simultaneously short-circuited.

151. Successive Phases of Short-circuit in Coils of a Slot.—The method described above may be used to investigate the order in which the coils occupying a given slot undergo commutation. Two distinct cases may be distinguished:

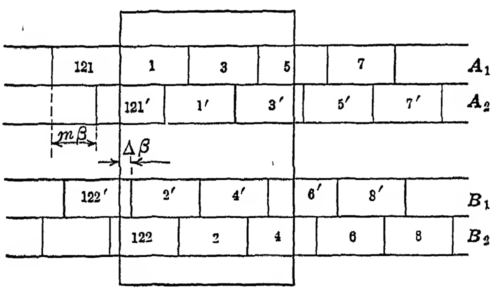


FIG. 236.—Diagram showing coils simultaneously short-circuited—wave winding.

1. Coil edges lying in the same radial plane (one above the other) enter and leave short-circuit simultaneously.

2. Coil edges lying in the same radial plane enter and leave short-circuit at different times.

Case 1.—If the coil edges are numbered in accordance with the system described in Art. 73 of Chap. III, and illustrated in Fig. 88, coil sides 1 and 2 of a two-layer winding will occupy the same radial plane, and so also will 3 and 4, 5 and 6, etc. Reference to Figs. 234 and 236 shows, therefore, that if coil sides 1 and 2, 3 and 4, or, in general, any two in the same radial plane, are to enter and leave short-circuit simultaneously, there must be

no displacement between the correspondingly numbered commutator segments; in other words, the condition to be satisfied is that

$$\Delta = \frac{S}{p} - \frac{1}{2}(y_1 - 1) = 0$$

or

$$y_1 = \frac{2S}{p} + 1 \tag{16}$$

For example, consider the case of a simplex lap winding having six coil edges per slot, a brush width of $2\frac{1}{2}$ segments, and $\Delta = 0$. With the help of a diagram like Fig. 234, but with Δ made equal to zero, it is readily shown that the successive phases of the short-circuiting of neighboring coils will follow the order shown in parts *a*, *b*, *c*, etc., of Fig. 237, where the shaded coils indicate short-circuit conditions. During a brief interval the condition shown in diagram *c* will exist, that is, all the coil edges in a slot will be simultaneously short-circuited; a little later, as in diagram *e*, six coil edges are again short-circuited, but four are in one slot and two in the next slot.

A study of Fig. 237 shows that when coil edges 1 and 2 leave short-circuit they are subject to the effect of mutual induction from the simultaneously short-circuited coils 3, 4, 5 and 6, all of which occupy the same slot. When coils 3 and 4 leave short-circuit they are subject to the mutual induction of coils 5 and 6, which are in the same slot, and of coils 7 and 8, which are in the next slot; obviously, because of this separation of the short-circuited group of coils, the inductive effect upon coils 3 and 4 will be smaller than in the case of coils 1 and 2. Similarly, when coils 5 and 6 leave short-circuit they are acted upon by the mutual induction due to the simultaneously short-circuited coils 7, 8, 9 and 10, all of which are in the slot adjacent to that occupied by 5 and 6, hence the inductive effect upon these two coils is still less than in the case of coils 3 and 4. The commutating conditions are, therefore, not the same in all of the winding elements, and their short-circuit current curves will have different forms.

An additional disturbing feature arises from the fact that when the successive coils of a slot, as 1–2, 3–4, 5–6, of Fig. 237 break contact with the brush, they are not identically situated with respect to the adjacent pole tip, consequently the e.m.fs. generated in

each of them during the final stage of the short-circuit (by cutting through the fringing field) will be different. This is due to the fact that the successive commutator segments are evenly spaced, while the coils, being grouped in slots, are not. Thus, in Fig. 237, coils 1 and 2 are ahead of 3 and 4, etc., with respect to the direction of rotation, and their short-circuit terminates when they are in a weaker field than that which acts upon 3 and 4 when the latter leave short-circuit. Similarly, coils 5 and 6 leave short-circuit when they are subjected to the action of a still stronger field than that which acts upon 3 and 4. If, therefore, the commutating e.m.f. acting upon coils 1 and 2 is just sufficient to overcome the e.m.f. of self- and mutual-induction therein, it will be more than sufficient to balance the smaller inductive e.m.f.

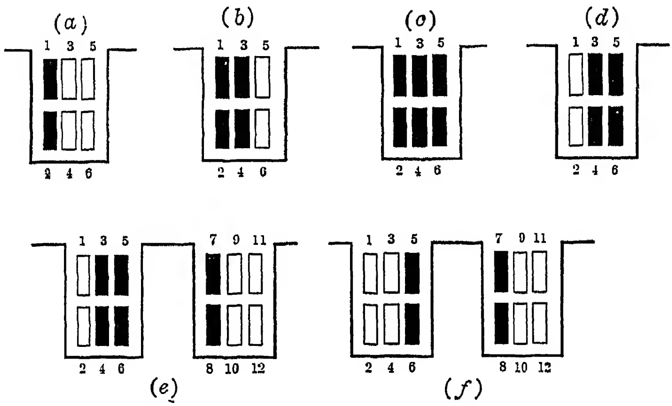


FIG. 237.—Successive phases of short-circuit in adjacent coils, $\Delta = 0$.

in 3 and 4, and much too great in coils 5 and 6. In the latter coils there will be a condition of overcommutation, and under these circumstances every third commutator segment may become blackened because of the possible excessive current density. In order that there may be no marked difference between the field intensities acting upon the various coils of a slot while they are undergoing commutation, the angle subtended by a slot should be small. For this reason the number of slots per pole should not be less than 12, and preferably greater than 12, and the angle subtended between the edges of a brush should not exceed one-twelfth of the angle from center to center of the poles.[1]

[1] Gray, Electrical Machine Design.

The order of commutation illustrated in Fig. 237 can occur only in full-pitch windings, since it is in such windings that the back pitch, y_1, is made nearly equal to $2S/p$. Chord windings (fractional pitch) are, therefore, characterized by the condition $\triangle \lessgtr 0$.

Case 2.—It follows from the above analysis that the second case arises when $\triangle \lessgtr 0$. An interesting variation of this case occurs when $\triangle = 1$, that is, when

$$y_1 = \frac{2S}{p} - 1 \tag{17}$$

Thus, if a winding has pitches that satisfy equation (17) and is arranged so that each slot contains six coil edges, the brush

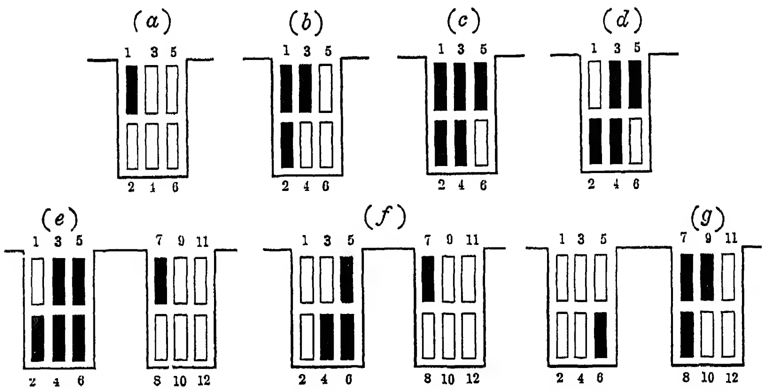


FIG. 238.—Successive phases of short-circuit in adjacent coils, $\Delta = 1$.

width being $2\frac{1}{2}$ commutator segments, the order of commutation of adjacent coils will be shown in Fig. 238. In this particular case, pairs of coils like 2 and 3, 4 and 5, etc., enter and leave short-circuit simultaneously.

152. Selective Commutation in Wave Windings.—A study of the simplex wave winding shown in Fig. 82 (p. 124) will show that the several brushes of one polarity are connected to each other not only by an external conductor but also through the winding by way of the coils that they short-circuit. The figure also shows that the resistances of these internal paths are not equal because of the varying areas of brush contact; further, the short-circuited coils are not at any instant identically located with respect to the

fringing fields through which they are moving, hence the e.m.fs. generated in them by rotation through the field, though small, are not the same in any two of them. Both of these facts are responsible for an unequal division of the total armature current between the several brushes. The unequal components of the total current shift from brush to brush in cyclical order, in such a way that Kirchhoff's laws are continuously satisfied. This shifting of the current values at the brushes in the case of wave windings is called selective commutation.

153. Duration of Short-circuit.—In the case of a simple ring winding the duration of short-circuit is simply the time required for a given point on the commutator to move through an arc equal to the width of a brush. But it will be seen from Figs. 234 and 236 that this simple relation does not hold in lap and wave windings, since a coil is short-circuited only when similarly numbered segments are simultaneously touched by the brush. These segments being displaced with respect to each other, the time of short-circuit may be either greater or less than in a ring winding.

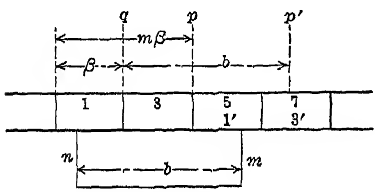


FIG. 239.—Diagram showing duration of short-circuit.

Consider the case of a multiplex lap winding, Fig. 239 (drawn to represent a duplex winding); the distance between corresponding edges of similarly numbered segments will be $m\beta = \frac{a}{p}\beta$, where β is the width of a segment. Short-circuit of coil 1 will endure while the edge m of the brush moves from position p until edge n reaches q. When n is at q, m will be at p', hence the short-circuit exists over the distance from p to p', which equals

$$b - (m\beta - \beta) = b + \beta\left(1 - \frac{a}{p}\right)$$

The time of short-circuit is then

$$T = \frac{b + \beta\left(1 - \frac{a}{p}\right)}{v_c} \tag{18}$$

where v_c is the peripheral velocity of the commutator.

An exactly similar result is obtained in the case of wave windings where the number of brush sets equals the number of poles. If one or more pairs of brush sets are omitted, the necessary correction can be applied by remembering that $m\beta$, Fig. 236, is the displacement corresponding to the distance between a given brush and the next brush of the same polarity. Therefore, if some of the brushes are removed, the term $m\beta = \frac{a}{p}\beta$ in the above equation for T must be multiplied by the number of double pole pitches in the region from which the brushes have been omitted.

In simplex lap windings $a/p = 1$, hence $T = b/v_c$, or the same as in a ring winding. T is less than this in multiplex lap windings. In wave windings, on the other hand, T is greater than in a ring winding, other things being equal, since $a/p < 1$.

154. Influence of Brush Width upon Average Reactance Voltage, and upon General Commutating Conditions.—The elementary relations discussed in Arts. 143, 144, and 145 with reference to a simple ring winding were developed on the assumption that the brush has the same width as a single segment of the commutator, so that only one element is short-circuited at a time by each brush; accordingly, the equations involve only the inductance of the coil undergoing commutation, since under the assumed conditions there is no mutual inductive effect upon it from neighboring coils. But in practice the brush width is always greater than the width of a single segment, consequently each coil as it undergoes commutation is affected by the mutual induction of simultaneously short-circuited coils as well as by its own self-inductance. The methods described in Arts. 147 to 153 provide means for determining the number and positions of the coils simultaneously short-circuited by wide brushes in the case of lap and wave windings, also the duration of the short-circuit period, and (provided the shape of the short-circuit current curve is known) the distribution of the current density under the brush. It remains, then, to amplify the elementary mathematical analysis so as to include the effect of wide brushes upon the average reactance voltage of the short-circuited elements of lap and wave windings.

It may as well be stated at the beginning that a rigorous treat-

ment of the general case covering any type of winding is practically impossible because there are so many interdependent factors entering into the problem that the mathematical difficulties are insurmountable. Fortunately, however, it is a relatively simple matter to derive formulas that are sufficiently accurate to serve as a guide, and which err on the side of safety.

Suppose, for example, that we consider first a *simplex lap winding of full pitch* having *two coil sides per slot* and in which the brush covers only *one segment*. Each element has $z = \frac{Z}{2S}$ turns. The coil edges belonging to simultaneously short-circuited elements will then lie one above the other, as indicated by the shading in Fig. 240*a*. The inductive effect upon a given element, C, is influenced not only by the change of current within itself but also by the change of current in the elements C' and C'', portions of which occupy the same slots as C; with the type of winding assumed, and provided that the magnetic circuits linked with all of the poles are symmetrical, the short-circuit current curves of elements C, C', and C'' will be identical, and the rate of change of current $\left(\frac{di}{dt}\right)$ will be the same in each element at the same instant. The total inductive e.m.f. at any instant will then be $(L + \Sigma M)\frac{di}{dt}$, where ΣM is the summation of the several coefficients of mutual induction, and the average reactance voltage will be

$$e_r = \frac{2i_0(L + \Sigma M)}{T} \tag{19}$$

which should be compared with equation (10).

It is possible to evaluate the combined coefficient $(L + \Sigma M)$ by a method due to Parshall and Hobart (see Art. 145). Thus, let

φ_s = number of lines of force that link with each inch of length of the "embedded" or slot part of coil C, per ampere-conductor of the group of conductors simultaneously short-circuited.

and let

φ_f = number of lines of force that link with each inch of "free" length of the end connections of coil C, per ampere-conductor of the group of conductors simultaneously short-circuited.

In each slot of length l' there are $2z$ conductors each carrying a current i, so that the flux linking one side of coil C is $\varphi_s \times l' \times 2z \times i$. In each group of end connections of length $\frac{1}{2}l_f$ there are z conductors, so that the flux linking one group of end connections is $\varphi_f \times \frac{1}{2}l_f \times z \times i$; the total flux linking both sides and both end connections of coil C is double the sum of the above fluxes, or it is

$$\varphi = 2zi\left(2\varphi_s l' + \frac{1}{2}\varphi_f l_f\right)$$

The equivalent inductance $(L + \Sigma M)$ is then the number of flux linkages per ampere divided by 10^8, or

$$L + \Sigma M = \frac{z\varphi}{i} \times 10^{-8} = 2z^2\left(2\varphi_s l' + \frac{1}{2}\varphi_f l_f\right) \times 10^{-8} \text{ henry} \quad (20)$$

Experiments made by Parshall and Hobart show that with average slot dimensions φ_s may be taken as 10 lines per ampere-conductor per inch of core (or 4 lines per ampere-conductor per centimeter of core) and φ_f as 2 lines per ampere-conductor per inch of free length (or 0.8 line per centimeter of free length). Accordingly, if inch units are used,

$$L + \Sigma M = 2z^2(20l' + l_f) \times 10^{-8} \quad (21)$$

Now let the brush width be increased until, let us say, the coil sides in three adjoining slots are simultaneously short-circuited, as indicated in Fig. 240*b*. The path of the leakage flux surrounding the entire group crosses three slots instead of only one as in Fig. 240*a*, hence the reluctance will be very nearly three times as great since the reluctance of the iron part of the path is negligible in comparison with that of the non-magnetic content of the slots; but the total m.m.f. acting around this leakage path is also three times as great as before, hence the actual flux linking an element is very nearly the same as before and $(L + \Sigma M)$ will have the same value as (21). On the other hand, the duration of the short-circuit is likewise three times as great as before, hence the average reactance is reduced to one-third of the value corresponding to the winding of Fig. 240*a*.

Consider windings like those indicated in Fig. 240*c* and *d*; in case (*c*), where the brush again covers only segment, the leakage flux is only one-third as great as in case (*a*), and the period of

commutation is the same, hence the average reactance voltage of (*c*) is one-third that of (*a*). In case (*d*), the leakage flux is the same as in (*a*) because both m.m.f. and reluctance have been increased in the ratio of three to one, while the period of commutation is three times as great, hence the average reactance voltage is again only one-third of that of (*a*).

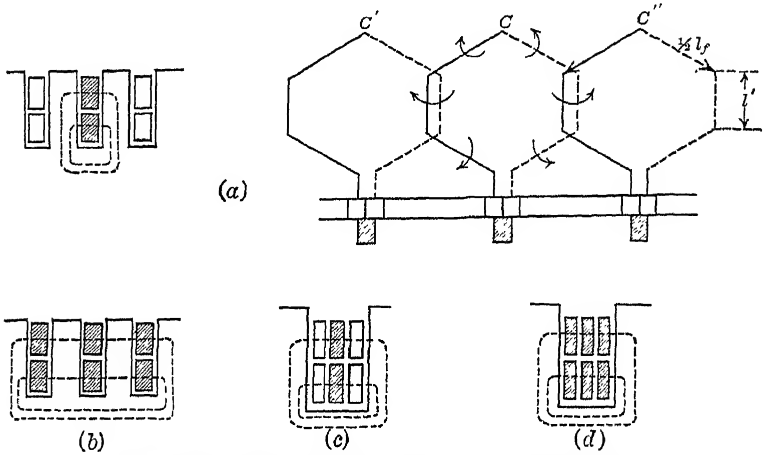


Fig. 240.—Effect of brush width upon commutation.

The above conclusions may be summarized in the formula

$$e_r = \frac{2i_0\,(L + \Sigma M)}{T} = \frac{2i_a}{a}\,\frac{L + \Sigma M}{T} \tag{22}$$

where

$$L + \Sigma M = 2z^2(20l' + l_f) \times 10^{-8}$$

and

$$T = \frac{b + \beta\left(1 - \frac{a}{p}\right)}{v_c}$$

In the case of simplex lap windings, $a = p$, hence $T = \frac{b}{v_c} = \frac{60}{nS} \times$ segments covered by brush, where n = r.p.m., and S = total number of commutator segments; hence for this special case, and under the assumption of full pitch elements,

$$e_r = 1.33 \times 10^{-8}\,\frac{z^2 nSi_a\,(l' + 0.05l_f)}{a \times \text{no. of segments covered by brush}} \tag{23}$$

l' and l_f being in *inches*. In this formula, if the number of seg-

ments covered by the brush is a mixed number (such as $2\frac{1}{2}$), the number to be substituted is the next higher integer.

If the shape of the slots departs materially from the average types used by Parshall and Hobart in their experiments, the empirical constants used in the above computation of $(L + \Sigma M)$ will have to be modified. More refined methods for calculating L and M are given in later articles in this chapter. But aside from this limitation it should be noted that in extending formula (23) to the case of wide brushes short-circuiting more than one element at a time, the assumption was tacitly made that the currents in all of the coils simultaneously undergoing commutation are at a given instant in the same phase of their variation and that their time rates of change are also equal at all times. This is clearly not the case in reality, and to the extent to which the actual facts depart from these assumed conditions the formula will be in error; as a matter of fact the changing currents in the neighboring short-circuited coils have a more or less differential effect upon one another, hence the results of formula (23) are more or less pessimistic and therefore err on the side of conservatism.

In the case of *fractional pitch* lap windings, some of the coil edges in the simultaneously short-circuited group will occupy positions in the manner shown in Fig. 238, parts *e*, *f*, and *g*. The leakage flux crossing the slot portions of a given coil will therefore be somewhat less than in the corresponding full pitch winding, hence the average reactance voltage will also be less, other things being the same. It is difficult to summarize all of the possibilities into a single formula; the best procedure is to make such a diagram as Fig. 238, selecting that particular grouping which gives the highest value of leakage flux, and estimating the leakage flux in the same general manner indicated above for the case of a full pitch winding, or by using the more accurate methods of Arts. 160 and 161.

In simplex *wave windings* where only *two brushes* are used, each brush short-circuits $p/2$ elements in series, as may be seen from Figs. 80 and 82, Chap. III. Accordingly formula (23) must be modified by multiplying it by $p/2$, or

$$e_r = 1.33 \times 10^{-8} \frac{z^2 n S i_a (l' + 0.05 l_f)}{a \times \text{no. segments covered by brush}} \times \frac{p}{2} \qquad (24)$$

In the case of wave windings in which all of the p possible brushes are used, each element is separately short-circuited during the period of commutation, and the $p/2$ elements comprising the group short-circuited by brushes of the same polarity are in parallel instead of in series. If the current divided equally between these $p/2$ elements, the average reactance voltage of each of them might be calculated from equation (23); but because of the so-called selective commutation in such windings, described in Art. 152, the average reactance voltage is higher than is indicated by that equation; on the other hand, equation (23) involves a period of commutation, T, that is somewhat too small, equation (18) showing that in wave windings T is somewhat greater than in simplex lap windings, other things being equal. These two factors therefore tend to counterbalance each other.

Summarizing the conclusions drawn from diagrams a, b, c, and d, Fig. 240, it will be seen that the average reactance voltage is the same in cases b, c, and d, and less in each of them than in case a. When the number of coil sides per slot is greater than two, as in cases c and d, an increase in brush width has no effect upon the average reactance voltage so long as the maximum number of simultaneously commutated coil sides does not exceed the number per slot. But if the brush width is increased until the number of simultaneously commutated coil sides exceeds the number per slot (compare cases b and a), there is a decrease in the average reactance voltage.

Variation of Potential Drop across Brush Width.—It has been shown in Art. 146 that if the commutation is linear the current density is uniform under the brush and that the loss of power is a minimum. If the commutation is non-linear it may be thought of as being the resultant of two currents in the local circuit, one of them following a linear law and the other a more or less complex law. The linear component combines with the main or working current of the armature to form a single current that flows straight across the contact surface on its way to (or from) the external circuit. Accordingly, if the commutation were linear, the drop of potential across the contact surface would be the same at all points, and there would be no difference of potential between points on the commutator that lie under the same brush. But in case the commutation is non-linear, the extra

component is equivalent to a circulating current within the local path formed by the short-circuited element and the brush, and under this condition the drop of potential across the brush contact will not be the same at all points.

Thus, in Fig. 241, suppose that one terminal of a low reading voltmeter is connected to the brush holder and the other is connected to the point of a lead pencil of medium hardness that can be touched successively to four or five equidistant points on the commutator between the heel and toe of the brush. With the machine running at normal speed and with full load, the plotted voltmeter readings may give such typical curves as A, B, or C. Curve B indicates a nearly uniform drop all along the arc of brush contact, hence nearly linear commutation. Curve A shows excessive drop and current density at the heel of the brush and a reversal of current flow at the toe, indicating a condition of overcommutation (see Art. 149); curve C is typical of under-commutation, the current density being too great at the toe of the brush. If a curve like A is obtained from an interpole machine the remedy is to reduce the excitation of the commutating pole; if the measurements give a curve like C, the excitation should be increased.

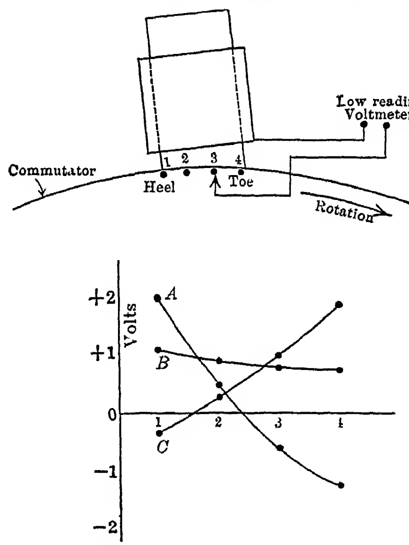


FIG. 241.—Measurement of brush contact drop.

155. Relations between Commutating E.M.F., Reactance Voltage and Brush Drop.[1]—It has already been pointed out that the principal condition to be fulfilled in order to insure satisfactory commutation is that the generated e.m.f. in the coil

[1] The student is advised to read a very excellent paper by B. G. Lamme, on Physical Limitations in D.-C. Commutating Machinery, Trans. A.I.E.E., Vol. XXXIV., Part II, p. 1739 (1915).

undergoing commutation shall as nearly as possible neutralize the e.m.f. of self- and mutual-induction throughout the commutation period. Consequently, if the e.m.f. due to self- and mutual-induction can be estimated to a reasonable degree of accuracy, and if this e.m.f. is found to be within limits that can be reached by the aid of such a device as a commutating pole, a possible difference between their values may, if it is sufficiently small, be absorbed in the resistance of the short-circuit path including the brush contact. On the other hand, if there is no special provision for generating a commutating e.m.f. in the short-circuited coil, as when the machine is required to operate with a fixed setting of the brushes near the neutral point in the fringing field of the main poles, the e.m.f. of self- and mutual-induction must be within the limit that can be absorbed by the resistance of the brush contact and of the short-circuited coil itself, as explained in Arts. 144 and 145 for the elementary case of a single coil.

It is clear, therefore, that the brush-contact resistance plays a very important part in the commutation process, particularly in large machines designed for heavy current output; for in such cases the brush-contact resistance is a large part of the resistance of the entire short-circuit path. In machines having relatively small current rating the cross-section of the armature conductors is correspondingly small and the resistance of the coil and leads becomes effective in limiting the short-circuit current, so that here the brush-contact resistance is of somewhat less importance than in large machines. The greater the resistance of the brush contact, the greater may be the difference between the e.m.f. of self- (and mutual-) induction and the commutating e.m.f., thereby enlarging the range of load within which commutation will be satisfactory. In general, the contact resistance of carbon brushes increases with the hardness of the material. A machine which fails to commutate satisfactorily when fitted with soft brushes may sometimes be improved by substituting brushes of a harder grade.

The resistance of the brush contact is not constant, but decreases with increasing current and with increasing temperature. When the average current density over the brush contact surface is about 35 to 40 amperes per sq. in., the drop of potential at the

contact surface is from 1 to 1.25 volts with ordinary carbons, and for higher values of current density the brush contact drop increases only very slowly. This is illustrated in Fig. 266, p. 355, data for which are obtained by reading the brush drop for various current densities after temperature conditions have become steady; for example, if the current density is suddenly raised from one value to a higher value, the brush drop does not immediately change in accordance with Fig. 266, but rises at first quite considerably and then falls to the value consistent with Fig. 266. This is due to the fact that the temperature does not rise immediately to its final value, hence the contact resistance falls to the new value only after a considerable time. It follows, therefore, that a momentary overload of considerable magnitude may occur without serious sparking at the commutator because of this tendency to maintain the original contact resistance; but if the overload continues, the contact resistance will soon fall to a point where it is no longer possible to limit the short-circuit current in the commutating element, and sparking will then begin.

It does not necessarily follow that commutation is satisfactory because of absence of sparking. The variation of current in the short-circuited coil may be such that there is no sparking and yet the current density may be excessive under certain portions of the brush, in the manner shown by curves A and C of Fig. 241. The loss of power due to irregular distribution of current under the brush may not be serious so far as it affects the efficiency of the machine, but it raises the temperature of the commutator and brushes, and this is detrimental for the reasons given in the preceding paragraph. Furthermore, excessive current density prevents the development of the smooth polish of the commutator and of the contact surface of the brush that is characteristic of good commutating conditions, and may result in pitting of the brush surface and blackening of the commutator.

The fact that the contact drop at the brush surface increases only very slowly with increasing current density imposes a definite limit upon the allowable difference between the commutating e.m.f. and the reactance voltage. For example, if this difference is 2 volts, the circulating current under the brush will reach a value corresponding to a contact drop if about 1 volt, since the

current must cross the contact surface twice; but if the unbalanced voltage is 3 volts, the circulating current will reach the much greater value corresponding to a contact drop of 1½ volts. With ordinary carbons, the brush drop will absorb an unbalanced voltage of from 2 to 2½ volts without excessive local circulating currents.

Up to a definite limit, the difference between the commutating e.m.f. and the reactance voltage will be equalized without sparking by the circulating current under the brush. The resultant current density may at times reach values that will cause glowing of the brushes but without sparking; but beyond a certain concentration of energy under the brush, sparking will result.

The difference between the commutating e.m.f. and the reactance voltage must be kept within proper bounds at all loads within the allowable working limits of the machine, from no load to the specified amount of overload. At no load there is no reactance voltage since the armature current is then zero (or very nearly zero), so that under this condition the commutating e.m.f. must be within the sparking limit.

In non-interpole machines the commutating e.m.f. is generated by the magnetic field in the interpolar space, and the intensity of this field is due to the resultant of the m.m.fs. of the field and armature windings which act in that region. If there were no saturation the commutating field would also be the resultant of the individual fields produced by the field winding and by the armature winding acting separately. In actual calculations this last fact is made use of because of its greater simplicity, and the disturbing effects of saturation must then be allowed for according to the judgment of the designer and his experience with the particular type of machine under consideration. The curves of flux distribution due to the field and armature windings can be determined by the methods described in Chap. V, and in the commutating zone will have the form indicated in Fig. 242, which represents the case of a shunt generator with a forward lead of the brushes. It will be seen that the strength of the commutating field decreases with increasing load, even if the field excitation remains constant, the change being unfortunately in the wrong direction inasmuch as the commutating e.m.f. should increase with the load. If the armature is magnetically

too powerful, or if the brushes are not given a sufficient lead, the commutating e.m.f. may even reverse under load, assuming as before that there are no special commutating devices present.

It follows, therefore, that in constant-speed generators and motors of the separately excited or shunt type, the commutating field and the e.m.f. generated by it decrease gradually with increasing load. This is shown in Fig. 243*a*, from which it may be concluded that at maximum load (generally taken as 25 per cent. overload) the reactance voltage must be considerably less than double the sparking limit in order that the difference between reactance voltage and commutating e.m.f. may remain within the sparking limit. On the other hand, in over-compounded generators the commutating e.m.f. increases with increasing load,

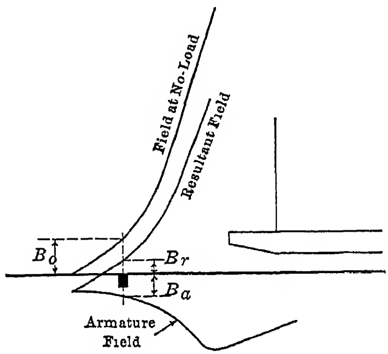


FIG. 242.—Commutating field distribution.

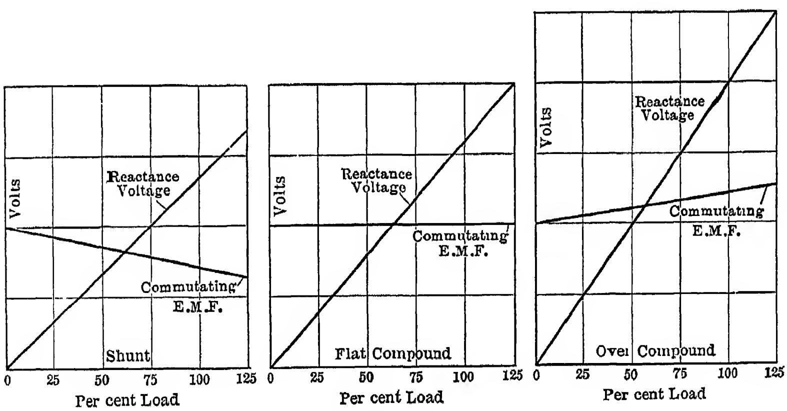


FIG. 243.—Unbalanced voltage in coil undergoing commutation.

so that in machines of this type the reactance voltage at maximum load may be more than double the sparking limit without introducing difficulties. In either case, if the excess of the reactance

voltage over the commutating e.m.f. at 125 per cent. load is equal to the commutating e.m.f. at no load, and the latter is within the sparking limit, the two e.m.fs. will exactly neutralize each other at approximately five-eighths of full load.

The above discussion explains the superior commutating properties of over-compounded or series-wound generators. Furthermore, if the armature cross-magnetizing effect in the commutating zone is neutralized, and a commutating field is set up such that it increases proportionally with the load, the reactance voltage can be almost exactly balanced at all loads and ideal commutating conditions will result. These effects can be secured by the use of *interpoles* or *commutating* poles, which are placed midway between the main poles and excited by the armature current (see Chap. IX).

In the case of wide brushes short-circuiting several coils, the drop of potential at a given point on the brush contact surface, as determined by a measurement such as is illustrated in Fig. 241, is not necessarily constant even under steady load conditions, but may pulsate in the manner indicated with respect to current density in Fig. 230. The voltmeter reading of Fig. 241 is then merely an average value. These pulsations result in high frequency currents under the brush, and these currents must be limited by the contact resistance to safe values. They are related to the form of the short-circuit curve, and the latter is in turn dependent upon the various voltages in the coil undergoing commutation. Experience seems to indicate that satisfactory operating results will be attained if the maximum e.m.f. generated in a short-circuited coil is limited to about 4 volts at no load. Referring to the symbols used in Fig. 242, the magnitude of this no-load e.m.f., per element, is

$$E_{c0} = \frac{Z}{2S} l'vB_0 \times 10^{-8}$$

where B_0 is the average strength of field in the commutating zone, $Z/2S$ is the number of turns per element, and v is the peripheral velocity in cm./sec. This assumes that the coils are of full pitch, so that both sides are at the same time cutting fields of equal intensity. In the case of fractional pitch windings it is necessary to add algebraically the unequal e.m.fs. generated

in each side of the coil. Under load conditions the commutating e.m.f. per element becomes

$$E_{cl} = \frac{Z}{2S} l'v\,(B_0 - B_a) \times 10^{-8} \tag{25}$$

with similar reservations concerning chorded windings. Hence,

$$E_{c0} = \frac{Z}{2S} l'v\,B_0 \times 10^{-8} < 4 \text{ volts.} \tag{26}$$

It is evident from these considerations that the value of B_0, the flux density in the commutating zone, must be sufficiently under control to allow the brushes to be placed in a position which will insure good commutation under all loads. To this end the field strength in the neighborhood of the commutating pole tip should shade off gradually instead of abruptly, a condition which can be realized fairly well by making the air-gap at the pole tips longer than it is under the central part of the pole shoes.

156. Pulsations of Commutating Field.—During the period of commutation the rotation of the armature periodically changes the positions of the teeth and slots with respect to the pole shoes, thereby giving rise to peripheral oscillations of the armature flux in the interpolar space. The changing current in the short-circuited coils produces a further pulsation of the flux in this region. There may also be pulsations of the flux as a whole, due to periodic changes in the reluctance of the magnetic circuit if the surface of the teeth presented to the poles does not remain constant. All of these effects are of high frequency and induce in the short-circuited coils e.m.fs. of rapidly changing direction which are superposed upon the main e.m.fs. considered in the preceding sections. They give rise to saw-tooth notches in the short-circuit current curves. Obviously these pulsations can be reduced by using numerous small slots with few coil edges per slot.

157. Reaction of Short-circuit Current upon Main Field.—*Flashing Over.*—Let a and b, Fig. 244, represent the initial and final positions of a coil undergoing commutation in a generator. In the a position the coil clearly exerts a demagnetizing magnetomotive force upon the main magnetic circuit, while in the b position the action is a magnetizing one. If commutation takes

place, on the average, in the geometrical neutral, and if the short-circuit current curve is symmetrical with respect to the point O (Fig. 223), the demagnetizing and magnetizing effects annul each other. But if the mid-point of the commutation period occurs when the coil is in the geometrical neutral, and the short-circuit current curve is not symmetrical, one or the other of the two effects will preponderate. Thus, in the case of overcommutation, the current will have the direction of the b position during the greater part of the time of short-circuit, hence there will be a resultant magnetizing action. In case of undercommutation the resultant action will be demagnetizing. Obviously, these statements are to be reversed in the case of a motor.

When a generator is running without load, currents will be set up in the coils short-circuited by the brushes. The direction of the current flow will depend upon the direction of the field in which the coils are moving, and, therefore, upon the direction of displacement of the brushes. With a forward lead of the brushes the current in the short-circuited coils will have the direction shown in the b position, Fig. 244, and will then exert a magnetizing effect; a backward lead of the brushes will evidently result in a demagnetizing action. The magnitude of these no-load short-circuit currents may be sufficiently great to materially influence the field flux, hence also the experimentally determined open-circuit characteristic.

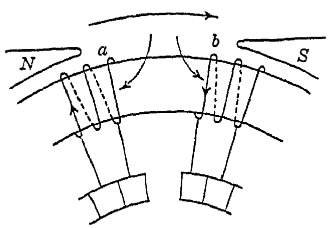


FIG. 244.—Magnetizing action of short-circuited elements.

If a generator is suddenly short-circuited at its main terminals when running with full excitation, the current in the coils undergoing commutation may rise to enormous values, and the inductance of the coils will tend to delay the reversal of this current, thereby producing a powerful demagnetizing effect. This will be particularly the case in commutating-pole machines. The natural effect of this demagnetizing action would be to weaken the main flux of the machine and therefore relieve the severity of the short-circuit; but the main flux of the machine represents a very considerable amount of stored energy, and this energy cannot instantly be changed. Consequently the main flux

decreases very slowly in spite of the large demagnetizing action of the coils undergoing commutation, and it follows that this demagnetizing action must be neutralized by a correspondingly largely increased current in the main field winding. Looked at in another way, the coils short-circuited by the brushes are inductively related to the field winding in much the same manner as are the primary and secondary windings of a transformer or induction coil. The sudden increase of current in the coils under the brushes caused by the principal short-circuit induces a reflected current impulse in the field coils. Since the resistance of the field winding has not been changed, the sudden increase in the exciting current will result in a marked rise of voltage at the terminals of the field winding, and this rise of voltage may be sufficient to puncture the insulation of the field winding.

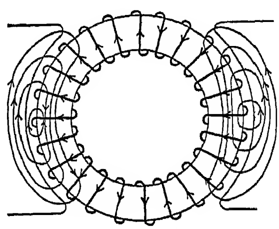


FIG. 245.—Linkage of cross-flux with armature winding elements.

If the voltage between adjacent segments of the commutator is too high, an arc may be established between them. When this condition exists from segment to segment between brushes of opposite polarity it is equivalent to a short-circuit of the entire machine, and is called a *flash over*. Flashing over may occasionally be caused by a dirty commutator, but is due chiefly to electromagnetic conditions in the machine itself.

The sudden increase of armature current due to a short-circuit or to an overload will be accompanied by a nearly proportional increase in the cross flux set up by the armature current, Fig. 245. In the process of building up, this flux reacts inductively upon the armature winding with which it links and induces in the winding elements an e.m.f. proportional to the time rate of change of the current. The more sudden the increase of armature current the greater will be the induced e.m.f. If the average value of the potential difference between adjacent segments is fairly high under normal load conditions, the additional e.m.f. may cause a spark discharge between the tips of adjacent commutator segments and so provide an ionized or conducting path for the main current. This induced e.m.f.

will be greatest in the elements located midway between the brushes. In machines that are normally subject to heavy rushes of current, as in motors used to drive reversing rolling mills, this tendency to flash over may be very serious, and in such cases special means must be employed to eliminate the danger. This is accomplished by the use of compensating windings which neutralize the armature cross flux, and which are described in Chap. IX.

158. Sparking Constants.—The expression

$$e_r = \frac{2i_0}{T} L$$

which has been shown to be of considerable importance can be put into another form involving the principal design constants of the machine, and may therefore be used as a check upon the magnitudes of the constants selected before the design has proceeded too far. Thus, it has been shown that

$$T = \frac{b + \beta\left(1 - \frac{a}{p}\right)}{v_c}$$

and

$$L = \frac{4\pi}{10^9} z^2 l' \times F$$

where F is a function of the dimensions of the machine. We have also

$$z = \frac{Z}{2S}$$

and

$$v_c = v \frac{d_{com}}{d}$$

where

v = peripheral velocity of armature
d = diameter of armature
d_{com} = diameter of commutator.

Also,

$$\frac{Zi_0}{\pi d} = q = \text{ampere-conductors per cm. of periphery.}$$

Substituting these values in the expression for e_r, we have

$$e_r = C\frac{Z}{S}l'vq \tag{27}$$

where

$$C = \frac{2\pi\beta F \times 10^{-9}}{b + \beta\left(1 - \frac{a}{p}\right)}$$

The quantity $\frac{Z}{S}l'vq$ may be considered as characteristic of commutating conditions; it has an average value, using metric units, of about 20×10^6. In English units (l in inches, v in feet per minute, and q in ampere-conductors per inch of periphery) it becomes approximately 40×10^6.

159. The Armature Flux Theory.—The distorted air-gap flux due to the magnetizing action of the armature current can be thought of as compounded of the fluxes which would be produced by the field and armature m.m.fs. acting separately. Strictly speaking, this is not exactly true when saturation of the iron part of the magnetic circuits exists, but it will serve as a first approximation to the truth. If, then, the brushes are set so that commutation takes place in the geometrical neutral, the short-circuited coils will not be acted upon by the field component of the flux, but the armature flux, being stationary in space, will generate e.m.fs. in them in exactly the same way that the main flux acts upon the conductors under the poles. The state of affairs in a generator will be as shown in Fig. 246. This figure brings out the fact that the e.m.f. generated by the armature flux in the short-circuited coil is in the same direction as the original current flow and therefore tends to prevent the desired reversal of current. Evidently this e.m.f. must be neutralized by an e.m.f. of opposite direction and somewhat greater magnitude in order that there may be a sufficient surplus e.m.f. to effect the reversal. This reversing e.m.f. may

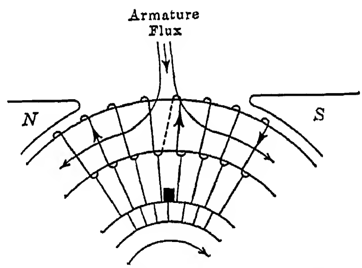


FIG. 246.—Production of armature flux.

be obtained by shifting the brushes until commutation takes place in a sufficiently intense part of the field flux; or the brushes may be kept in the neutral axis and the armature m.m.f. wiped out by the opposing m.m.f. of an interpole or its equivalent (see Chap. IX).

A full treatment of the subject of commutation based on this analysis has been worked out by B. G. Lamme.[1] It has the advantage that it emphasizes the physical phenomena involved in the commutation process. It does not involve the explicit consideration of the e.m.fs. of self- and mutual-induction, though these are implicitly involved in the e.m.fs. generated by cutting the stationary armature flux. For example, in the paper referred to there occurs the following statement:

> "According to the usual theory, during the commutation of the coil the local magnetic flux due to the coil is assumed to be reversed. However, in the zone in which the commutation occurs, certain of the magnetic fluxes may remain practically constant in value and direction during the entire period of commutation. This is but one instance, of which there are several, to show where there is apparent contradiction of fact in the usual mathematical assumptions made in treating this problem."

The reversal of the self-excited flux which is responsible for the e.m.f. of self-induction is not, however, a mere mathematical abstraction in the inductance theory; for while this flux does reverse *with respect to the coil*, it does not reverse its direction in space as will be clear from Fig. 244, which shows a short-circuited coil in two successive positions, before and after the reversal of the current. This is a consequence of the motion of the coil. If, however, this self-excited flux is considered as producing the e.m.f. of self-induction, it should be eliminated in the computation of the commutating flux; in other words, the latter is then to be taken as the resultant of the field flux and that part of the armature flux which is due to the armature turns outside of the commutating zone.

160.[2] Calculation of the Self-inductance, L, in Slotted Armatures.—The self-inductance of a coil has been shown to be equal

[1] A Theory of Commutation and Its Application to Interpole Machines, Trans. A.I.E.E., Vol. XXX, Part 3, 1911, p. 2359.

[2] Arts. 160 and 161 may be omitted if design is to be studied separately.

to the number of flux linkages per ampere, divided by 10^8. In the case of an armature coil embedded in a slot, the self-excited flux linking with the coil may be separated into three parts:

1. The flux crossing the slot from wall to wall of the teeth and completing its path through the core, as indicated by φ_1, Fig. 247.
2. The flux passing from tip to tip of the teeth within the space between pole tips as indicated by φ_2.
3. The flux φ_3 linking with the end connections beyond the edges of the core.

The number of linkages due to each of these fluxes will now be computed separately.

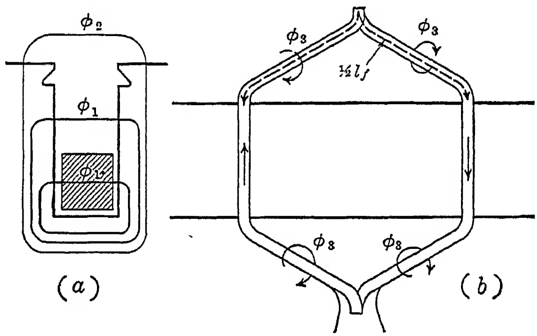


FIG. 247.—Paths of leakage flux surrounding coil.

1. SLOT LEAKAGE FLUX.—Practically without exception the windings of all direct-current machines are arranged in two layers, one side of each coil being in the top layer, the other side in the bottom layer. The magnitude and distribution of the flux is therefore not the same on the two sides of a coil.

a. Coil Edge Occupying the Bottom of a Slot, Fig. 248.—The coil edge contains $z = Z/2S$ conductors, whose total m.m.f. per unit current is $\frac{4\pi}{10}z$ gilberts. The m.m.f. acting upon an elementary tube dx will then be

$$\frac{4\pi}{10}\frac{x}{h_1}z,$$

assuming that the lines of force pass straight across the slot. The flux produced in this elementary path is then

$$d\varphi'_1 = \frac{\dfrac{4\pi}{10}\dfrac{x}{h_1}z}{\dfrac{b_s}{l'dx}}$$

where l' is the corrected length of the armature core, all dimentions being in centimeters. The denominator of the above expression represents the reluctance of the air part of the path, that of the iron part being negligible in comparison. This flux links with $\frac{x}{h_1}z$ conductors, hence the number of linkages due to it is

$$\frac{4\pi}{10}\left(\frac{x}{h_1}z\right)^2\frac{l'dx}{b_s}$$

and this may be reduced to henrys by dividing by 10^8.

$$\therefore dL'_{1b} = \frac{4\pi}{10}\left(\frac{x}{h_1}z\right)^2\frac{l'dx}{b_s} \times 10^{-8}$$

The total number of linkages through the entire depth of the coil is found by integrating in x from 0 to h_1 or

$$L'_{1b} = \frac{4\pi}{10^9}\frac{z^2l'}{h_1{}^2b_s}\int_0^{h_1} x^2dx = \frac{4\pi}{10^9}z^2l'\frac{h_1}{3b_s} \tag{28}$$

Above the coil the m.m.f. has the constant value $\frac{4\pi}{10}z$. Within the region h_2 the flux is uniformly distributed and has the magnitude

$$\varphi''_1 = \frac{\dfrac{4\pi}{10}z}{\dfrac{b_s}{h_2l'}}$$

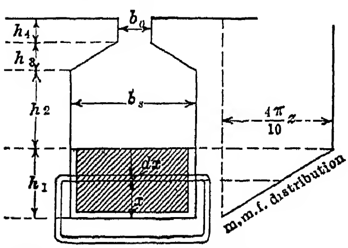


FIG. 248.—Slot leakage flux, coil occupying bottom of slot.

and since it links with all of the conductors,

$$L''_{1b} = \frac{4\pi}{10^9}z^2l'\frac{h_2}{b_s} \tag{29}$$

In the same way the inductances due to the flux in the regions h_3 and h_4 are

$$L'''_{1b} = \frac{\frac{4\pi}{10}z}{\frac{b_0 + b_s}{2h_3 l'}} z \times 10^{-8} = \frac{4\pi}{10^9} z^2 l' \frac{2h_3}{b_0 + b_s} \tag{30}$$

and

$$L^{\mathrm{IV}}_{1b} = \frac{4\pi}{10^9} z^2 l' \frac{h_4}{b_0} \tag{31}$$

The total inductance due to slot leakage is

$$L_{1b} = L'_{1b} + L''_{1b} + L'''_{1t} + L^{\mathrm{IV}}_{1b} = \frac{4\pi}{10^9} z^2 l' \left(\frac{h_1}{3b_s} + \frac{h_2}{b_s} + \frac{2h_3}{b_0 + b_s} + \frac{h_4}{b_0}\right) \tag{32}$$

In the case of straight slots, Fig. 249*a*, this reduces to

$$L_{1b} = \frac{4\pi}{10^9} z^2 l' \left(\frac{h_1}{3b_s} + \frac{h_2}{b_s}\right) \tag{33}$$

and with the shape of slot Fig. 249*b* it becomes

$$L_{1b} = \frac{4\pi}{10^9} z^2 l' \left(\frac{h_1}{3b_s} + \frac{h_2}{b_s} + \frac{2h_3}{b_0 + b_s} + \frac{h_4}{b_s}\right) \tag{34}$$

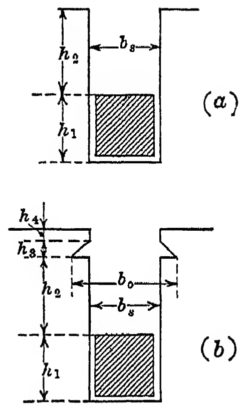


FIG. 249.—Coil occupying straight slots.

b. Coil Edge Occupying the Top of a Slot, Fig. 250.—Using the same methods as in case *a*, the resulting expressions for L_{1t} are identical with those for L_{1b} except that h_2 is replaced by h'_2. The total inductance of the coil due to slot leakage is then

$$L_1 = L_{1b} + L_{1t}, \tag{35}$$

or less than twice L_{1b}.

2. TOOTH-TIP LEAKAGE FLUX.—Assume that the lines of force from tip to tip of teeth are made up of a straight portion, b_0, and of two quadrants of circles, as in Fig. 251. The flux in an elementary path dx is

$$d\varphi_2 = \frac{\frac{4\pi}{10}z}{\frac{b_0 + \pi x}{l'dx}}$$

hence the total inductance for both sides of the coil is

$$L_2 = 2 \times \frac{4\pi}{10^9} z^2 l' \int_0^{\frac{1}{2}(\tau - b)} \frac{dx}{b_0 + \pi x} = \frac{4\pi}{10^9} z^2 l' \times 1.46 \log_{10}\left[1 + \frac{\pi(\tau - b)}{2b_0}\right] \quad (36)$$

The expression $(\tau - b)$ represents the distance between pole tips, and the superior limit of the integral is taken to be half of this amount on the assumpt on that the coils undergoing commutation are approximately midway between pole tips. Values of L_2 calculated by the above equation will be somewhat too large because no account has been taken of the effect of neighboring slot openings in reducing the flux.[1]

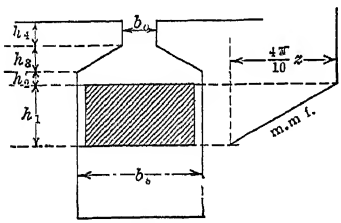


FIG. 250.—Coil occupying top of slot.

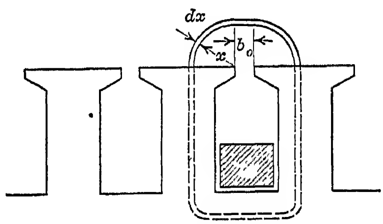


FIG. 251.—Tooth-tip leakage.

3. END CONNECTION LEAKAGE FLUX.—Various approximate formulas have been developed for calculating the inductance due to the end connections.

Niethammer[2] gives

$$L_3 = z^2 l_f \left[0.4 \log_{10}\left(\frac{l_f}{s}\right) - 0.1\right] \times 10^{-8} \quad (37)$$

where s is the diagonal of the rectangular coil section (including insulation between turns) and l_f (see Fig. 247b) is the total free length per element.

[1] The limit of integration used in the above equation has been checked by numerous tests, and gives results that agree fairly well with actual measurements. Arnold (Die Gleichstrommaschine) uses the entire pole pitch as the superior limit; Gray (Electric Machine Design) uses only one tooth width.

[2] Elektrische Maschinen Apparate u. Anlagen, 1, p 139. Stuttgart, 1904.

Arnold[1] gives

$$L_3 = z^2 l_f \left[0.46 \log_{10} \left(\frac{\frac{1}{2} l_f}{d_s} \right) - 0.092 \right] \times 10^{-8}$$

$$= z^2 l_f \left[0.46 \log_{10} \left(\frac{l_f}{d_s} \right) - 0.23 \right] \times 10^{-8} \qquad (38)$$

where d_s is the diameter of a circle whose circumference equals the perimeter of the coil section including the insulation between turns (see Fig. 252); *i.e.*,

$$d_s = \frac{2(a+b)}{\pi}$$

Hobart calculates the end-connection leakage on the basis of a flux of 0.8 line per ampere-conductor per cm. of free length.

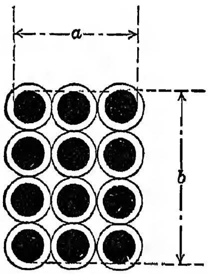


FIG. 252.—Cross section of coil.

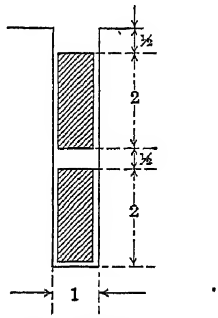


FIG. 253.—Relative slot dimensions, particular case.

The total inductance of a winding element is the sum of the inductances due to the several parts of the leakage, or

$$L = L_1 + L_2 + L_3 =$$

$$\frac{4\pi}{10^9} z^2 l' \left[\left(\frac{2h_1}{3b_s} + \frac{h_2 + h_2'}{b_s} + \frac{4h_3}{b_0 + b_s} + \frac{2h_4}{b_0} \right) + \right.$$

$$\left. 1.46 \log_{10} \left(1 + \frac{\pi}{2} \cdot \frac{\tau - b}{b_0} \right) \right] + \frac{z^2 l_f}{10^8} \left[0.4 \log_{10} \frac{l_f}{s} - 0.1 \right] \qquad (39)$$

Parshall and Hobart have published[2] the results of measurements of the inductance of armature coils of commercial machines from which they deduce that, on the average, the flux linked with the coils is at the rate of 4 c.g.s. lines per ampere-conductor

[1] Die Gleichstrommaschine, Vol. I, p. 376, 2nd ed.

[2] Electric Generators, 1900.

per cm. of "embedded" length of wire (10 lines per inch), and 0.8 c.g.s. lines per ampere-conductor per cm. of "free" length (2 lines per inch). These values check fairly well with the results of the foregoing formulas when customary dimensions are inserted. Thus, consider a machine with straight open slots whose ratio of depth to width is 5 : 1 (Fig. 253) and in which $\frac{\tau - b}{b_0}$ is approximately 10, which is equivalent to about five slots in the space between poles.

Taking $z = 1$ and $l' = 1$ cm.,

$$L_{1b} = \frac{4\pi}{10}(2/3 + 3) \times 10^{-8}$$

$$L_{1t} = \frac{4\pi}{10}(2/3 + 1/2) \times 10^{-8},$$

or an average of $\frac{4\pi}{10}(2/3 + 1\frac{3}{4}) \times 10^{-8} = 3.05 \times 10^{-8}$. The value of L_2 for one side of the coil, with $z = 1$ and $l' = 1$, is

$$\frac{4\pi}{10} \times 0.73 \log_{10}[1 + 15.7] \times 10^{-8} = 1.11 \times 10^{-8}$$

or a total of 4.16×10^{-8} henrys, corresponding to 4.16 lines per ampere-conductor per cm. of length. Parshall and Hobart's method is rapid and simple, but it is open to the objection that the designer must exercise great discretion in selecting the proper unit value of flux to fit the dimensions of his machine.

161.[1] **Calculation of the Mutual Inductance, *M*.**—The mutual inductance of two coils is equal to the number of flux linkages with one of them when a current of 1 ampere flows through the other, divided by 10^8.

The previous discussion of the simultaneous short-circuiting of several coils indicates that there are two cases to be considered: one in which the coils in question occupy the same slot, the other in which they lie in different slots.

1. Coils Occupying the Same Slot.—In this case two distinct conditions are possible: (*a*) the coil edges lie side by side; (*b*) the coil edges lie one above the other, or one in the top layer, the other in the bottom layer.

(*a*) It is clear that if the coil edges lie side by side in the slots,

[1] See foot-note, p. 336.

and therefore also throughout their entire lengths, there is no great error in writing

$$M = L$$

(*b*) Since the coil edges are in different layers, the end connections run in opposite directions, hence the mutual inductance is due only to the slot and tooth-tip fluxes.

In Fig. 254 the cross-hatched areas represent the two sides of a coil. On the left-hand side the inducing coil is at the bottom of the slot and exerts a m.m.f. of $\frac{4\pi}{10}z$ gilberts per ampere on the elementary tube dx; the slot flux linkages are

$$\int_0^{h_1} \frac{\frac{4\pi}{10}z}{\frac{b_s}{l'dx}}\left(z\frac{x}{h_1}\right) + \frac{4\pi}{10}z^2l'\frac{h'_2}{b_s} = \frac{4\pi}{10}z^2l'\left[\frac{h_1}{2b_s} + \frac{h'_2}{b_s}\right]$$

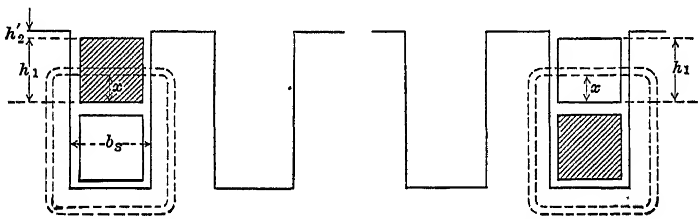


FIG. 254.—Flux leakage paths, mutual inductance.

On the right-hand side, the slot flux linkages are

$$\int_0^{h_1} \frac{\frac{4\pi}{10}\left(z\frac{x}{h_1}\right)}{\frac{b_s}{l'dx}}z + \frac{4\pi}{10}z^2l'\frac{h'_2}{b_s} = \frac{4\pi}{10}z^2l'\left[\frac{h_1}{2b_s} + \frac{h'_2}{b_s}\right]$$

or the same as on the other side of the coil.

The linkages due to the tooth-tip leakage flux are obviously the same as in the calculation of L. Finally, therefore,

$$M = \frac{4\pi}{10^9}z^2l'\left\{\left(\frac{h_1}{b_s} + \frac{2h'_2}{b_s}\right) + 1.46\log_{10}\left[1 + \frac{\pi}{2}\frac{\tau - b}{b_0}\right]\right\} \quad (40)$$

2. COILS OCCUPYING DIFFERENT SLOTS.—(*a*) *Both Coils in the Same Layer.*—In this case the sides of the coils will be parallel to each other throughout their entire lengths, and the inter-

linked flux will consist of tooth-tip leakage flux along the embedded portion and end-connection flux along the free lengths. Considering the tooth-tip leakage first, coil edge 1, Fig. 255, acts upon paths surrounding coil edge 2 with a m.m.f. of $\frac{4\pi}{10}z$ gilberts per ampere. In the tube dx the flux linkages are then

$$\frac{\frac{4\pi}{10}z}{\frac{b_0 + \pi(t + x)}{l'dx}}z$$

and the mutual inductance due to this flux on both sides of the coil is

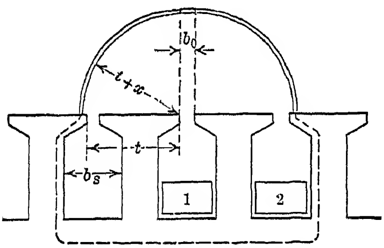


Fig. 255.—Mutual inductance, coils in adjacent slots.

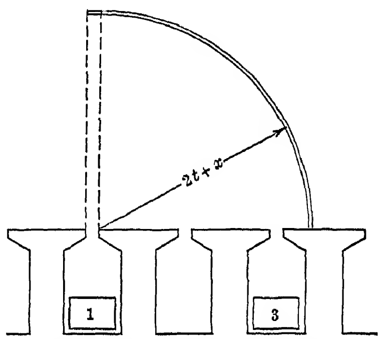


Fig. 256.—Mutual inductance, coils not in adjacent slots.

$$M'_{12} = 2 \times \frac{4\pi}{10^9} z^2 l' \int_0^{\frac{\tau - b}{2} - t} \frac{dx}{b_0 + \pi(t + x)} =$$

$$\frac{4\pi}{10^9} z^2 l' \frac{4.6}{\pi} \log_{10} \frac{b_0 + \frac{\pi}{2}(\tau - b)}{b_0 + \pi t} \quad (41)$$

This value is somewhat too large since in carrying out the integration continuously to the pole tips the effect of the slot openings is ignored.

The mutual inductance due to end-connection leakage is difficult to estimate. Arnold recommends taking it as one-half of the corresponding self-inductance of a single coil. On this basis

$$M''_{12} = z^2 l_f \left[0.2 \log_{10}\left(\frac{l_f}{s}\right) - 0.05\right] \times 10^{-8} \tag{42}$$

and

$$M_{12} = M'_{12} + M''_{12} \tag{43}$$

When the coils considered are not in adjoining slots, but are placed as in Fig. 256, the above equations become

$$M'_{13} = 2 \times \frac{4\pi}{10^9} z^2 l' \int_0^{\frac{\tau - b}{2} - 2t} \frac{dx}{b_0 + \pi(2t + x)} =$$

$$\frac{4\pi}{10^9} z^2 l' \frac{4.6}{\pi} \log_{10} \frac{b_0 + \frac{\pi}{2}(\tau - b)}{b_0 + 2\pi t} \tag{44}$$

and

$$M_{13}'' = \tfrac{1}{4} L_3 = z^2 l_f \left[0.1 \log_{10}\left(\frac{l_f}{s}\right) - 0.025\right] \times 10^{-8} \tag{45}$$

$$M_{13} = M'_{13} + M''_{13} \tag{46}$$

It is not necessary to carry the computation beyond the case shown in Fig. 256 for the reason that the numerical values become relatively small and the brushes are seldom so wide that coils are simultaneously short-circuited in more than three consecutive slots.

(b) *Coils not in the Same Layer.*—In this case the mutual inductances M'_{12} and M'_{13}, due to tooth-tip leakage, remain the same as before; the end-connection leakage reduces to zero because the coils separate and run in opposite directions after leaving the slots. Then

$$M_{12} = M'_{12} = \frac{4\pi}{10^9} z^2 l' \frac{4.6}{\pi} \log_{10} \frac{b_0 + \frac{\pi}{2}(\tau - b)}{b_0 + \pi t} \tag{47}$$

and

$$M_{13} = M'_{13} = \frac{4\pi}{10^9} z^2 l' \frac{4.6}{\pi} \log_{10} \frac{b_0 + \frac{\pi}{2}(\tau - b)}{b_0 + 2\pi t} \tag{48}$$

These values are somewhat too large inasmuch as the effect of the slot openings has been neglected.

PROBLEMS

1. The short-circuit current curve of a machine is plotted to such a scale that the ordinate i_0 (Fig. 223) is equal in length to T, and the curve itself is made up of components one which is linear and the other is a semicircle drawn on T as a diameter. By what per cent. does the energy loss at the brush contact exceed the energy loss of a similar machine in which commutation is linear? What is the rate of change of the commutated current at the end of the period of commutation?

2. The semi-circular form of the extra current of Problem 1 is replaced by a parabolic curve which has zero value at the beginning and end of the commutation period and which reaches a maximum value of $i_0/2$ at the mid-point of the period. Find the per cent. increase of energy loss at the brush contact over that due to linear commutation. What is the final rate of change of the commutated current?

3. Construct a curve showing the variation of current density at five points equally spaced along the brush contact arc for the case of a machine which has a brush covering $3\frac{1}{2}$ commutator segments and in which the short-circuit current curve has the shape specified in Problem 2.

4. The armature of a 4-pole machine has a simplex wave winding arranged in 63 slots, 4 conductors per slot. The commutator has a diameter of 10 in. and the brushes are $1\frac{1}{4}$ in. thick in the tangential direction. Construct diagrams showing the sequence of commutation in adjacent slots.

5. The armature of a 10-pole machine has a triplex wave winding of 1234 conductors arranged in 206 slots. The commutator has 617 segments and a diameter of $51\frac{1}{2}$ in., and the brush thickness in the tangential direction is $1\frac{1}{4}$ in. Construct diagrams showing the sequence of commutation in adjacent slots.

6. The machine of Problem 4 runs at 1170 r.p.m., and that of Problem 5 runs at 100 r.p.m. Find the duration of the period of commutation in each case, assuming (*a*) that all p sets of brushes are used, (*b*) that only two brushes are used.

7. The diameter of the armature of Problem 4 is 12 in., the total length of armature core is 5 in., and the slots are 0.3 in. wide by 1.0 in. deep. The conductors consist of No. 4 d.c.c. round wires placed one above the other. The thickness of insulation at the bottom of the slot, and also that between the elements of the top and bottom layer, is 15 mils. Assuming that the total length of the end connection joining consecutive conductors is three times the pole pitch, and that the pole arc is 70 per cent. of the pole pitch, find the total inductance of an element, taking into account the mutual induction due to neighboring short-circuited coils.

8. An armature has semi-closed circular slots of radius r cm., as shown in the right-hand diagram of Fig. 54, Chapter II. The opening at the top of the slot is r_1 cm. wide and r_2 cm. deep, the center of the circular slot being $(r + r_2)$ cm. below the periphery. If the slot contains z conductors, distributed uniformly over the cross-section of the circle, what is the inductance due to slot leakage, assuming that the lines of force pass straight across the slot (horizontally in Fig. 54)?

CHAPTER IX

COMPENSATION OF ARMATURE REACTION AND IMPROVEMENT OF COMMUTATION

162. Principle of Compensation.—The cross or transverse magnetizing action of the armature current is the primary cause of the field distortion which in turn necessitates the shifting of the brushes and thereby brings into existence the demagnetizing action of the armature. Clearly, then, if the transverse magnetomotive force of the armature were balanced by an equal and opposite magnetomotive force having the same distribution in space, the distortion of the field would be completely eliminated and brush displacement would be unnecessary except, possibly, to assist in commutation. Moreover, if the armature magnetomotive force is overcompensated, there will exist in the neutral zone a component of flux having the proper direction to reverse the current in the short-circuited coils, and the brushes could then be permanently fixed in the geometrical neutral axis.

If the ratio of compensating ampere-turns to armature ampere-turns is unity, nearly complete neutralization of the armature flux will result; if the ratio is slightly greater than unity, there will exist in the commutating zone a reversing flux which increases proportionally with the armature current (unless saturation of the iron of the magnetic circuit sets in) which is precisely the condition necessary to secure good commutation at all loads. In either case, whether the above ratio is unity or greater than unity, the compensating winding must be traversed by the main armature current or a fractional part thereof; consequently the compensating winding is connected in series with the armature, and may or may not be provided with a diverting shunt, as in the case of the series winding of a compound machine.

The problem of compensating armature reaction then consists of two parts; one, the prevention of field distortion in order to minimize the danger of flashing-over at the commutator, as dis-

cussed in Art. 157; the other, the production of a commutating e.m.f. for the purpose of neutralizing the reactance voltage of the short-circuited coils and reversing the current in them. Of these two the latter is usually the more important.

163. Compensating Devices.—In a German patent granted to Menges in 1884 there is the first exposition of the principle of compensating armature reaction. The patent specifications call for the use of a stationary compensating winding wound around the armature at the sides of the poles and traversed by the armature current, or a part of it, in such a direction as to oppose the magnetizing action of the armature. Later, in 1892, H. J. Ryan and M. E. Thompson experimented extensively along similar lines and developed the method shown diagrammatically in Fig. 257 for the case of a bipolar generator. The compensating winding is embedded in slots in the pole faces. The space distribution of the compensating winding being very nearly the same as that of the armature winding, the neutralization of armature reaction is practically complete. Since in the case shown in Fig. 257 the current in the armature conductors is half of the total current, the number of turns in the compensating winding should be half of the effective number of cross-magnetizing armature turns. In larger multipolar machines the number of compensating turns per pole should be $1/a$ times the armature turns per pole, where a is the number of current paths through the armature.

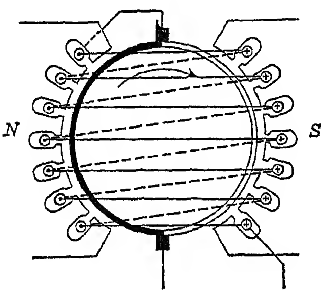


Fig. 257.—Diagram of compensating winding.

Fig. 258 illustrates the construction of the field frame of a machine of this type as built by the Ridgway Dynamo and Engine Company. The entire magnetic circuit is built up of sheet steel stampings, the main ring or yoke being clamped between cast-iron frames; the main pole pieces that carry the coils of the shunt winding are bolted to the yoke, and the cores that carry the compensating winding are held by the bolts which pass through the main poles and by the wedges which hold the commutating lugs in position. These wedges also serve to reduce the cross-

section of the magnetic path from pole to pole, and so keep down magnetic leakage.

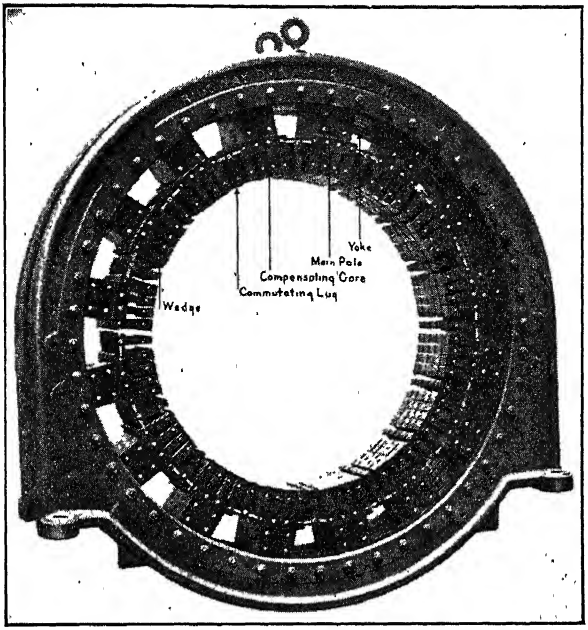


Fig. 258.—Frame of Ridgway generator, showing slotted pole face.

Fig. 259 illustrates diagrammatically a portion of the magnetic circuit of a machine embodying the above device. Under load

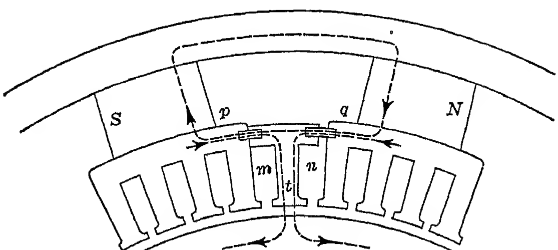


Fig. 259.—Flux paths in poles of compensated machine.

conditions the magnetomotive force of the compensating winding acts in the directions shown by the dotted lines, assuming generator action and clockwise rotation. Section q of the bridge

is acted upon by two m.m.fs. having the same direction, but the flux is not materially increased on account of the initial saturation of the iron; and section p is acted upon by two m.m.fs. of opposite direction. The result is that the central tooth t is acted upon by a resultant m.m.f. which makes it a north pole under the assumed conditions, hence it produces a local field of

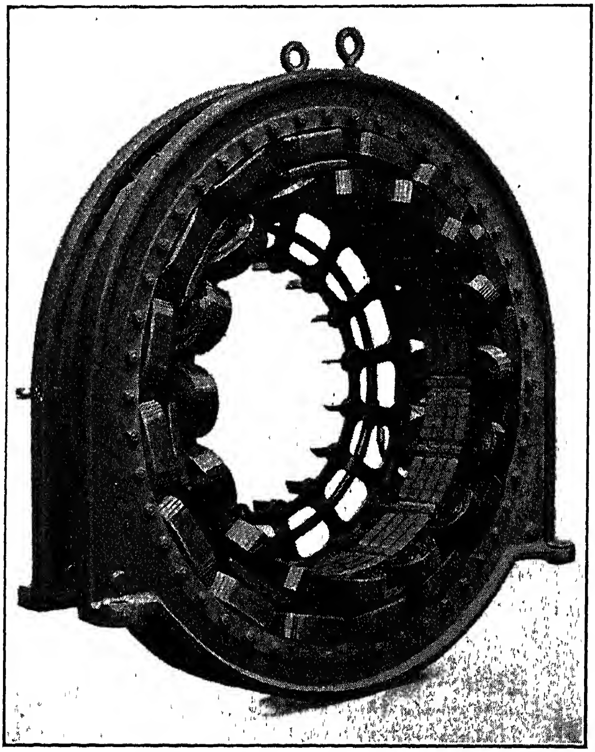


FIG. 260.—Frame of Ridgway generator, coils in place.

the proper direction to assist the reversal of the current in the short-circuited coils. This effect is accentuated by making slots m and n somewhat larger than the others (see also Fig. 258) and winding in them more than the normal number of conductors. The arrangement of the compensating winding and main field winding is shown in Fig. 260.

Closely akin to the Thompson-Ryan device is an arrangement

due to Deri. Instead of using a field frame of the salient pole type with the addition of a slotted ring, the field structure consists of a slotted cylinder wound with two independent sets of coils and concentrically surrounding the armature, as indicated in Fig. 261. The main winding, M, produces poles whose axes are indicated by the arrows; the compensating winding, C, sets

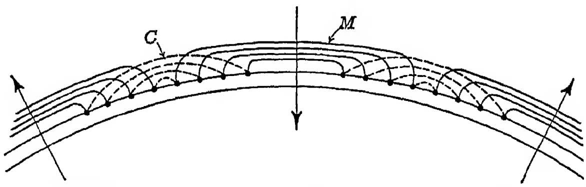


FIG. 261.—Deri's arrangement of main and compensating windings.

up a magnetomotive force acting along axes midway between the poles, and in opposition to the armature m.m.f. The field structure closely resembles the stator of an induction motor, and since the reluctance of the magnetic path is the same along all diametral paths, the compensation can be made complete.

164. Commutating Devices.—The entire prevention of field distortion is not necessary for successful commutation. The

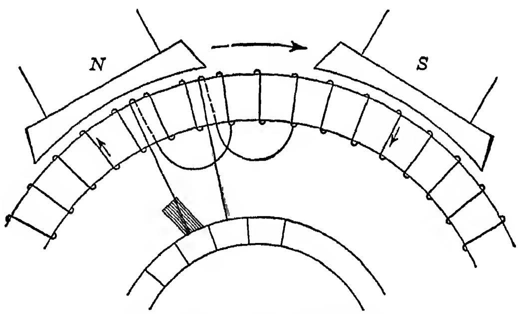


FIG. 262.—Sayers' winding.

principal consideration is to insure the presence of a commutating field of sufficient intensity to generate in the short-circuited coil an e.m.f. large enough to neutralize the reactance voltage.

Thus, in the Sayers winding, Fig. 262, there is no compensation of armature reaction, but the reversing e.m.f. is introduced into the short-circuited coil by an auxiliary winding which is so placed as to cut a part of the main flux during the commuta-

tion period and which, during that interval, is in series with the coil undergoing commutation. At all other times the auxiliary coil is not in circuit. The auxiliary coils are in reality merely extensions of the commutator leads which have been wound around the armature. The main coils are connected to auxiliary coils which lie *behind* them with respect to the direction of rotation so that commutation is not dependent upon the flux density at the leading pole tip, as in the ordinary machine, but upon that at the trailing pole tip; and since the field intensity at the latter increases with increasing current, the commutating e.m.f. increases with it. A limit is set to this automatic adjustment of commutating conditions by the saturation of the trailing pole tip. The Sayers device is of historical rather than practical interest, as is also that of Swinburne.[1] In the latter, small U-shaped electromagnets, Fig. 263, excited by the main current, are placed in the

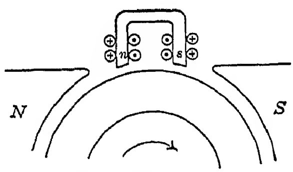


FIG. 263.—Swinburne's commutating device.

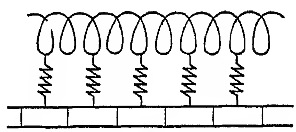


FIG. 264.—High resistance leads to commutator.

neutral zone. Another method that has been used for the prevention of commutation difficulties consists of the insertion in the circuit of the short-circuited coil of an auxiliary resistance which serves to limit the current and, therefore, the energy that must be handled at the brush contact. The simplest method that suggests itself for this purpose is the use of commutator leads of high specific resistance, as indicated in Fig. 264; this arrangement is used in certain types of alternating-current motors. It is, however, open to the serious objection that the main current must flow through a set of these extra resistors with consequent heating and loss of efficiency.

Instead of inserting resistance in the circuit in the above manner, the brushes may be so constructed as to interpose more resistance in the path of the short-circuit current than in that of

[1] Journal Inst. of Electrical Engineers, 1890, p. 106.

the main current, by making them of alternate longitudinal layers of carbon and copper. The copper provides a path of high conductance so far as the main current is concerned, while the short-circuit current must pass transversely through the higher resistance of the carbon and copper in series. It is possible, however, that brushes of this type may fail to operate satisfactorily for the reason that the short-circuit current may pass from one copper layer to the next by way of the commutator surface instead of through the intermediate layer of carbon. A better design, due to Young and Dunn,[1] provides for the final rupture of the short-circuit at an auxiliary carbon brush insulated from the main brush, in the manner indicated in Fig. 265. This has the effect of considerably increasing the resistance of the circuit at the last stage of the commutation process.

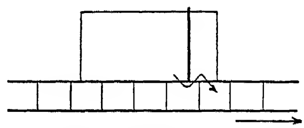


Fig. 265.—Special brush with auxiliary insulated section.

At the present time, however, the device that is used to the practical exclusion of all others, wherever commutating conditions present special difficulties, is the *interpole* or *commutating pole*, the principal features of which are discussed later. The structural arrangement of the interpole machine is shown in Fig. 67, Chap. II. The magnetomotive force of the interpoles it so adjusted that the m.m.f. of the armature is slightly overcompensated. The final adjustment for sparkless operation is made by varying the air-gap under the interpoles by means of shims, or by means of a shunt around the interpole winding.

165. Commutation in Machines having no Auxiliary Devices.—It goes without saying that the mechanical construction of the commutator and brush rigging must be such as to insure perfect contact and absence of vibration. The natural period of vibration of the brushes and brush holders should differ from that of any vibration which may possibly be impressed upon them by the motion of the commutator, in order that mechanical resonance may not occur. The copper of the commutator should be of uniform quality so that a true cylindrical surface may be preserved, and the mica insulation between segments should have a rate of wear as nearly as possible the same as that of the copper.

[1] Electrical World, 1905, p. 481.

Reference has already been made in the preceding chapter to the more important magnetic and electrical factors concerned in the commutation process, to which due regard must be paid to insure sparkless operation. For the sake of completeness they are here recapitulated and in addition there is given a summary of the methods adopted to secure them.

1. The commutating e.m.f. generated in the coil under a brush must not be greater than 4 to 4½ volts, or

$$E_{c0} = \frac{Z}{2S} l' v B_0 \times 10^{-8} < 4 \text{ to } 4\tfrac{1}{2} \text{ volts.}$$

This equation is based upon the assumption of full-pitch windings; in the case of fractional pitch windings the two sides of a coil undergoing commutation are in fields of unequal strength, sometimes of the same polarity, so that the e.m.fs. generated in them may tend to annul each other. This results in a decrease of the short-circuit e.m.f. At the same time there is a decrease in the demagnetizing action of the armature. Both effects indicate that chording the windings is advantageous, but a distinct limit is set by the fact that in such windings at least one side of the short-circuited coil will lie close to a pole tip where the field intensity changes sharply; this feature greatly restricts the zone through which the brushes may be rocked, and is incompatible with the requirement of a fringing field of gradual slope.

2. The average reactance voltage

$$e_r = \frac{2i_0}{T}(L + \Sigma M) < 2i_0 R_b$$

should be less than 1 volt, as determined by the relation $\frac{R_b T}{L} > 1$. The limiting value of e_r is set by the voltage drop at the brush contact, $2i_0R_b$, which is of the order of 1 volt. The harder the grade of carbon used in the brushes the greater will be the drop. Consequently hard carbon should be used where resistance commutation is necessary. Fig. 266 (taken from Gray's Electrical Machine Design) shows the variation of brush contact drop with current density for an average carbon. The tendency toward constant drop with increasing current density is obvious. The contact drop in the direction from commutator

to brush is generally somewhat higher than that in the direction from brush to commutator.

3. The brush width should not be too great, in order to cut down the mutual induction of the simultaneously short-circuited coils. In practice, the brush width seldom exceeds 3.5 times the width of a commutator segment. To secure sufficient contact area to carry the current the axial length of the brushes is adjusted so that the average current density is in the neighborhood of 30 amperes per sq. in. (5 amperes per sq. cm.) for hard carbons and as high as 65 amperes per sq. in. (10 amperes per

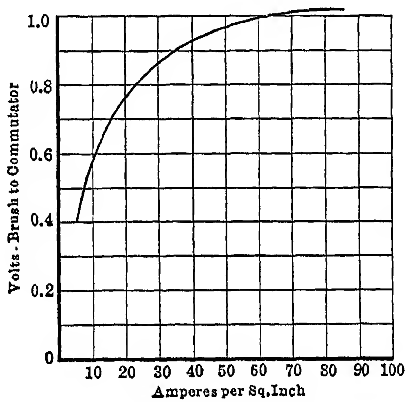


FIG. 266.—Relation between brush drop and current density, average carbon.

sq. cm.) for soft carbons. An average of about 40 amperes per sq. in. is customary. Considerably higher densities are permissible with metal (copper) brushes, up to about 160 amperes per sq. in.

Since the number of simultaneously short-circuited coils is given by

$$n_{sc} = \frac{b}{\beta} \cdot \frac{p}{a}$$

and since the ratio $\frac{b}{\beta}$ is fixed by usual practice at from 2 to 3.5, it follows that p/a must be kept small if special difficulties are encountered. The ratio p/a is a maximum in series (two-circuit) windings, hence if the reactance is too large the remedy is a series-parallel or full-parallel winding.

4. The inductance of the coils can be kept within limits by reducing the number of turns per element; or conversely, for a given number of armature conductors, by increasing the number of commutator segments. The value of L can also be made small by selecting a relatively short axial length of armature. Both of these considerations point to the desirability of a design in which the ratio of diameter to length of armature core is relatively large. A large number of commutator segments involves a moderate average value of volts per segment, $V \div \frac{S}{p} = \frac{pV}{S}$, which value, in simplex lap windings, should not exceed 20 volts and should be less than 15 volts, if possible. The maximum difference of potential between adjacent segments in machines of the series types should never exceed 40 volts, otherwise there is the possibility that the machine will flash over.

The selection of a large diameter carries with it the possibility of using a large number of slots each containing but few coil sides, thereby reducing the mutually inductive action. The presence of numerous armature slots means further that there will be a sufficient number of them between pole tips to cut down oscillations of the commutating flux and minimize pulsations of the flux as a whole.

5. It was shown in Chap. V that in order to prevent the reversal of the commutating field by the distorting action of the armature it is necessary to observe the relation

$$\text{armature ampere-turns per pole} \gtrless 1.1 \text{ field ampere-turns per pole},$$

though in most cases the factor is 0.8 to 0.9 instead of 1.1.

In other words, the main field should be powerful, or "stiff," in comparison with the armature field. Since the armature magnetomotive force acts upon a different path from that subjected to the main field excitation, it is clear that the disturbing effect of armature reaction can be reduced by introducing additional reluctance into the transverse magnetic circuit of the armature. If at the same time those parts of the magnetic circuit which are influenced by both the armature and field excitation are saturated at no load, the load current will have little effect in producing distortion; the saturation acts, of course, as an increase of reluctance.

The first of the above methods is not as simple as it appears at first sight; the armature magnetomotive force acts upon a path which includes the most important part of the main magnetic circuit, namely the air-gap and the iron parts adjacent

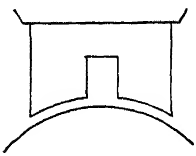


Fig. 267.

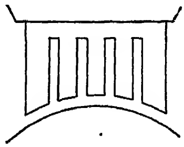


Fig. 268.

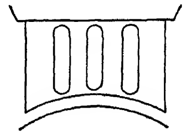


Fig. 269.

Figs. 267, 268, 269.—Longitudinal slotting of pole cores.

thereto. The addition of reluctance to the path of the armature m.m.f. therefore adds more or less to the reluctance of the main circuit, hence requires more field excitation and increases the cost of the machine. Designs embodying this principle

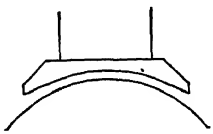


Fig. 270.

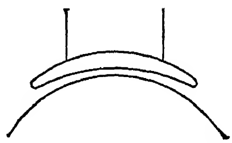


Fig. 271.

Figs. 270 and 271.—Chamfered and eccentric pole faces.

alone involve a longitudinal slotting of the pole cores, as shown in Figs. 267, 268 and 269. Their effectiveness is open to question inasmuch as the armature flux will tend to pass around behind the slots rather than across them. The most effective

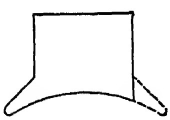


Fig. 272.

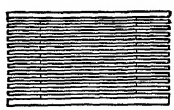


Fig. 273.

Figs. 272 and 273.—Construction of laminated pole.

designs are those which include both the features of additional reluctance and saturation of the iron; the extra reluctance is usually obtained by chamfering the pole tip, as in Fig. 270, or by making the bore of the pole faces eccentric, as in Fig. 271. The saturation feature is most important at the trailing tip in the case of generators and at the leading tip

in the case of motors; the desired saturation is obtained by using a long thin tip, or, in the case of laminated poles, by using a stamping of the form of Fig. 272, in which case the laminations are built up to the required thickness in such a manner that the projecting tips are alternately on opposite sides; a plan view of the pole face as seen from the armature surface is then like Fig. 273. It is, of course, not necessary that the entire pole be laminated to secure this construction, as the pole shoe alone may be built of stampings and bolted to a solid pole core.

Fig. 274 illustrates the pole construction of a Lundell generator which embodies a number of these features; it is, however, only adapted to machines that run in one direction.

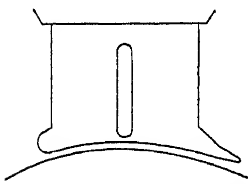


FIG. 274.—Pole of Lundell generator.

Saturation of the armature teeth acts in the same way as saturation of the pole tips. The tooth density is therefore purposely made high, often as high as 140,000 lines per. sq. in. (21,000 per sq. cm.). This is particularly true in the case of motors which are required to operate with the brushes fixed at the geometrical neutral, as in railway motors.

6. The requirement of a fringing commutating field of gradually changing intensity is met by properly shaping the tips of the pole shoes. The length of the air-gap at the pole tips can be computed from the equation

$$\delta' = \frac{(1.25 \text{ to } 2)\left[\frac{\beta Z i_a}{360 a} - 2AT_t\right]}{1.6\, B_g}$$

as shown in Chap. V.

166. Commutating Poles.—Commutating poles, or interpoles, have been extensively adopted for the improvement of commutation in generators and motors in which sparkless operation would be difficult or impossible of attainment under ordinary conditions. Examples of such machines are turbo-generators and shunt motors which are required to have a wide range of speed, as discussed in Chaps. VI and VII; they are also extensively used in series railway motors. Commutating poles obviate the necessity for the various expedients commonly employed in ordinary machines. They are superior to forms of construction involving compensating windings in the pole

faces because of their greater simplicity, but cannot take the place of compensating windings in machines subject to sudden changes of load over a wide range, as in reversing mill motors.

Complete neutralization of the armature flux is not possible by the use of interpoles for the reason that the space distributions of the m.m.fs. of armature and interpoles are different. This is not objectionable, however, except where complete neturalization is required to prevent flashing over, since in most cases the essential feature is the production, in the commutating zone only, of a reversing flux of proper intensity; the flux distribution outside of this zone is usually of minor importance. Since the ratio of interpole ampere-turns to armature ampere-turns is constant, the difference between them will always be proportional to the armature current, hence the commutating flux would satisfy the requirement of being proportional to the armature current at all loads provided the reluctance of the interpole circuit were constant; unfortunately, however, the interpoles tend to become saturated under heavy loads, with the result that the commutating flux does not increase in strict proportion to the armature current.

In multipolar machines having a two-circuit armature winding and in which only two brush sets are used, as in street railway motors, it is not necessary to have as many interpoles as there are main poles; for in this type of winding each brush short-circuits $p/2$ elements in series, and a correct value of commutating e.m.f. generated in any single portion of the circuit will be just as effective as several smaller e.m.fs. generated in different parts of the circuit. Railway motors of the usual four-pole type are often built with only two interpoles.

The presence of interpoles increases the magnetic leakage of the main poles, the calculation of the leakage factor being made in the manner outlined in Chap. IV. To reduce the leakage to a minimum, both the breadth and length of the interpoles should be kept as small as possible, and the span of the main poles made smaller than in ordinary machines; the ratio of pole arc to pole pitch is usually between 0.60 and 0.65, instead of 0.70. The span of the interpole should be equal to, or slightly greater than, the distance moved over by a slot while the coils in it are undergoing commutation. The axial length of the interpole can be made less than that of the main poles, for it is immaterial in what portion of the coil the neutralizing e.m.f. is

generated; but if the interpole is shortened, the intensity of the field under it must be greater than under a pole of full length in the ratio of full length to actual length, with due regard to fringing of the flux.

167. Winding of Commutating Poles.—The calculation of the winding to be placed on the commutating poles presents no special difficulties. It is necessary to provide a sufficient number of ampere-turns to balance those of the armature and to supply the m.m.f. require to drive the commutating flux through the

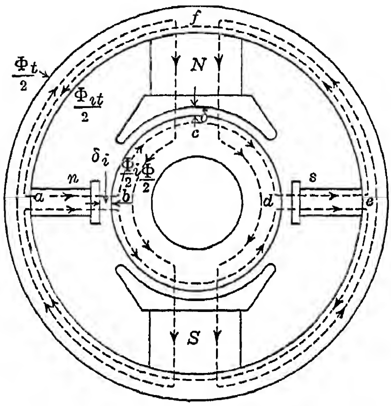


FIG. 275.—Magnetic circuits in interpole machine.

transverse path *n*, *s*, Fig. 275, taking into account the m.m.f. supplied by the main poles, *N*, *S*, in those parts of the path of the lines of induction which are common to both magnetic circuits. The figure represents a bipolar generator revolving in the clockwise direction; if the machine were a motor revolving in the same direction, the polarity of the interpoles would have to be reversed, a condition that would be met automatically upon reversing the current through the armature, since the two windings are in series.

Fig. 275 reveals the fact that the m.m.fs. of the main and commutating poles act in the same direction in two of the four quadrants of the armature core and of the yoke, and in opposite directions in the other two quadrants. If there were no saturation each m.m.f. would produce a proportional flux, in which case the flux in the armature core would be $\frac{\Phi + \Phi_i}{2}$ in two of the

quadrants, and $\frac{\Phi - \Phi_i}{2}$ in the other two, where Φ_i is the working flux produced by the interpole. Similarly, in two of the quadrants of the yoke the flux would be $\frac{\Phi_t + \Phi_{it}}{2}$, and $\frac{\Phi_t - \Phi_{it}}{2}$ in the other two; here $\Phi_t = \nu\Phi$ where ν is the coefficient of dispersion of the main poles, and $\Phi_{it} = \nu_i\Phi_i$ where ν_i is the coefficient of dispersion of the interpoles. The magnitude of the commutating flux Φ_i is given by

$$\Phi_i = B_{ig}\, b'_i\, l'_i$$

where B_{ig}, the flux density in the gap under the interpole, is determined by the value of the commutating e.m.f. to be generated, and b'_i and l'_i are respectively the corrected breadth and length of the interpole; these corrected lengths are greater than the actual lengths by from 3 to 4 times the air-gap, δ_i, under the interpole. Corresponding to this value of Φ_i and to that of the main flux Φ there will be definite flux densities in each part of the closed magnetic circuit *abcdefa*, and to each flux density there will correspond a definite number of ampere-turns which may be determined from the appropriate *B-H* curves. Thus, let

AT_{ic} = ampere-turns required by the two interpole cores and shoes
AT_{ig} = ampere-turns for the two interpole air-gaps, δ_i
AT_{it} = ampere-turns for the two sets of teeth opposite the interpoles
AT'_a = ampere-turns for the armature core, b to c
AT''_a = ampere-turns for the armature core, c to d
AT'_y = ampere-turns for the yoke, e to f
AT''_y = ampere-turns for the yoke, f to a

Then, in the closed magnetic circuit, *abcdefa*, the algebraic sum of all the m.m.fs. must be zero, in accordance with Kirchhoff's law. In this circuit there act, in addition to the m.m.fs. listed above, that due to the two interpole windings, AT_i, and that due to the armature, AT_{arm}, where

$$AT_{arm} = \frac{2}{p}\,\frac{Z}{2}\,i_a = \frac{Zi_a}{\pi d}\cdot\frac{\pi d}{p} = q\tau$$

$$\therefore AT_i = q_T + AT_{ic} + AT_{ig} + AT_{it} - AT'_a + AT''_a + AT'_y - AT''_y$$

It follows, then, that the number of turns to be wound on each interpole is $\frac{1}{2}\frac{AT_i}{i_a}$, provided the interpole coils are not shunted. A larger number of turns may be used if a diverting shunt is placed across the interpole winding.

The above discussion applies directly to the bipolar machine of Fig. 275, and with obvious modifications applies also to multipolar machines.

168.[1] **Effect of Commutating Poles upon Coil Inductance.**— The presence of commutating poles causes an increase in the inductance of the short-circuited coils under them. In the expression $L = L_1 + L_2 + L_3$ (Chap. VIII), the term L_2, due

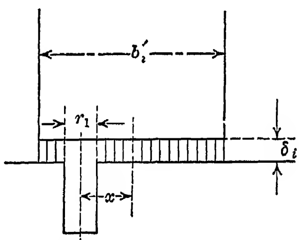


FIG. 276 —Tooth-tip flux under interpole.

to tooth-tip leakage, is affected. Its value may be computed as follows:

Suppose the center of the slot containing a coil edge to be a distance x cm. from the center of the commutating pole of corrected breadth b'_i, Fig. 276. The tooth-tip flux within the limits of the pole is

$$\frac{\frac{4\pi}{10}z}{\frac{\delta'_i}{l'_i\left[\frac{b'_i}{2}+\left(x-\frac{r_1}{2}\right)\right]}+\frac{\delta'_i}{l'_i\left[\frac{b'_i}{2}-\left(x-\frac{r_1}{2}\right)\right]}} = \frac{4\pi}{10}z\,\frac{l'_i}{4\delta'_i}\left[\frac{(b'_i-r_1)^2-4x^2}{b'_i-r_1}\right]$$

[1] See foot-note 2, p. 336.

and the average inductance, considering both sides of the coil, is

$$L_2 = 2 \times \frac{4\pi}{10^9} z^2 \frac{l'_i}{4\delta'_i} \int_0^{\frac{b'_i - b_0}{2}} \left\{ (b'_i - r_1) - \frac{4x^2}{b'_i - r_1} \right\} dx$$

$$= \frac{4\pi}{10^9} z^2 l'_i \frac{(b'_i - r_1)^2}{6\delta'_i}$$

In the above equations δ'_i is the corrected gap length, $\delta_i \frac{t}{t - \sigma r_1}$, all dimensions being expressed in centimeters.

If $l' < l'_i$, there must be added to the above expression a term

$$\frac{4\pi}{10^9} z^2 (l' - l'_i) \times 1.46 \log_{10} \left[1 + \frac{\pi(\tau - b)}{2r_1}\right]$$

169. Compounding Effect of Commutating Poles.—In Chap. V, it was shown that a forward lead of the brushes in the case of a generator produces a demagnetizing effect and consequently reduces the generated e.m.f., while a backward lead causes a compounding action. If the generator is provided with commutating poles, these effects of brush displacement are accentuated, as may be seen from Fig. 277. Thus, Fig. 277*a* represents the conditions when the brushes are in the geometrical neutral axis; the armature winding between adjacent brushes of opposite polarity is then acted upon by the flux due to a main pole only, the effect of the oppositely directed fluxes due to the interpoles (shown by the hatched areas) being to annul each other. If the brushes are displaced in the direction of rotation, Fig. 277*b*, the total flux is reduced by the difference between the hatched areas *efh* and *bcd* and in addition by the sum of areas *abc* and *efg*. In the same manner a backward displacement of the brushes, Fig. 277*c*, results in an increase of the flux linked with the armature winding.

Similar considerations show that in the case of commutating pole motors a forward lead of the brushes will increase the flux and therefore reduce the speed, while a backward lead will decrease the flux and raise the speed. In Chap. VII it was shown that a considerable backward displacement of the brushes of a commutating pole motor may result in a continuous succession of reversals of rotation. If actual reversal of direction does not occur, pulsation of speed may result, for since the effect of a backward displacement is to weaken the active field, there will

be a corresponding decrease of counter e.m.f. and an increase of armature current to produce the necessary acceleration of the armature. The increased current further strengthens the interpoles, and so still further weakens the field and accelerates the armature. But the counter e.m.f. is proportional to the active flux and to the speed. The tendency to accelerate the armature

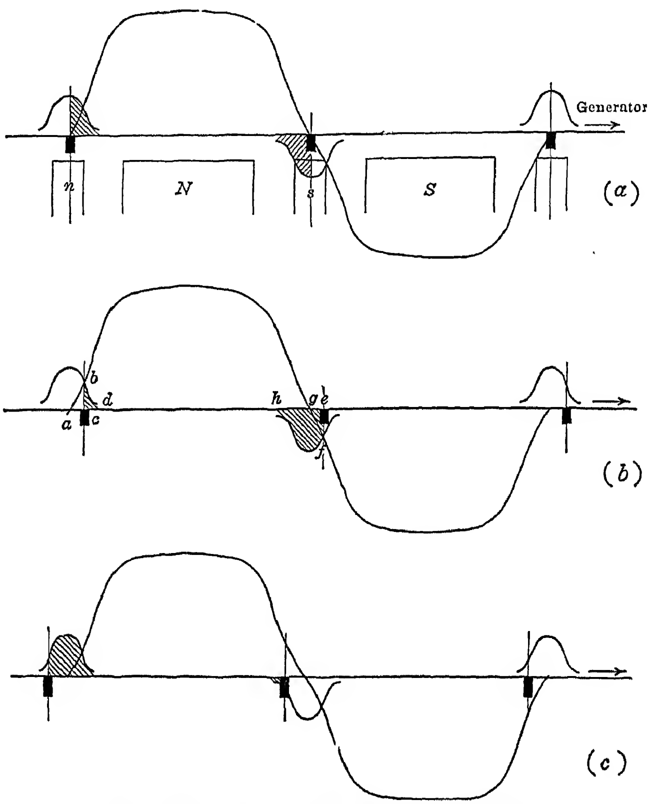


Fig. 277.—Compounding effect of interpoles.

will then continue until the decrease of counter e.m.f. due to reduced flux is offset by the increase due to greater speed. The decrease of field strength cannot, however, go on indefinitely, because the interpoles eventually become saturated, but up to the time that saturation of the interpoles sets in, the speed has been continuously increasing, and the momentum of the armature tends to maintain the speed even after the flux

has reached a practically constant value; especially will this be true if the rotating parts have large moment of inertia. The result will be a rapid increase of counter e.m.f., possibly to a value greater than the line voltage, in which case the machine would become a generator drawing upon its kinetic energy of rotation to send current back to the supply line. The speed under these conditions would rapidly fall, causing such a reduction of counter e.m.f. that the armature current again rises, thereby producing increased speed, and so repeating the above described cycle of changes.

CHAPTER X

EFFICIENCY, RATING AND HEATING[1]

170. Conventional and Measured Efficiency.—The efficiency of a machine is defined as the ratio of the power delivered by the machine to the power received by it. Naturally both input and output must be expressed in terms of the same unit of power before computing their ratio; thus in the case of generators the input is mechanical and the output is electrical, while in motors the input is electrical and the output is mechanical. The usual units of power are the watt, the kilowatt, and the horse-power, where

$$1 \text{ h.p.} = 33{,}000 \text{ ft.-lb. per min.} = 746 \text{ watts}$$

$$1 \text{ kw.} = 1000 \text{ watts} = 1.34 \text{ h.p.}$$

In ordinary practice the rating of generators is given in terms of kilowatts available at the terminals at the specified voltage of the machine. In the case of motors it is customary to express the rating in terms of the number of horse-power available at the shaft (except in the case of railway motors); the Standardization Rules of the American Institute of Electrical Engineers recommend that motor output ratings be expressed in kilowatts, but this recommendation is not followed by the Electric Power Club, which is an association of all the principle motor manufacturers of the United States.

Two distinct efficiencies are recognized in engineering specifications, the *conventional* efficiency and the *directly measured* efficiency. The conventional efficiency is used unless otherwise specified. In either case, when the efficiency is referred to without specific reference to the load conditions, it is to be understood that it is the efficiency at the rated load of the machine, and at a temperature of 75° C.

[1] The student is referred to the Standardization Rules of the American Institute of Electrical Engineers for further particulars on these subjects. See Trans. A I.E.E., Vol. XL, p. 1559 (1921).

The conventional efficiency is computed from the fundamental relations

$$\text{input} = \text{output} + \text{losses}$$
$$\text{output} = \text{input} - \text{losses},$$

whence

$$\text{efficiency} = \frac{\text{output}}{\text{input}} = \frac{\text{output}}{\text{output} + \text{losses}} = \frac{\text{input} - \text{losses}}{\text{input}} \quad (1)$$

If it were possible to measure all of the losses corresponding to particular values of the load throughout the working range of the machine, the efficiency computed from the above equation would be the true efficiency; but for reasons to be explained later it is not possible to measure all of the individual losses with entire accuracy, and in such cases conventional values are arbitrarily assigned to one or more of them, hence the origin of the term conventional efficiency.

The directly measured efficiency is obtained from actual measurements of input and output. For example, in the case of direct-current generators of small or moderate size the mechanical input can be measured by some form of transmission dynamometer and the output by suitable ammeters and voltmeters; in the case of direct-current motors the power input can be measured by ammeters and voltmeters, and the output by a Prony brake or its equivalent. In making direct measurements, electrical power must always be measured at the line terminals of the machine; mechanical power, as in the case of motors, must be measured at the pulley, gearing, or coupling on the rotor shaft, thus excluding the loss in the belt or in the gears (except that in railway motors the mechanical output is measured at the car axle, thereby including gear loss in the amount chargeable to the motor). The method of direct measurement is impracticable beyond definite limits of machine ratings for the double reason that it is not feasible to design suitable dynamometers and brakes and that even if testing facilities were available the cost of supplying the necessary energy would be prohibitive.

Machines which are too large to be tested by means of dynamometers or brakes may under certain conditions be subjected to direct measurement of efficiency by means of the circulating power method, also called the Hopkinson or "loading-back"

method, provided two machines of the same type and rating are available. Thus, if the two machines are of the shunt type, they are mechanically coupled, preferably by direct connection, and one of them is started as a motor from a supply circuit of the rated voltage of the machines. The excitation of the other machine is adjusted until its terminal voltage is equal to that of the supply line, and it is then also connected to the line in such a manner that the e.m.fs. of the two machines are in opposition in the local circuit between them. By adjusting the excitation of one or both of the two machines, any amount of current may be made to circulate between them; one machine runs as a motor and drives the other as generator; the latter returns power to the line, so that the line is called upon to supply only the sum of the losses of the two machines, and this total loss is readily measurable. It is not quite true that the total loss is divided equally between the two machines, for since one of them is a motor and the other a generator their excitations will not be exactly the same, hence the fluxes and the corresponding core losses will be appreciably different.

171. Losses in Direct-current Generators and Motors.— The determination of the efficiency of a generator or motor at any particular output requires either (1) a knowledge of the corresponding input, as in the directly-measured efficiency method; or (2) a knowledge of the sum of the component losses at that value of output, as indicated in equation (1). In the first method, the sum of all the losses is equal to the excess of input over output, though the degree of accuracy attainable in such a determination of losses is not very high because a small per cent. of error in measuring either input or output will result in a much greater per cent. of error in their difference. In the second method, which leads to the determination of the conventional efficiency, the component losses are of three types, (*A*) accurately measurable or determinable losses, (*B*) approximately measurable or determinable losses, and (*C*) indeterminable losses; it is to the losses of the third type, and to some extent to those of the second, that conventional values are assigned in computing the conventional efficiency.

The individual losses in direct-current machines of the commutating type may be grouped under the following headings:

1. The *ohmic losses*, due to the heating effect of the current flowing through the resistances of

(*a*) the armature winding;
(*b*) the field windings, and rheostat if present;
(*c*) the brushes and brush contacts.

2. The *core losses*, due to

(*a*) hysteresis in the armature core and teeth;
(*b*) eddy currents in the armature core, teeth, and pole faces.

3. The *mechanical losses*, due to

(*a*) bearing friction;
(*b*) friction between the moving parts and air, called "windage;"
(*c*) brush friction.

4. *Stray load losses* caused by

(*a*) eddy currents in the armature conductors;
(*b* the short-circuit currents in the coils undergoing commutation;
(*c*) pulsations of the flux set up by the currents in the commutated coils and by the variation of reluctance of the main magnetic circuit due to the presence of teeth and slots;
(*d*) eddy currents in the end plates of the armature core, in non-insulated bolts through the laminated core, etc.;
(*e*) distortion of flux in armature core and teeth, produced by armature reaction.

With respect to the principal classification of losses, the component losses are grouped as follows in the Institute Standardization Rules:

A. Accurately Measurable or Determinable.

1. No-load core losses, including eddy currents in the conductors at no load.
2. Load i^2r (ohmic) losses in the windings. No-load i^2r losses in the windings.

B. Approximately Measurable or Determinable.

1. Brush friction loss.
2. Brush-contact loss.
3. Losses due to windage and bearing friction.

C. Indeterminable.

1. Core loss due to flux distortion.
2. Eddy current losses in conductors due to transverse (cross) fluxes occasioned by the load currents.
3. Eddy current losses in conductors due to tooth saturation resulting from distortion of the main flux.
4. Tooth frequency (core) losses due to flux distortion under load.
5. Short-circuit loss of commutation.

Inspection of the list of individual losses shows that some of them may be constant, or nearly constant, within the working range of the machine, while others are variable with the load; and that those losses which are substantially constant in one type of machine may be variable in another, depending upon its characteristics. For example, the friction and windage loss is constant in constant-speed generators and motors, but is variable in such machines as series motors; the i^2r loss is practically constant in the field winding of a shunt motor and in the shunt winding of a long-shunt compound generator adjusted for constant terminal voltage, but is variable in all series field windings and in the armature; the core loss is nearly constant in constant-flux machines, but variable in series motors and generators, where the flux changes with the load.

172. Efficiency and Losses in Constant-potential, Constant-speed Machines.—For the purpose of computing the conventional efficiency in constant-potential, constant-speed machines such as shunt generators and motors, the losses are grouped into two classes, namely, (1) those which remain substantially constant at all loads, and (2) those which are variable with the load.

The constant loss includes iron or core losses, friction and windage, and i^2r or ohmic losses in the shunt winding (including rheostat if present). The ohmic loss in the shunt field winding and rheostat will not change with the load if it is connected to the main terminals, leaving out of account a possible change in the resistance of the winding due to change of temperature; but if the machine is a constant-potential short-shunt compound generator, the difference of potential between the terminals of the shunt winding will rise slightly with increasing load, and correspondingly increase the shunt field loss.

The variable loss includes ohmic (i^2r) losses in the armature and in field windings in series therewith, such as series field coils and commutating and compensating windings.

The Institute rules specify that the indeterminable stray load losses are to be assigned a value of zero per cent., that is, they are to be neglected in computing conventional efficiency. Further, that the brush contact drop, which in reality varies with the load, is to be taken as constant in ordinary machines, and computed on the assumption that there is a brush contact drop of 1 volt at each brush set, or a drop of 2 volts for the entire machine, without regard to the amount of current flowing. In the case of low voltage starting motors and lighting generators for automobiles, these values of brush drop do not apply, but are to be replaced by experimentally determined values.

The constant loss of a machine of the shunt type can be determined experimentally in a simple manner by running it as a motor, without load, at its rated voltage V and measuring the no-load current input to its armature $(i_a)_0$ and to the field winding (i_s). The total current input under this no-load condition is $(i_a)_0 + i_s$, and the total power input is $V[(i_a)_0 + i_s]$. The only loss in addition to the constant loss is the ohmic loss in the armature $(i_a)_0{}^2r_a$, and since the output is zero, the constant loss is

$$P_c = V[(i_a)_0 + i_s] - (i_a)^2{}_0 r_a \tag{2}$$

This test should be made after the machine has been running under load for a sufficient time to raise its temperature to the normal operating value; and the value of the armature resistance, r_a, should also be measured at working temperature. The brushes should be placed so that commutation takes place in the neutral axis.

The value of the constant loss determined by equation (2) includes the core loss, friction and windage, and ohmic loss in the shunt winding. The latter is equal to Vi_s, so that the combined value of core loss, friction and windage is given by

$$P_c - Vi_s = V(i_a)_0 - (i_a)^2{}_0 r_a. \tag{3}$$

In case it is desired to determine separately the value of core loss on the one hand and friction and windage on the other, the following procedure may be used:.

1. Drive the machine under test from an independent motor which has previously been calibrated so that its output is known for given values of impressed voltage, speed and excitation. The machine under test must have its brushes removed and must be without field excitation. The output of the driving motor will then be equal to the bearing friction and windage of the machine under test.

2. Replace the brushes of the machine under test, and take an additional reading of the output of the driving motor, the speed being the same as before. The difference between the reading of this test and that of (1) is then equal to the brush friction.

3. With the conditions the same as in (2), except that the excitation of the machine under test is adjusted to a known amount, take a new reading of the output of the driving motor. The difference between this reading and that obtained in test (2) is then the core loss of the machine under test for the particular value of the excitation used; by taking a series of readings for varying amounts of excitation, the core loss can be found for any desired excitation.[1]

The core loss of a machine of the constant-flux, constant-speed type does not remain absolutely constant under varying conditions of load, as is assumed in computing conventional efficiency. The hysteresis loss in any given element of volume of the core varies as the 1.6th power of the flux density, while the eddy current loss varies as the square of the flux density (see Art. 178). The effect of armature reaction is to distort the flux, thereby increasing the flux density in some portions of the core and decreasing it in others; the net result of this shift is to increase the total core loss even though the total flux remains unaltered. This change in the core loss between no load and full load is a part of the so-called stray load losses.

[1] If the above tests are carried out on a machine having a parallel-wound armature, then if the magnetic circuits are not all exactly the same, an alternating current will flow through the armature winding without regard to the presence or absence of brushes. A comparatively small amount of magnetic unbalancing may set up an alternating current of large magnitude, of the order of full-load current or even more. If this is the case the core loss determined by the above method will include the copper loss due to the unbalanced alternating current.

As an example of the method of computing conventional efficiency, let it be assumed that the following data have been determined from a 250-kw., 550-volt, flat compound long-shunt generator (current output at full load, 455 amperes).

Constant loss:

Core loss	3500 watts
Friction and windage	2000 watts
Shunt field loss, including rheostat	1250 watts
Total	6750 watts
Resistance of armature and series field (hot)	0.03 ohm.

For any given value of current output, i, the armature current is

$$i_a = i + i_s = i + \frac{1250}{550} = i + 2.27$$

The output in watts is given by $550 \times i$, and the losses are: constant loss, 6750 watts; variable loss, including copper loss in armature and series field, $i^2_a \times 0.03$; brush contact loss, $2 \times i_a$. Hence for any output current, i, the efficiency is

$$\eta = \frac{550i}{550i + 6750 + (i + 2.27)^2 \times 0.03 + 2(i + 2.27)}$$

Assuming values of i and substituting in this equation, data are obtained from which the curves of Fig. 278 are plotted.

It will be observed that the efficiency rises rapidly from zero value at no load and approaches a maximum value; if the calculations were carried beyond the limits shown in the figure, the efficiency would begin to fall for the reason that the total losses increase at a greater rate than the first power of the current output.

In the above problem, illustrative of the case of a generator, it will be noted that the output is proportional to the line current, assuming that the terminal voltage is constant. If the voltage is not constant, line current will not be proportional to power output, and if it is desired to plot efficiency and losses in terms of power output instead of in terms of current, suitable corrections would have to be made. For example, suppose that a machine has a rising external characteristic, the terminal voltage rising

linearly from 110 volts at no-load to 115 volts at full-load current; at full-load current output the power output is $115i$ watts; at half of full-load current output the power output is $112.5 \times \frac{1}{2}i = 56.75i$, which is 49.3 per cent. of full-load output, instead of exactly 50 per cent.

In computing motor performance from the measured no-load losses, it must be remembered that the current input is not proportional to the power delivered by the motor. Thus, considering a shunt motor, the power input is Vi and the power output is

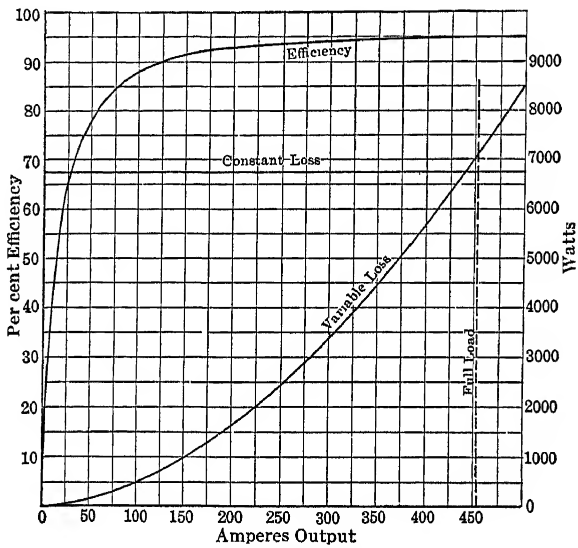


FIG. 278.—Losses and efficiency.

$$P = Vi - P_c - i_a^2 r_a$$

But

$$i = i_a + i_s$$

and

$$P_c = \text{core loss} + \text{friction and windage loss} + \text{shunt field loss}$$
$$= P_{h+e} + P_f + Vi_s$$

where

$$P_{h+e} = \text{core loss}$$
$$P_f = \text{friction and windage loss}$$

It follows, therefore, that

$$
\begin{aligned}
P &= V(i_a + i_s) - P_{h+e} - P_f - Vi_s - i_a^2 r_a \\
&= Vi_a - (P_{h+e} + P_f) - i_a^2 r
\end{aligned}
$$

and

$$i_a = \frac{V}{2r_a} \pm \sqrt{\left(\frac{V}{2r_a}\right)^2 - \frac{P + P_{h+e} + P_f}{r_a}}$$

In this equation the minus sign before the radical is the only one that has a physical meaning. As an example, consider a 110-volt, 10-h.p. motor which has an armature resistance of 0.11 ohm, a field resistance of 40 ohms, and which takes an armature current of 3.1 amperes at no load. The combined value of core loss and friction and windage is then

$$P_{h+e} + P_f = 110 \times 3.1 - (3.1)^2 \times 0.11 = 340 \text{ watts}$$

The armature current intake at full load (P = 10 h.p. = 7460 watts) is

$$i_a = \frac{110}{2 \times 0.11} - \sqrt{\left(\frac{110}{2 \times 0.11}\right)^2 - \frac{7460 + 340}{0.11}} = 76.8 \text{ amp.}$$

and the line current is

$$i = i_a + i_s = 76.8 + \frac{110}{40} = 79.55 \text{ amp.}$$

When the machine is delivering half of its rated load, or 5 h.p. = 3730 watts, the line current is found to be 42.25 amp., or considerably more than half the line current at full load.

The efficiency discussed in the preceding articles may be thought of as the "over-all" efficiency of the machine, since it is the ratio of net output to gross input. But in analyzing the operation of a generator or a motor, two additional subsidiary values of efficiency may be distinguished, as follows:

In a generator, the total mechanical power input is not converted into electrical power, but a portion is dissipated as mechanical loss in friction and windage and in core loss; the core losses in this case act as a brake, and are in fact equivalent to an increase in the frictional resistance. The difference between the total mechanical input and these losses is then converted into electrical form in the armature; a portion of the electrical power thus developed is lost in the ohmic resistance of the various

windings of the machine and in the brush contact resistance, and the balance is available as net output. It follows, therefore, that

electrical power developed = *mechanical input* − *friction and windage* − *core loss*

so that the *efficiency of conversion* is

$$\eta_c = \frac{\text{electrical power developed}}{\text{mechanical power input}}$$
$$= \frac{\text{output} + \text{ohmic losses}}{\text{mechanical power input}} \tag{4}$$

Similarly, the ratio of net electrical power output to the total electrical power developed in the armature may be called the *electrical efficiency*, η_e, and

$$\eta_e = \frac{\text{electrical power output}}{\text{electrical power developed}} = \frac{\text{output}}{\text{output} + \text{ohmic losses}} \tag{5}$$

Comparison of equations (4) and (5) with the expression for overall efficiency,

$$\eta = \frac{\text{output}}{\text{input}}$$

shows that

$$\eta = \eta_c\eta_e \tag{6}$$

In the case of motors, a part of the electrical power input is initially lost in the ohmic resistance of the windings and brush contacts, and the balance is converted into mechanical power; but of the total mechanical power developed, a part is lost in friction and windage and in core loss, the latter again acting as a brake on the machine and being equivalent to an increase in the friction load. Consequently the following relations hold:

mechanical power developed = *electrical input* − *ohmic losses*

and

mechanical output = *mechanical power developed* − *friction and windage* − *core loss.*

Hence the *efficiency of conversion* is

$$\eta_c = \frac{\text{mechanical power developed}}{\text{electrical power input}}$$
$$= \frac{\text{electrical input} - \text{ohmic losses}}{\text{input}} \tag{7}$$

and the *mechanical efficiency* of the motor is

$$\eta_m = \frac{\text{mechanical power output}}{\text{mechanical power developed}}$$

$$= \frac{\text{output}}{\text{output} + \text{friction and windage} + \text{core loss}} \tag{8}$$

It follows here also that

$$\eta = \eta_c \eta_m \tag{9}$$

173. Condition for Maximum Efficiency.—In any machine in which the total losses comprise a part that remains constant independently of the load, and a part that varies as the square of the load, *maximum efficiency* will occur at that particular value of load at which the fixed and variable losses are equal. Thus, if the load (or output) is P, the constant loss is P_c, and the variable loss is kP^2, where k is a constant,

$$\eta = \frac{P}{P + P_c + kP^2} \tag{10}$$

Differentiating η with respect to the load P, and equating the result to zero to find the condition for maximum value of η, we have

$$\frac{d\eta}{dP} = \frac{(P + P_c + kP^2) - P(1 + 2kP)}{(P + P_c + kP^2)^2} = 0$$

whence

$$P_c = kP^2 \tag{11}$$

and the maximum efficiency is

$$\eta_{max} = \frac{P}{P + 2P_c} \tag{12}$$

The above relations are very nearly those which occur in machines of the constant-potential, constant-speed type, such as shunt generators and motors. The constant loss includes friction, windage, ohmic loss in the field winding, and core loss; the variable loss is equal to $i_a^2 r_a$ plus the brush contact loss (conventional value $= 2i_a$ watts). If the brush contact loss is absorbed into the ohmic loss in the armature by assigning to r_a a suitably increased average value, the variable loss is seen to be nearly proportional to i_a^2, and i_a is in turn nearly proportional to the output. Examining the actual conditions somewhat more in detail, the facts with regard to the shunt *generator* may be summarized as follows:

Let V be the terminal voltage, and let i, i_a, and i_s represent respectively the line current, armature current and shunt field current; then

$$i_a = i + i_s$$

and

$$\eta = \frac{Vi}{Vi + P_c + i_a^2 r_a} \tag{13}$$

where P_c is the constant loss and r_a is the value of armature resistance modified to include brush contact resistance. Since i_s is generally only a small per cent. of i at values of the load approaching full-load rating, very little error will be introduced by writing

$$\eta = \frac{Vi}{Vi + P_c + i^2 r_a}$$

whence

$$\frac{d\eta}{di} = \frac{(Vi + P_c + i^2 r_a)V - Vi(V + 2ir_a)}{(Vi + P_c + i^2 r_a)^2}$$

and if

$$\frac{d\eta}{di} = 0,$$

$$P_c = i^2 r_a \cong i_a^2 r_a \tag{14}$$

or the efficiency is a maximum for that value of current output at which the constant and variable losses are equal.

Similarly, in a shunt *motor*, the input is Vi, and the output is

$$Vi - P_c - i_a^2 r_a$$

whence the efficiency is

$$\eta = \frac{Vi - P_c - i_a^2 r_a}{Vi} \cong \frac{Vi - P_c - i^2 r_a}{Vi} \tag{15}$$

and

$$\frac{d\eta}{di} = \frac{Vi(V - 2ir_a) - (Vi - P_c - i^2 r_a)V}{V^2 i^2}$$

and since for maximum efficiency $\frac{d\eta}{di} = 0$, maximum efficiency will occur when

$$P_c = i^2 r_a$$

In this case the differentiation is made with respect to the input current, i, as independent variable, whereas in the case of the generator the output current was the independent variable

The above conclusions are not entirely rigorous for either the generator or motor, but they are sufficiently accurate for practical purposes. A somewhat more accurate analysis might be based on the fact that the variable loss is made up of parts which depend upon the first power of the current as well as upon the second power; thus, the core loss P_{h+e} (due to hysteresis and eddy currents), instead of remaining constant, may be assumed to vary linearly with the load current, or

$$P_{h+e} = (P_{h+e})_0 \pm c\,i \tag{16}$$

where $(P_{h+e})_0$ is the core loss at no load, and c is a constant; the brush contact loss is nearly proportional to i_a, and therefore also to the line current, i; the shunt field loss is $\frac{V^2}{r_s}$ in plain shunt and in long-shunt compound machines, and is

$$\frac{(V \pm ir_f)^2}{r_s}$$

in short-shunt machines, the positive sign being used in the case of generators, the negative sign in the case of motors; the series field loss is $i_a^2 r_f$ in long-shunt machines, and $i^2 r_f$ in short-shunt machines. The summation of all the losses therefore includes a constant term, a term that varies directly with the line current, and a term that varies as the square of the current; hence the efficiency is given by an expression of the form

$$\begin{aligned} \eta &= \frac{Vi}{Vi + P_c + C_1 i + C_2 i^2} \\ &= \frac{V}{V + \frac{P_c}{i} + C_1 + C_2 i} \end{aligned} \tag{17}$$

For maximum efficiency the denominator must be a minimum, hence differentiating the denominator and equating to zero, the condition for maximum efficiency is found to be

$$-\frac{P_c}{i^2} + C_2 = 0$$

or

$$P_c = C_2 i^2 \tag{18}$$

from which it follows that for maximum efficiency the constant

loss should be equal to that part of the variable loss which varies as the square of the line current.

174. Location of Point of Maximum Efficiency.—From the preceding article it is clear that by a proper choice of the relation between the fixed and the variable losses the point of maximum efficiency may be made to fall at any desired output. For example, assume that the total losses consist of a constant term, P_c, and a term variable with the square of the load. Let the rated full-load output be P, and let the constant loss be xP, where x is any fractional part of the output, and let the variable loss at full load be yP, where y is any fractional part of the output. Then the efficiency at full load is

$$\eta = \frac{1}{1 + x + y} \tag{19}$$

Let zP be the output at which it is required that the efficiency be a maximum. The variable loss will be $z^2(yP)$, and for maximum efficiency

$$z^2(yP) = xP$$

or

$$z = \sqrt{\frac{x}{y}} \tag{20}$$

For example, let it be required to divide the total losses in such a way that the maximum efficiency shall occur at three-fourths load and that the efficiency at rated full load shall be 85 per cent. Then

$$\frac{1}{1+ x+ y} = 0.85$$

$$z = \sqrt{\frac{x}{y}} = 0.75$$

from which $x = 6.35$ per cent. and $y = 11.3$ per cent.

It is seen that if the fixed losses, represented by x, are relatively large, and the variable copper losses, represented by y, are small, maximum efficiency will probably occur beyond full load. To make the maximum efficiency occur at a fractional part of full load, the copper losses should be large compared with the fixed losses. Thus, if it is known that a machine is to be operated for considerable periods at light loads and only

occasionally at full load or overloads, it should be so designed as to have a relatively high armature resistance in order to make the efficiency a maximum at or near the point of average load.

175. All-day Efficiency.—*The all-day efficiency* of a machine is the ratio of the net *energy output* to the total *energy input* during a working day. Inasmuch as charges for electrical service are based largely on energy consumption (kilowatt-

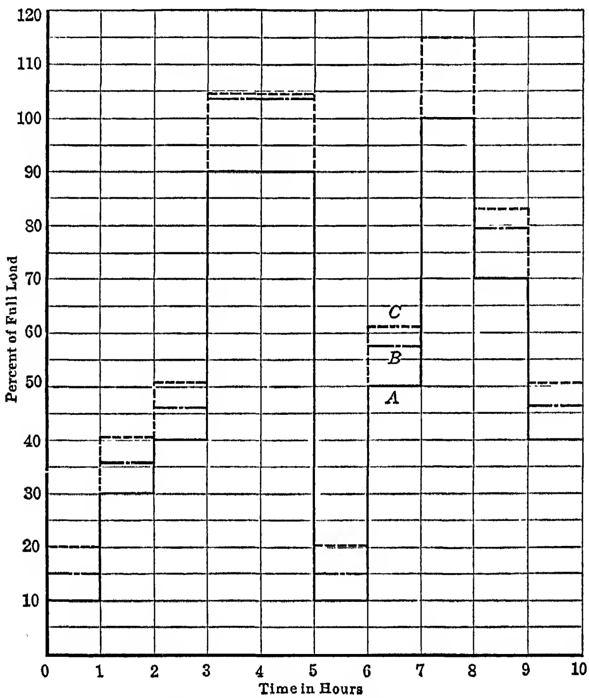


FIG. 279.—*A*, Load curve; *B* and *C*, curves of power input.

hours), it is important that the all-day efficiency of a motor that runs continuously should be as high as possible. The all-day efficiency of a machine is dependent to a large extent upon the shape of its *load curve A*, Fig. 279, and is also affected by the ratio of its fixed and variable losses. The ordinates of the load curve represent power output and the abscissas represent time, so that the area under the curve is proportional to the energy output. If this load is carried by a motor whose fixed losses are 5 per cent., and whose variable losses are 10 per cent.

of its rated output, the power input will vary as shown by curve *B*; while if the fixed and variable losses are, respectively, 10 and 5 per cent. of the rated output, the power input will be given by curve *C*. In the former case the all-day efficiency is 85.8 per cent., in the latter 81.7 per cent. The difference between the two becomes greater and greater, in favor of the machine with the lower fixed loss, as the period of light load increases; for example, if the machine runs for nine hours at 10 per cent. load, and one hour at full load, the all-day efficiency of the first machine is 75.7 per cent. as against 64.3 per cent. for the second.

176. Efficiency and Losses in Variable-flux, Variable-speed Machines.—In machines in which the speed or the flux, or both of them, are inherently variable under operating conditions, as for example in series motors, the core loss and friction and windage loss are also variable. The combined value of core loss and friction and windage loss in a series motor can be found experimentally by the following method:

Separately excite the field winding from any suitable source, and start the motor, without any load upon it, by gradually increasing the voltage impressed upon the armature. After the motor starts, increase the excitation until the field current has the highest value it will have under load conditions, and adjust the armature voltage to a value at which a reading is desired. Note the field current, armature current and voltage, and the speed. Keeping the excitation constant, adjust the armature voltage to two or three different values, and for each setting take readings as before. The armature voltages ordinarily used in this test are 250, 400, and 550 volts for 550-volt motors, and 300, 450, and 650 volts for 650-volt motors. Repeat this series of readings for a number of other values of field current, down to the lowest field current consistent with safe speed. For each setting the combined value of core loss and friction and windage loss will be equal to the power input to the armature minus the ohmic loss in the armature winding and at the brush contact.

If it is desired to separate the total loss thus determined into friction and windage and core losses, the machine should be run as a series motor without load and at reduced voltage. By varying the impressed voltage through a sufficient range to cover the working range of speed, and taking simultaneous readings of

impressed voltage, current and speed, the friction and windage corresponding to any speed may be taken as equal to the power

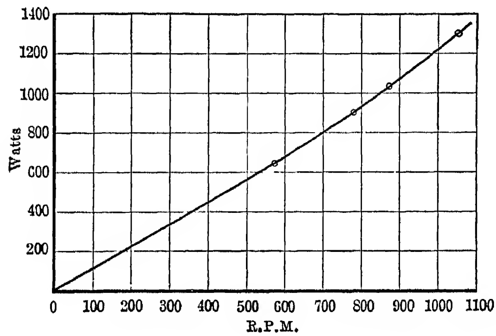


FIG. 280.—Friction and windage as a function of speed. Railway motor test.

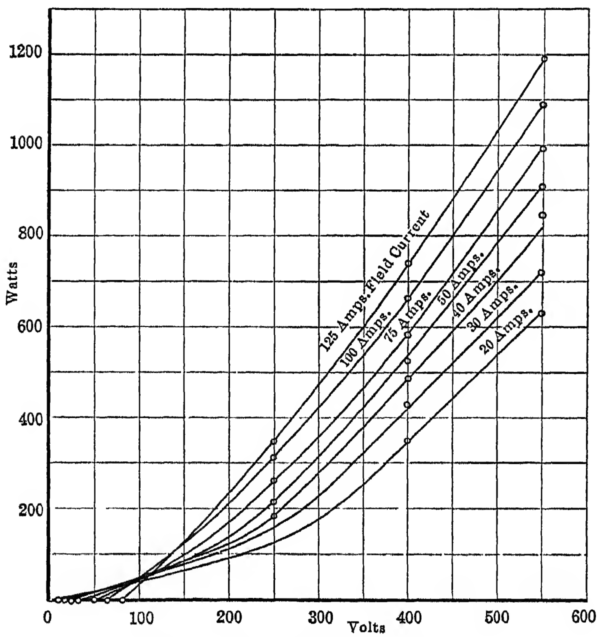


FIG. 281.—Core loss of series motor.

supplied, less the ohmic losses in the armature and field windings, the core loss being negligible under these test conditions. The core loss at any given speed is then equal to the difference be-

tween the total loss as determined by the first test, and the friction and windage loss at the same speed as found in the second test. In this manner curves like Figs. 280 and 281 may be determined;[1] data taken from these curves can then be used to calculate the efficiency at any load.

177. The Ohmic Losses.—The ohmic or copper (i^2r) losses are accurately measurable or can be determined by calculation from the design data of the machine. The method consists of determining the resistance of each part of the winding at the working temperature, 75 deg. C., and computing the loss by forming the product i^2r. For details of the various methods of measuring resistance the student is referred to the numerous text-books and hand-books dealing with electrical measurements;[2] but a very common method for measuring resistances that are not too low is the drop-of-potential method, which consists of passing a known value of current through the winding whose resistance is required, and reading the corresponding drop of potential through the winding by means of a voltmeter of suitable range. When this method is used to measure the resistance of highly inductive circuits like the shunt field windings of large machines, the reading of the ammeter should not be taken until the current has reached a steady value; for the high inductance of such a circuit will cause the current to reach its final value only after an appreciable time, sometimes two or three minutes, and a premature reading will result in an apparant resistance considerably higher than the true value. After the readings of current and voltage have been taken, the voltmeter must first be disconnected before opening the circuit, otherwise the excessive voltage developed by the collapse of the magnetic field will destroy the instrument.

Measurements of very low resistances such as those of armature windings and series field windings of large machines cannot be made with sufficient accuracy by the drop-of-potential method because of the small voltage drop through such a circuit. Some form of low resistance bridge, such as the Thomson double bridge, should be used.

[1] Motor and Generator Testing, Westinghouse Elec. and Mfg. Co.; Sec. 8, pp. 8 and 11. July, 1913.

[2] See Electrical Measurements, by Frank A. Laws.

Computations of the i^2r loss in a winding are based upon the fact that the resistance of a circuit is proportional to its length and inversely as its cross-section. That is,

$$r = \rho \frac{l}{s} \tag{21}$$

where ρ is the specific resistance, or resistivity, of the material at the temperature of the conductor, l is the length of the conductor, and s is its cross-section. The resistivity at a temperature t deg. C. is equal to

$$\rho = \rho_0(1 + 0.00427t) \tag{22}$$

where ρ_0 is the resistivity at zero deg. C. and 0.00427 is the temperature coefficient of increase of resistance per degree C. rise from an initial temperature of zero deg. C. If length is expressed in feet and cross-section in circular mils, $\rho_0 = 9.59$ ohms for commercial copper; if these dimensions are expressed in centimeters and square millimeters, respectively, $\rho_0 =$ 0.016 ohm. If the initial temperature differs from zero, the temperature coefficient is to be computed from the formula $1/(234.5 + t)$; thus, at an initial temperature of $t = 40$ deg. C., the temperature coefficient of increase of resistance per degree Centigrade rise is $1/274.5 = 0.00364$.

The following is a summary of formulas for computing the i^2r loss in the various parts of the machine circuit.

(*a*) *Armature.* The ohmic loss in the armature of any type of generator or motor is

$$P_{ca} = i_a{}^2r_a$$

where

$$r_a = \rho_0 \frac{l_a}{s_a} (1 + 0.00427t) \frac{1}{a^2} \tag{23}$$

in which formula

l_a = total length of wire on the armature
s_a = cross-section of armature conductors
t = working temperature of the armature copper in degrees Centigrade, and which is to be taken as 75 deg. C.
a = number of armature circuits in parallel.

(*b*) The *field* copper loss in separately excited machines is

25

$$P_{cf} = i_f{}^2r_f \text{ watts} \tag{24}$$

and in plain series generators and motors is

$$P_{cf} = i_a^2 r_f \text{ watts} \tag{25}$$

where r_f represents the combined resistance of the field winding and its regulating shunt if the latter is used.

In shunt machines the field loss is

$$P_{cf} = i_s^2 r_s = \frac{V^2}{r_s} = Vi_s \text{ watts} \tag{26}$$

where r_s includes the resistance of the regulating rheostat if one is used.

In long-shunt compound machines, the field loss (total) is

$$P_{cf} = i_a^2 r_f + i_s^2 r_s \text{ watts} \tag{27}$$

where $i_a = i + i_s$ in case the machine is a generator, and $i_a = i - i_s$ in case it is a motor.

In short-shunt compound machines the total field loss is

$$P_{cf} = i^2 r_f + i_s^2 r_s \text{ watts} \tag{28}$$

the above relations again holding between i, i_a and i_s.

(c) The *ohmic loss at the commutator* depends upon the drop of potential at the transition surface between commutator and brushes, as well as upon the amount of current flowing. If the drop of potential at each brush is Δe, the loss is

$$P_{cc} = 2i_a \Delta e \text{ watts} \tag{29}$$

With the usual type of carbon brushes, Δe is approximately 1 volt when the brush current density has values common in ordinary practice (see Fig. 266). Values of Δe cited by the A.I.E.E. Standardization Rules are shown by the following table:

Grade of brush	Volts drop across one brush contact (average of positive and negative brushes)
Hard carbon	1.1
Soft carbon	0.9
Graphite	0.5 to 0.8
Metal-graphite types	0.15 to 0.5 (the former for largest proportion of metal)

In a pamphlet[1] issued by the Westinghouse Electric and Manufacturing Company, the following formula is given for computing the total brush contact drop

$$2\triangle e = \frac{i_a}{20 \times \text{total brush area in sq. in.}} + 1 \qquad (30)$$

178. The Core Losses.—

HYSTERESIS LOSS.—

In the Armature Core.—The relative motion between the armature core and the magnetic field produces a periodic reversal of the magnetism of the core, thereby giving rise to a loss

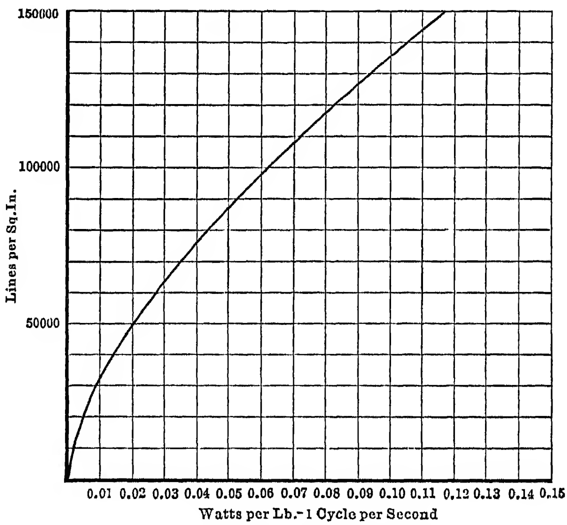


FIG. 282.—Curve of hysteresis loss.

of power through molecular friction in the mass of the armature core. This *hysteresis* loss can be represented by an empirical equation due to Steinmetz

$$P_{ha} = \eta f V B_a^{1.6} \text{watts} \qquad (31)$$

where

η = a constant depending upon the material of the core

$f = \frac{pn}{120}$ = the number of magnetic cycles per second

[1] Motor and Generator Testing.

V = the volume of the core
B_a = the maximum value of the flux density in the core.

If metric units are used in the above equation (volume in cubic centimeters and flux density in lines per sq. cm.), $\eta = 0.0021 \times 10^{-7}$ for ordinary sheet steel; if volume is expressed in cubic inches and flux density in lines per sq. in., $\eta = 0.0017 \times 10^{-7}$. Since the weight, W, of the core is proportional to its volume, the equation for the hysteresis loss can also be written

$$P_{ha} = \eta f W B_a^{1.6} \text{ watts} \tag{32}$$

in which case $\eta = 0.0062 \times 10^{-7}$ if British units are used (weight in pounds, flux density in lines per sq. in.).

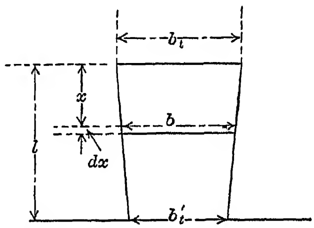


FIG. 283.—Computation of hysteresis loss in teeth.

The curve of Fig. 282 shows the variation of the hysteresis loss, expressed in watts per pound per cycle per second, as a function of the flux density expressed in lines per sq. in., using the above value of η.

In the Armature Teeth.—The flux density varies from section to section because of the taper of the teeth, and it is not correct to compute the hysteresis loss by substituting an average value of flux density in the above equation.

Consider an element dx, Fig. 283, at a distance x below the tip of the tooth; its volume is

$$dV = blkdx = \left(b_t - \frac{b_t - b'_t}{l_t}x\right)lkdx$$

where k is the lamination factor (from 0.85 to 0.90). Assuming that the total flux is the same at all sections of the tooth, the flux density will vary inversely as the width of the section, or

$$B = \frac{b_t}{b}B_t$$

where B_t is the actual, or corrected, flux density at the top of the tooth. The hysteresis loss in the element is then

$$dP_{ht} = \eta f B^{1.6} dV = \eta f k l b_t^{1.6} B_t^{1.6} \frac{dx}{b^{0.6}}$$

and the total loss per tooth is

$$P_{ht} = \eta k f l b_t^{1.6} B_t^{1.6} \int_0^{l_t} \frac{dx}{\left(b_t - \frac{b_t - b'_t}{l_t} x\right)^{0.6}}$$

$$= 2.5 \eta k f l b_t^{1.6} B_t^{1.6} (b_t^{0.4} - b'^{0.4}_t) \frac{l_t}{b_t - b'_t} \qquad (33)$$

Since the volume of a tooth is

$$V_t = \frac{b_t + b'_t}{2} k l_t l$$

the above expression can be written

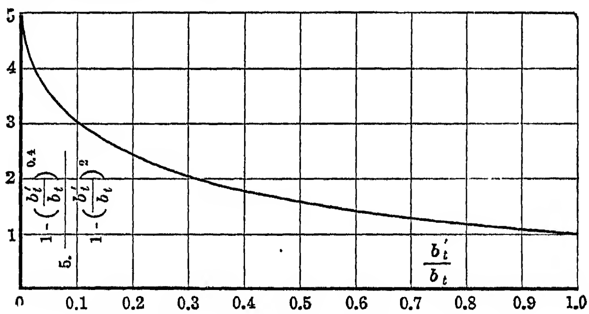


FIG. 284.—Correction factor, hysteresis loss in teeth.

$$P_{ht} = \eta f V_t B_t^{1.6} \times 5 \frac{1 - \left(\frac{b'_t}{b_t}\right)^{0.4}}{1 - \left(\frac{b'_t}{b_t}\right)^2} \qquad (34)$$

In other words, the expression for the hysteresis loss in the teeth is similar to the general expression, but with the addition of the factor

$$5 \frac{1 - \left(\frac{b'_t}{b_t}\right)^{0.4}}{1 - \left(\frac{b'_t}{b_t}\right)^2}$$

The ordinates of Fig. 284 give the value of this factor for various values of b'_t / b_t.

EDDY CURRENT LOSSES.—That part of the core loss due to eddy

or Foucault currents can be approximately calculated by the formula derived below, but it is more usual to determine the loss under known experimental conditions for reasons that will appear later.

Consider a radial element, Q, Fig. 285, of one of the armature core stampings. Let the thickness of the stamping be t, and let ct be the radial depth of the core, where c is a numeric. When the element is in the vertical position OA, the flux passing through its lateral walls is a maximum, and when it is in the horizontal axis OB the flux is zero. This change of flux occurs four times per revolution in a bipolar machine, or, in general, four times per magnetic cycle. The changing flux induces an alternating e.m.f.

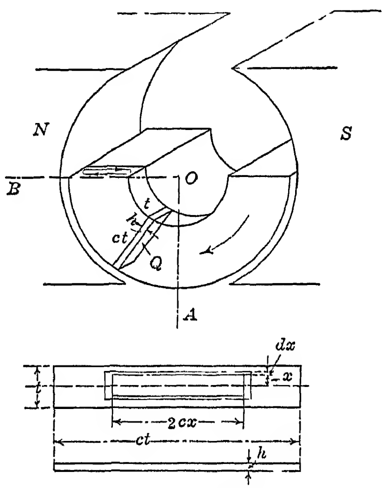


FIG. 285.—Elementary paths of eddy currents.

and sets up a corresponding alternating current which may be assumed to flow in paths like those indicated in the lower part of the figure; an elementary current path is then bounded by similar rectangles of widths $2x$ and $2(x + dx)$, and lengths $2cx$ and $2c(x + dx)$, respectively. The change of flux through such an elementary circuit will be $4B_a \times 4cx^2$ lines per magnetic cycle, where B_a is the maximum flux density in the core, or $16B_acx^2f$ lines per second, where f is the number of magnetic cycles per second. The average e.m.f. in the elementary circuit will be $16B_acfx^2 \times 10^{-8}$ volts. The resistance of the path is

$$\gamma\left[\frac{4cx}{hdx} + \frac{4x}{hcdx}\right]$$

where γ is the specific resistance of the material of the core. The loss in the elementary path is

$$i^2r = \frac{e^2}{r} = \frac{(16B_acfx^2\cdot 10^{-8})^2}{\frac{4\gamma x}{hdx}\left(c + \frac{1}{c}\right)} = \frac{64hB_a{}^2f^2x^3dx}{\gamma}\cdot\frac{c^3}{c^2+1}\cdot 10^{-16}$$

and the total loss is

$$P_{ea} = \frac{64hB_a{}^2f^2}{\gamma\times 10^{16}}\,\frac{c^3}{c^2+1}\int_0^{t/2} x^3dx = \frac{hB_a{}^2f^2t^4}{\gamma\times 10^{16}}\,\frac{c^3}{c^2+1}$$

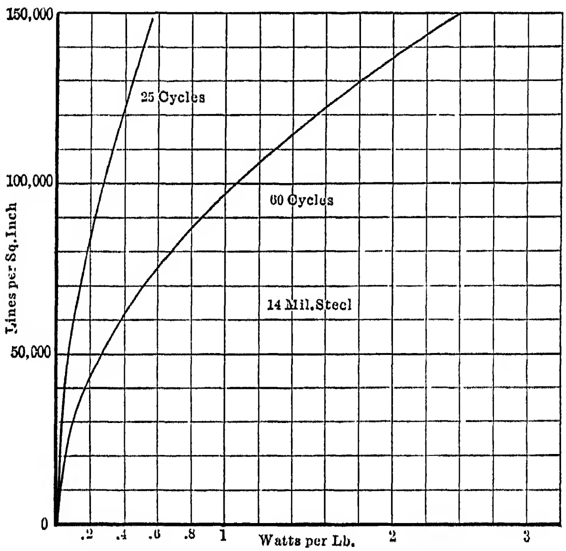


FIG. 286.—Curves of eddy current loss.

But hct^2 is the volume of the element, hence the loss is

$$P_{ea} = \frac{B_a{}^2f^2t^2}{\gamma\times 10^{16}}\cdot\frac{c^2}{c^2+1}\times(\text{volume of tooth})\text{ watts} \qquad (35)$$

This equation shows that the eddy current loss varies as the square of the flux density, the square of the frequency of the magnetic reversals, and the square of the thickness of the laminations; and inversely as the specific resistance of the core material. The equation cannot, however, be relied upon for accurate results, because the actual distribution of the current may differ considerably from the assumed distribution, and the laminations

are not perfectly insulated from each other, as has been tacitly assumed. Due to these causes the actual measured loss will be from 50 to 100 per cent. greater than that computed from the formula. Thus, assuming

$$
\begin{aligned}
B_a &= 10{,}000 \text{ gausses} \\
f &= 60 \text{ cycles} \\
t &= 14 \text{ mils} = 0.0356 \text{ cm.} \\
\gamma &= 12 \times 10^{-6} \text{ ohms per cm. cube} \\
\frac{c^2}{c^2+1} &= 1 \text{ (nearly)}
\end{aligned}
$$

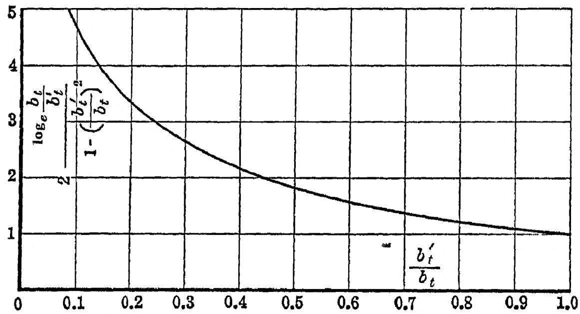


FIG. 287.—Correction factor, eddy current loss in teeth.

the loss in watts per pound by the above formula is 0.22, while the observed value for these data in the case of annealed sheet steel is 0.44 watt per pound. Fig. 286 shows the variation of eddy current loss with flux density at frequencies of 25 and 60 cycles per second and for laminations 14 mils thick. The loss at other frequencies and thicknesses can then be computed by observing that the loss varies as the squares of these quantities.

Eddy Current Loss in the Teeth.—Referring to Fig. 283, the eddy current loss in an elementary section of a tooth is

$$
\begin{aligned}
dP_{et} &= \epsilon f^2 t^2 B^2 \times \text{volume} = \epsilon f^2 t^2 B^2 bkldx \\
&= \epsilon f^2 t^2 b_t^2 B_t^2 kl \frac{dx}{b}
\end{aligned}
$$

where ϵ is the eddy current constant. Integrating,

$$P_{et} = \epsilon f^2 t^2 b_t^2 B_t^2 kl \int_o^{lt} \frac{dx}{b_t - \frac{b_t - b'_t}{l_t} x}$$

$$= \epsilon f^2 t^2 b_t^2 B_t^2 kl \frac{l_t}{b_t - b'_t} \log_e \frac{b_t}{b'_t}$$

$$= \epsilon f^2 t^2 B_t^2 \times \text{volume of tooth} \times 2 \frac{\log_e \frac{b_t}{b'_t}}{1 - (b'_t/b_t)^2} \qquad (36)$$

This equation differs from the original equation (35), in that it contains the additional factor

$$2 \frac{log_e b_t/b'_t}{1 - (b'_t/b_t)^2}$$

the value of which is shown as a function of b'_t/b_t in Fig. 287.

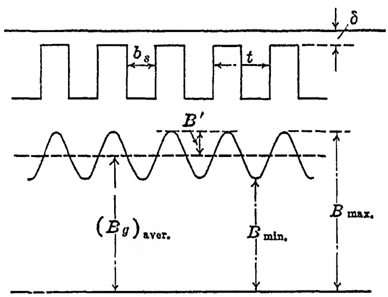


FIG. 288.—Variation of flux density opposite teeth and slots.

Eddy Current Loss in the Pole Faces.—Reference has been made in Chap. II to the cause of the eddy current loss in the pole faces. This loss is confined to a relatively thin layer at the face of the pole because the direction of the induced eddy currents is always such as to damp out the flux pulsations that produce them (Lenz's law).

The flux pulsation at any given point in the pole face will pass through a complete cycle of changes in the time required for a point on the armature to move over a distance equal to the tooth pitch, that is, in a time $t' = \frac{t}{\pi dn/60}$ seconds. This gives a frequency of $f_t = \frac{1}{t'} = \frac{\pi dn}{60t}$ = *number of teeth* $\times$ *rev. per sec.*

Fig. 288 represents the variation of flux density at the pole face on the assumption that the curve of distribution is sinus-

oidal. The amplitude of the pulsation is $B' = \frac{B_{max} - B_{min}}{2}$. Then if

v = peripheral velocity of the armature in centimeters per second

μ = permeability of the material of pole face

ρ = specific resistance of material of pole face in absolute electromagnetic units

the pole face loss in watts per sq. cm. is[1]

$$P_p = \frac{B'^2}{8\pi}\sqrt{\frac{v^3t}{\mu\rho}} \times 10^{-7} = k^2\frac{B_g^{\,2}}{8\pi}\sqrt{\frac{v^3t}{\mu\rho}} \times 10^{-7} \qquad (37)$$

where $k^2 = \left(\frac{B'}{B_g}\right)^2$ is a function of the relative dimensions of the air-gap, teeth and slots. Adams[2] has worked out the curve of Fig. 289 as giving fairly satisfactory values of k^2 in terms of the ratio b_s/δ. If British units are used (B_g in lines per sq. in., v in feet per second, t in inches and μ and ρ as above) the loss in watts per sq. in. of pole face is given as

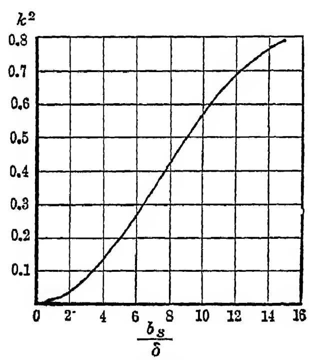


FIG. 289.—Constant for calculation of pole-face loss.

$$P_p = 1.65 \times 10^{-7}\, k^2 B_g^{\,2}\sqrt{\frac{v^3t}{\mu\rho}} \qquad (38)$$

Total Core Loss.—Except in the case of special designs, it is more convenient to have access to the combined values of hysteresis and eddy current losses than to compute each of these losses separately. Test methods lend themselves readily to the determination of the total core loss, and from the results of such tests curves like Fig. 290[3] can be prepared.

179. Mechanical Losses.—

Bearing Friction and Windage.—While it is possible to compute the loss due to bearing friction, the loss due to windage involves so many complex variables that calculation of its magnitude is im-

[1] Potier, L'Industrie Electrique, 1905, p. 35.
Rüdenberg, Elektrotechnische Zeitschrift, Vol. XXVI, p. 181, 1905.

[2] Adams, Lanier, Pope and Schooley, Trans. A.I.E.E., Vol. XXVIII, p. 1133, 1909.

[3] From Gray's Electrical Machine Design, p. 102.

possible. As it is likewise impossible to separate the combined value of the two losses as obtained by test measurements, they are always grouped as *friction and windage* loss. This loss varies from 1 to 3 per cent. of the rated capacity in high-speed machines of moderate capacity, and from 0.8 to 2 per cent. in low-speed machines of moderate size. In large direct-connected machines the loss will be from ½ to 1 per cent. In very high-speed machines, such as turbo-generators, the loss due to windage will be increased.

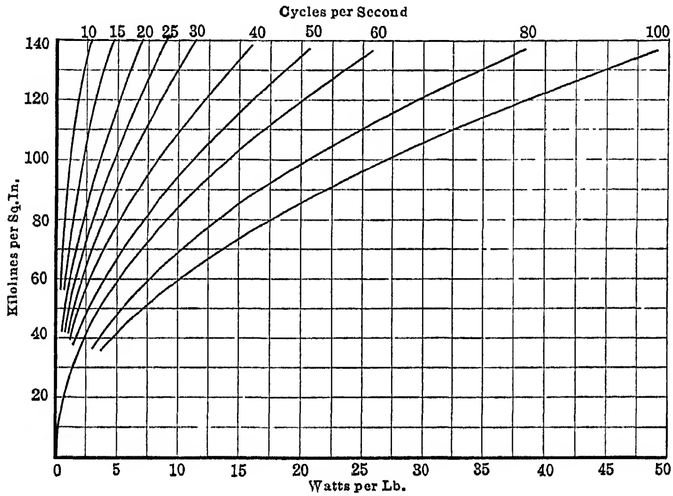


FIG. 290.—Total core loss.

Friction loss in the bearings varies with the $\frac{3}{2}$ power of the peripheral velocity of the shaft in the bearings, up to velocities of about 1800 ft. per minute; at higher velocities it varies directly with the velocity. The windage loss, as in the case of fans, varies as the third power of the speed. But in both cases these losses are independent of the load on the machine.

Commutator Friction Loss.—

Let

d_{com} = diameter of the commutator, inches
A_b = total area of brush contact, sq. in.
p_c = brush pressure (lb. per sq. in.)
f = coefficient of friction.

Then the brush friction loss, in watts, is

$$P_{bf} = \frac{\pi d_{com} n}{12} \cdot \frac{A_b p_c f}{33{,}000} \times 746 = 0.0059 d_{com} n A_b p_c f \text{ watts} \quad (39)$$

Ordinarily the value of p_c is from 1.5 to 2 lb. per sq. in., and f is about 0.3 for carbon brushes and 0.2 for metal brushes.

180. Stray Load Losses.—

Eddy Currents in the Armature Conductors.—When large, solid armature conductors are used in open slots, different portions of the same section of the conductor may be simultaneously in fields of different strength. Under these conditions e.m.fs. of different magnitudes will be generated from point to point of the cross-section, and eddy currents will result. The loss due to these eddy currents may be from 5 to 15 per cent. of the loss due to the ohmic resistance. This is equivalent to saying that, so far as the armature copper loss is concerned, the effective armature resistance is from 5 to 15 per cent. greater than the true resistance. This loss may be minimized by stranding the conductors or using smaller conductors in parallel.

Miscellaneous Losses Due to Short-circuited Currents, Etc.—These are minor losses, and cannot be computed. In testing, they are absorbed in the amount attributed to friction, windage, and core losses.

Summary of Losses

Copper Loss:	
Armature	$i_a^2 r_a$
Field: separately excited	$i_f^2 r_f$
series	$i_a^2 r_f$
shunt	$i_s^2 r_s$
compound, long shunt	$i_a^2 r_f + i_s^2 r_s$
compound, short shunt	$i^2 r_f + i_s^2 r_s$
Commutator	$2i_a \cdot \Delta e$
Core Loss:	
Hysteresis: armature core	$\eta f V B_a^{1.6}$
armature teeth	$\eta f V_t B_t^{1.6} \times 5 \dfrac{1 - \left(\dfrac{b'_t}{b_t}\right)^{0.4}}{1 - \left(\dfrac{b'_t}{b_t}\right)^2}$
Eddy currents: armature core	$\epsilon f^2 t^2 B_a^2 V$
armature teeth	$\epsilon f^2 t^2 B_t^2 V_t \times 2 \dfrac{\log_e b_t/b'_t}{1 - (b'_t/b_t)^2}$
pole faces	$k^2 B_g^2 \sqrt{\dfrac{v^3 t}{\mu \rho}} \times \text{constant}$
Mechanical Losses:	
Bearing friction and windage	½ to 3 per cent. depending upon size and speed.
Brush friction	$0.0059 d_{com} n A_b p_c f$

STRAY LOAD LOSSES:
Eddy currents in armature conductors...... (0.05 to 0.15) $i_a^2r_a$
Losses due to currents in short-circuited coils, etc.

181. Rating and Capacity.—The rating, or rated output, of a machine is based on, but does not exceed, the maximum load which can be taken from it under prescribed conditions of test. If these prescribed conditions are those of the A.I.E.E. Standardization Rules, the rating is said to be the Institute rating; if the prescribed conditions are those of the International Electrical Commission, the rating is said to be the I.E.C. rating. The *capacity* of a machine, expressed in terms of its output, is the load or duty it will carry for a specified time, or continuously, without exceeding certain temperature limitations, as described in the next article.

The Standardization Rules of the A.I.E.E. define two kinds of ratings, namely, *continuous rating*, and *short-time rating*, the latter applying to machines designed for discontinuous or intermittent service. In determining the continuous rating, the machine is subjected to a *heat run*, or test under load conditions, for a sufficient length of time to bring about a constant difference of temperature, of prescribed amount, between the machine and the surrounding air. If the load on the machine is normal full load, it may take from six to eighteen hours to reach stationary temperature conditions, but the time may be reduced by overloading the machine to a reasonable extent during the preliminary period. By taking temperature readings at more or less regular intervals during the test, and plotting rise of temperature against time, the shape of the curve so obtained will indicate to what extent the load should be increased or decreased. The *short-time* rating of machines intended to operate intermittently, that is, with more or less frequent stops of sufficient duration to allow cooling to occur, is the load that the machine will carry for a specified, limited period, without exceeding prescribed conditions of test.

Machines which operate on a cycle of duty that is repeated more or less regularly, as in elevator service, are rated in terms of an *equivalent load* which may be based either on a continuous or short-time test, but selected to simulate as closely as possible the thermal conditions of actual service. The standard durations of short-time equivalent tests are 5, 10, 15, 30, 60 and 120 minutes.

The Standardization Rules specify that the rated output of both generators and motors shall be expressed in kilowatts, thus marking a departure from the practice of rating motors in terms of horse-power. For practical purposes, the horse-power rating, if used, may be taken as four-thirds of the kilowatt rating.

182. Allowable Operating Temperatures.—Theoretically, the output of a generator is limited only by the possibility of sufficiently reducing the resistance of the receiver circuit, at the same time maintaining the generated e.m.f. and supplying the driving power; practically, however, the capacity of the machine is limited by the ability of the insulation to withstand without deterioration, and for long periods, the maximum temperature caused by the heating due to i^2r and other losses, though in some cases the load limit may be determined by commutating conditions. For each kind of insulating material there is a limiting temperature above which deterioration is very rapid, but so far as the useful life of the insulation is concerned there seems to be no particular advantage in operating at temperatures below the safe limits. In case the machine is designed to operate at temperatures well within the safe limits, there will be a margin between its rating and its capacity, hence these terms are not synonymous. If the safe limits of temperature are exceeded, the deterioration of the insulation is rapid, the damage increasing with the duration and extent of the excess temperature.

In the Standardization Rules of the American Institute of Electrical Engineers in force prior to the adoption of the revised rules of 1916, it was specified that the allowable *rise* of temperature of the parts of a machine (excepting railway motors) should be as follows: armature and field windings, 50° C.; commutator, 55° C.; bearings, 40° C. These rises of temperature were based upon standard conditions of a room temperature of 25° C., a barometric pressure of 760 mm., and normal conditions of ventilation. It was further provided that if the room temperature differed from 25° C., the observed rise of temperature should be corrected by ½ per cent. for each degree difference between room temperature and 25° C., the correction to be added to the observed rise if the room temperature was below 25° C., and subtracted if it was higher.

In later rules, emphasis is placed upon the highest permissible temperature of the hottest spot as well as upon the maximum

rise of temperature. The rise of temperature of air-cooled machines (excluding railway motors) is based upon an ambient temperature of 40° C., but it is specified that the observed rise of temperature must never exceed the limits given in the following table, whatever the ambient temperature during the test.

<table>
<tr><th rowspan="2">Method</th><th rowspan="2"></th><th colspan="3">Temperature rise</th></tr>
<tr><th>Class A material</th><th>Class B material</th><th>Class C material</th></tr>
<tr><td>Thermometer</td><td></td><td>50° C.</td><td>70° C.</td><td rowspan="5">Limit not Specified</td></tr>
<tr><td>Resistance</td><td></td><td>55° C.</td><td>75° C.</td></tr>
<tr><td rowspan="3">Embedded Detector</td><td>For windings with two coil-sides per slot, with detectors between top and bottom coil sides and between coil sides and core.</td><td>60° C.</td><td>80° C.</td></tr>
<tr><td rowspan="2">For windings with one coil-side per slot with detectors between coil sides and core and between coil sides and wedge</td><td>55° C.</td><td>75° C.</td></tr>
<tr><td colspan="2">Minus 1° C. for every 1000 volts of terminal pressure of the machine above 5000 volts.</td></tr>
</table>

Class A includes cotton, silk, paper and similar materials when so treated as to increase the thermal limit, or when permanently immersed in oil; also enameled wire. When these materials are not treated, impregnated, or immersed in oil, they are not included in Class A, and the observable temperature rise must be 15° C. below the limits fixed for these materials when impregnated.

Class B includes mica, asbestos and other materials capable of resisting high temperatures, in which any Class A material or binder is used for structural purposes only, and may be destroyed without impairing the insulation or mechanical qualities of the insulation. (The word "impair" is used in the sense of causing any change which could disqualify the insulation for continuous service.)

Class C includes materials like mica, porcelain, quartz, etc., capable of resisting higher temperatures than those of Class B.

The above temperature limits recognize the advances made in the art of constructing insulating materials since the adoption of the superseded rules. While it is known that Class B insulating materials can be supplied to withstand maximum temperatures of 150° C. and even higher, the limit has for the present

been set at 125° C., pending the accumulation of more extensive data at higher temperatures; machines designed for maximum operating temperatures in excess of 125° C. must be subject to special guarantees by the manufacturer.

The temperature limits of commutators must not exceed the values given in the tables for the insulation employed, either in the commutator or in any insulation whose life would be affected by the heat of the commutator. These limits are intended only to protect the insulation of the commutator and of adjacent parts, and are not intended as a criterion of successful commutation.

The new rules abolish the requirement of a correction of the observed rise of temperature due to a difference between the ambient temperature at the time of the test and the standard reference temperature (except in the case of air-blast transformers which are not considered in this text.) This is due to the fact that numerous tests have shown that the effect of variations of the ambient temperature is small, obscure and of doubtful direction. It is, however, recommended that tests be conducted at ambient temperatures not lower than 15° C. The effect of high altitude in increasing the temperature rise of some types of machinery is recognized by recommending a reduction of the normal permissible temperature rise to the extent of 1 per cent. for each 100 meters by which the altitude exceeds 1000 meters.[1] In the case of machines intended for operation at an altitude of 1000 meters or less, a test at any altitude less than 1000 meters is satisfactory and no temperature correction is necessary.

Three methods of determining temperatures of the various parts of a machine are specified, one or the other of these methods being adequate for commercial tests.

1. *Thermometer method,* including measurements by mercury or alcohol thermometers, by resistance thermometers, or by thermocouples, any of these instruments being applied to the hottest *accessible* part of the completed machine. When this method is used, the hottest-spot temperature is estimated by adding a conventional correction of 15° C. to the highest temperature observed, except that when the thermometer is applied directly to the surface of a bare winding, such as an edgewise strip conductor or copper casting, the correction is 5° C. instead of 15° C.

The ambient temperature is to be measured by means of

[1] Water-cooled oil transformers are exempt from this reduction.

several thermometers placed at different points around and halfway up the machine at a distance of 1 to 2 meters, and protected from drafts and abnormal heat radiation. To this end the thermometers are to be immersed in oil in a suitable heavy metal cup, for example, a massive metal cylinder with a hole drilled partly through it. This hole is filled with oil, and must be sufficiently deep to insure complete immersion of the bulb of the thermometer. The smallest size of oil cup permitted by the rules consists of a metal cylinder 25 mm. (1 inch) in diameter and 50 mm. (2 inches) high, but the size of the oil cup must be increased with that of the machine under test. The object of thus increasing the size of the oil cup is to avoid errors in the calculations of temperature rise due to the time lag between changes of temperature of the machine and the surrounding air, this time lag being greater the greater the size of the machine.

Where machines are partly below the floor line in pits, the temperature of the armature is referred to a weighted mean of the pit and room temperatures, the weight assigned to each being based on the relative proportions of the machine in and above the pit. The temperature of the portion of the field structure constantly in the pit must be referred to the ambient temperature in the pit.

2. *Resistance Method.*—This method consists in the determination of the temperature of windings by measurement of their increase of resistance; when this method is used, careful check readings must be taken by means of thermometers, but without disassembling the machine, in order to increase the probability of revealing the highest observable temperature. Whichever method yields the highest temperature, that temperature shall be taken as the highest observable temperature and a hottest-spot correction of 10° C. added thereto. This method is not permitted in the case of low resistance field coils where the joints and connections form a considerable part of the total resistance.

In the case of resistance measurements the temperature coefficient of copper is to be computed from the formula $1/(234.5 + t)$, where t is the initial temperature in degrees Centigrade. From this it follows that the rise of temperature of a winding is given by the formula

$$\theta = (234.5 + t)\left(\frac{R_{t+\theta}}{R_t} - 1\right) \qquad (40)$$

where

$R_{t+\theta}$ = resistance of winding at $(t + \theta)$ degrees

R_t = resistance of winding at t degrees.

3. *Embedded Temperature-detector Method.*—This method involves the use of thermocouples or resistance coils located as nearly as possible at the estimated hottest spot, but is to be used only with coils placed in slots. The thermocouples or resistance coils are built into the machine, and a sufficient number shall be employed to insure locating the hottest spot. They should be placed in at least two sets of locations, one between the coils and core; and one between the top and bottom coils in the case of two-layer windings, or between the coil and wedge in single-layer windings. Detectors of this kind will assume a temperature practically equal to that of the adjacent coils. A correction of 5° C. is to be added to the highest reading in the case of two-layer windings with detectors between coils and between coils and slots; and a 10° correction in the case of single-layer windings, plus 1° C. for each 1000 volts above 5000 volts terminal voltage (single-layer windings are commonly used in alternators, seldom or never in direct-current machines).

183. Heating of Railway Motors.—Operating conditions in the case of railway motors are much more severe than in ordinary motors because of restricted space and the nature of the service. It is therefore good practice to permit higher working temperatures for short periods than in other types of machines. Further, the variable nature of the load makes it more difficult to give a definite rating to railway motors. The *nominal rating* of a railway motor is, therefore, arbitrarily defined as the mechanical output at the car or locomotive axle, measured in kilowatts, which causes a rise of temperature above the surrounding air, by thermometer, not exceeding 90° C. at the commutator and 75° C. at any other normally accessible part, after one hour's continuous run at its rated voltage on a stand with the covers arranged to secure maximum ventilation without external blower. The rise in temperature, as measured by resistance, shall not exceed 100° C. The statement of the nominal rating must also include the corresponding voltage and armature speed.

The *continuous ratings* of a railway motor are defined as the inputs in amperes at which it may be operated continuously at one-half, three-fourths and full voltage respectively, without exceeding the specified temperature rises tabulated below, when operated on stand test with the motor covers and cooling system, if any, arranged as in service. The system of ventilation must be defined, and if cooling is by means of forced draft the volume of air on which the rating is based must be given.

STAND-TEST TEMPERATURE RISES OF RAILWAY MOTORS

Class of material	Temperature rises of windings	
	By thermometer method	By resistance method
A	65° C.	85° C.
B	80° C.	105° C.

The temperatures obtained on stand test, with current and voltage adjusted to give losses equal to those in service will be higher than in actual service because of the absence of the ventilation due to the motion of the car or train. In general, the temperature rise in actual service will be from 75 to 90 per cent. of the temperature rise on stand test in the case of enclosed motors, the losses being the same in both cases; and from 90 to 100 per cent. in ventilated motors.

184. Temperature Specifications of Electric Power Club.—The Electric Power Club, composed of the principal manufacturing companies of the United States, and successor to the American Association of Motor Manufacturers, has adopted a set of standard specifications for generators and motors which differ in some respects from the Standardization Rules of the A.I.E.E. The following specifications are taken from the edition of the regulations adopted by the Club published in March, 1922.

Temperature Ratings

1. There may be two ratings for open type motors and generators with Class A insulation and continuous time ratings as follows:

(*a*) A rating giving a 40-deg. C. temperature rise guarantee under continuous operation with a two-hour, 25 per cent. overload guarantee at 55 deg. C., to be designated and known as the 40-deg. rating.

(*b*) A rating giving a 50-deg. C. temperature rise guarantee under continuous operation without overload temperature guarantee, to be designated and known as the 50-deg. rating.

2. Machines having 40-deg. ratings are designed for all classes of service, including those in which an overload capacity of 25 per cent. for two hours is desired.

3. Machines having 50-deg. ratings are designed for conditions in which the load requirements are accurately known, and under which the machine will not be subjected to load in excess of its rating. Other ratings without overload temperature guarantee, which are designed for these same conditions of service, are:

Type	Class of insulation	Time rating	Temperature rating deg. C.
Open	A	Any	50
Semi-enclosed	A	Any	50
Enclosed	A	Any	55
Open	B	Any	70
Semi-enclosed	B	Any	70
Enclosed	B	Any	75

185. Output Equation.—A definite relation, originally derived by G. Kapp, exists between the rating, speed and dimensions of the armature. This relation, when expressed in algebraic form, is commonly referred to as the *output equation*. Thus, let

V = rated terminal voltage

i_a = rated armature current

ψ = ratio of pole arc to pole pitch

q = ampere-conductors per unit length of armature periphery.

Since

$$V = \frac{p}{a}\frac{\Phi Z n}{60 \times 10^8} \text{ (nearly)}$$

and

$$\Phi = B_g b l = \frac{\pi d \psi}{p} B_g l \text{ (nearly)}$$

the power output of the machine in kilowatts is

$$KW = \frac{V i_a}{1000} = \frac{\pi^2 \psi B_g q}{60 \times 10^{11}} d^2 l n = \xi d^2 l n \tag{41}$$

where

$$\xi = \frac{\pi^2 \psi B_g q}{60 \times 10^{11}} \tag{42}$$

is called the *output coefficient.* The numerical value of this coefficient depends upon the "design constants" of the machine, ψ, B_g and q, but principally upon B_g and q since the range of values of ψ is limited. B_g is clearly a measure of the degree of utilization of the magnetic material of the machine; similarly, q is in part a measure of the specific utilization of armature copper, for it is closely related to the thermal characteristics, as has been shown by Adams.[1] Thus let q be expressed in ampere-

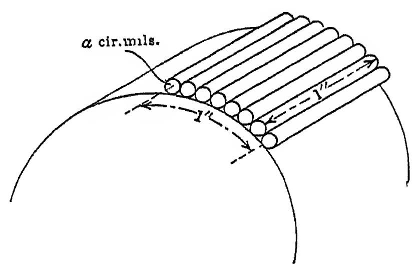


FIG. 291.—Calculation of copper loss per sq. in. of armature surface.

conductors per inch of armature periphery, and let h be the current density in the armature conductors expressed in circular mils per ampere. Let Fig. 291 represent a portion of the armature surface (shown as a smooth-core type for convenience) of 1 inch square. Each conductor will carry a current of $\frac{i_a}{a}$ amperes, and its cross-section will be $\frac{i_a}{a} h$ circular mils; its resistance per inch of length is

$$r = \rho \frac{\text{length}}{\text{area}} = 1 \times \frac{1}{\frac{i_a}{a} h} = \frac{a}{i_a h} \text{ ohms}$$

[1] Trans. A.I.E.E., Vol. XXIV., p. 653, 1905.

since the specific resistance of copper at the working temperature of the armature is very nearly 1 ohm per circular mil-inch. The i^2r loss per conductor is then

$$\left(\frac{i_a}{a}\right)^2 \frac{a}{i_a h} = \frac{i_a}{ah} \text{ watts}$$

But the number of conductors per inch of armature periphery is $\frac{q}{i_a/a} = \frac{aq}{i_a}$, hence the i^2r loss per sq. in. of armature surface is

$$\frac{i_a}{ah} \times \frac{aq}{i_a} = \frac{q}{h} \text{ watts per sq. in.} \qquad (43)$$

The value of q varies from about 400 in machines of 20 kw. or less, up to about 800 to 850 in machines of 1000 kw. capacity. The ratio q/h (watts per sq. in. due to copper loss) is generally in the neighborhood of unity for ordinary peripheral velocities of 2500 feet per minute, but may be as high as 2.5 in large machines running at high peripheral speeds (6000 feet per minute) where the ventilation is more effective. Values of B_g range from about 40,000 lines per sq. in. in small machines up to 60,000 lines per sq. in. in large machines. The value of ξ generally lies between 0.000015 (small machines) and 0.000056 (large machines).

186. Heating and Cooling Curves.—The energy losses in any machine are converted into heat and cause a rise of temperature whose final value depends upon the heat capacity of the materials of the structure and upon the facility with which the heat may be radiated or otherwise dissipated. The temperature will become stationary when the rate of heat generation becomes equal to the rate of dissipation.

It is of interest to derive the law of heating and cooling of a homogeneous body for the reason that it throws light on the conditions obtaining in the more complex structure of a generator or motor.

Let

Q = heat generated per second, in kg-calories
s = specific heat of the substance = amount of heat required to raise 1 kg. 1° C.
W = weight of the body in kg.
A = radiating surface in sq. cm.
α = coefficient of cooling = amount of heat in kg-cal.

dissipated per second per sq. cm. of radiating surface per degree difference of temperature between body and surrounding medium

θ = temperature of body in degrees Centigrade

θ_1 = temperature of surrounding medium in degrees Centigrade.

1. *Heating of the body.*

In a time dt the temperature will increase by $d\theta$ degrees. During this interval the heat liberated amounts to Qdt kg-calories, and the body absorbs $sWd\theta$ kg-calories. The remainder will be dissipated, to the amount $A\alpha(\theta - \theta_1)dt$ kg-calories, so that

$$Qdt = sWd\theta + A\alpha(\theta - \theta_1)dt \tag{44}$$

Transposing,

$$dt = \frac{sWd\theta}{Q - A\alpha(\theta - \theta_1)}$$

Assuming that $\theta = \theta_1$ when $t = 0$,

$$\int_0^t dt = sW \int_{\theta_1}^{\theta} \frac{d\theta}{Q - A\alpha(\theta - \theta_1)}$$

which gives

$$\theta - \theta_1 = \frac{Q}{\alpha A}\left(1 - \epsilon^{-\frac{\alpha A}{sW}t}\right) \tag{45}$$

When $t = \infty$,

$$(\theta - \theta_1)_{t=\infty} = \frac{Q}{\alpha A} \tag{46}$$

and this is the limiting temperature rise of the body. The last equation (46) may be written

$$Q = \alpha A(\theta - \theta_1)_{t=\infty} \tag{47}$$

which expresses the fact that when the temperature becomes stationary the rates of heat production and dissipation are equal.

2. *Cooling of the body.*

In this case no heat is being developed, consequently $Q = 0$ and the fundamental equation becomes

$$0 = sWd\theta + A\alpha(\theta - \theta_1)dt \tag{48}$$

If the temperature is Θ degrees when $t = 0$,

$$\int_0^t dt = -sW\int_\Theta^\theta \frac{d\theta}{A\alpha(\theta - \theta_1)}$$

which gives

$$\theta - \theta_1 = (\Theta - \theta_1)\epsilon^{-\frac{\alpha A}{sW}t} \tag{49}$$

as the equation of the cooling curve. If $\Theta - \theta_1 = (\theta - \theta_1)_{t=\infty} = \frac{Q}{\alpha A}$, that is, if the temperature at the beginning of cooling is equal to the limiting temperature at the end of the heating period,

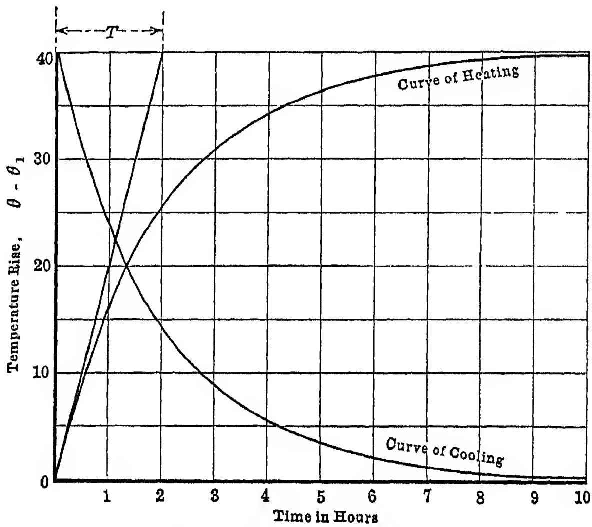


FIG. 292.—Heating and cooling curves.

the equation of the cooling curve is the same as the variable part of the heating equation, but with a change of sign. Hence, in this case, the heating and cooling curves are of the same logarithmic shape, but one is turned upside down with respect to the other, as shown in Fig. 292.

Differentiating the heating equation

$$\theta - \theta_1 = \frac{Q}{\alpha A}\left(1 - \epsilon^{-\frac{\alpha A}{sW}t}\right)$$

and substituting $t = 0$ in the result, the slope of the curve at the origin is found to be

$$\left(\frac{d\theta}{dt}\right)_{t=0} = \frac{Q}{sW} \tag{50}$$

that is, dependent upon the mass and material of the body, but not upon its cooling area or the nature of the radiating surface. In fact, at the first instant, all of the heat is absorbed and none of it is radiated; hence, the slope of the curve at the origin gives the rate at which the temperature would rise if all the heat were absorbed. If the temperature continued to rise at this rate, the limiting temperature rise, $\frac{Q}{\alpha A}$, would be reached in a time $T = \frac{sW}{\alpha A}$ seconds; T is called the *time constant* of the body.
The heating equation can then be written

$$\theta - \theta_1 = \frac{Q}{\alpha A}\left(1 - \epsilon^{-\frac{t}{T}}\right)$$

To substitute numerical values in the above equations, the following relations obtain

$$\begin{aligned} Q &= 0.2386 \times (\text{loss in kw.}) \text{ kg-cal. per sec.} \\ &= 0.527 \times (\text{loss in kw.}) \text{ lb-cal. per sec.} \\ s &= 0.11 \text{ for iron} \\ s &= 0.09 \text{ for copper.} \end{aligned}$$

The value of α may be found from the experimentally determined[1] fact that when air is blown across the bare (or thinly varnished) surface of an iron core its surface temperature will rise 1° C. when the radiation is 0.0038 $(1 + 0.25v)$ watts per sq. cm. of surface, where v is the velocity of the air in meters per second; this is equivalent to 0.0245 $(1 + 0.00127v)$ watts per sq. in. if v is in feet per minute. From this it follows that

$$\alpha = 0.906\ (1 + 0.250v) \times 10^{-6} \text{ kg. cal. per sec. per sq. cm. per } 1^\circ \text{ C.} \tag{51}$$

where v is given in meters per second; or

$$\alpha = 12.89\ (1 + 0.00127v) \times 10^{-6} \text{ lb-cal. per sec. per sq. in. per } 1^\circ \text{ C.} \tag{52}$$

where v is expressed in feet per minute.

The experiments of Ott also showed that if the surface of the core is coated with a thick double layer of varnish the radiation

[1] Ludwig Ott, London Electrician, 1907, p. 805.

is 0.0030 $(1 + 0.107\, v)$ watts per sq. cm. per 1° C. rise of surface temperature, v being in meters per second; or 0.0194 $(1 + 0.00054v)$ watts per sq. in. per 1° C, v being in feet per minute.

Temperature rise computed from the above formula will not agree in general with the observed rise in actual machines because it neglects the transfer of heat from the winding to the core, or *vice versa;* likewise, the irregular distribution of heat evolution and the thermal capacity of the insulation. But in general terms it will be true that the ultimate rise of temperature can be expressed by the equation

$$\theta - \theta_1 = \text{constant} \times \frac{\text{watts dissipated}}{\text{radiating surface}} \tag{53}$$

where the constant is in each case to be determined by experiment.

187. Heating of the Armature.—The experimental results of Ott referred to above may be changed to a form applicable to the rotating part of the machine. Taking the value of the radiation for bare or thinly varnished surfaces, the temperature rise is given by

$$\theta - \theta_1 = \frac{w}{a}\,\frac{460}{1 + 0.25v} \tag{54}$$

where

w = total watts dissipated

a = total radiating surface, sq. cm.

v = peripheral velocity of armature, meters per second

while for a heavily varnished surface

$$\theta - \theta_1 = \frac{w}{a}\,\frac{333}{1 + 0.107v} \tag{55}$$

Accordingly, the rise of temperature for a radiation of 1 watt per sq. cm. is found by putting $\frac{w}{a} = 1$. The temperature rise (in degrees Centigrade) for a radiation of 1 watt per sq. in., expressing v in feet per minute, is

$$\frac{71.3}{1 + 0.00127v} \text{ for a bare surface}$$

$$\frac{52}{1 + 0.00054v} \text{ for a heavily varnished surface}$$

Other writers give the value of this constant as follows:

	Metric units	English units
Kapp.	$\dfrac{550}{1 + 0.1v}$	$\dfrac{85}{1 + 0.00051v}$
Arnold.	$\dfrac{300}{1 + 0.1v}$	$\dfrac{46.5}{1 + 0.00051v}$
Esson.	$\dfrac{354}{1 + 0.0006v}$	
Wilson.	$\dfrac{640}{1 + 0.18v}$	$\dfrac{99}{1 + 0.00091v}$
Thompson	$\dfrac{645}{1 + 0.3\sqrt{v}}$	$\dfrac{100}{1 + 0.0213\sqrt{v}}$

The above expressions are embodied in Fig. 293, which shows the rise of temperature per watt per sq. in. as a function of the pe-

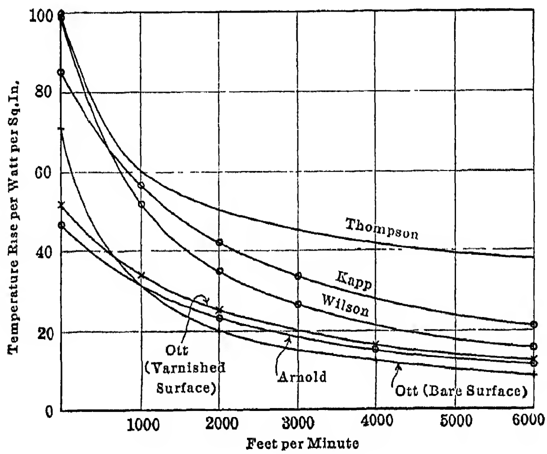


FIG. 293.—Relation between temperature rise and peripheral velocity of armature.

ripheral velocity in feet per second. It will be observed that the curve represented by Arnold's formula lies nearly midway between the two corresponding to Ott's researches, at least for values of v within the usual limits of practice.

It should be noted that all of the above formulas for computing rise of temperature are more or less uncertain, unless applied to a machine of the same type as that from which the constants were experimentally determined. A large part of the differences between the curves of Fig. 293 is due to the fact that there is no

absolute agreement as to what constitutes the radiating surface. Some writers specify the outer cylindrical surface only, but including the surface of the end connections as well as of the core; others include the exposed sides of the core in addition to the outer cylindrical surface. Evidently all exposed surfaces are useful in radiating heat, but not to the same extent. The magnitude and direction of the flow of heat from the interior to the exterior of a mass will depend upon the heat conductivity in different directions; and since the conductivity along the laminations is much greater than across them (Ott found it to be from 50 to 100 times greater) it follows that unless the core is very deep the greater part of the heat will be dissipated from the cylindrical surface. It has been pointed out[1] that a rational equation for the rise of temperature of an armature should be of the form

$$\theta - \theta_1 = \frac{w}{\Sigma a_1 + c\Sigma a_2} \cdot \frac{C}{1 + bv} \tag{56}$$

where

Σa_1 = sum of cylindrical cooling surfaces
Σa_2 = sum of end surfaces
c = a variable coefficient less than unity.

The value of c will be smaller the greater the ratio of heat conductivity along the laminations to that across them.

The Arnold formula for rise of armature temperature is

$$\theta - \theta_1 = \frac{w}{a} \frac{(40 \text{ to } 70)}{1 + 0.00051\, v} \tag{57}$$

where a and v are expressed in square inches and feet per minute, respectively. The numerical coefficient in the numerator is to be taken near the lower limit of its range when the ventilation is good. In using this formula, however, it should be noted that w does not include the watts dissipated in the end connections, nor does a include their surface; in other words, the rise of temperature of the armature core is to be distinguished from that of the end connections. Consequently, to estimate the rise of temperature of the core, the value of w to be inserted in the formula is

[1] Ott, London Electrician, 1907, p. 805.

$$w = \text{total core loss} + \frac{\text{embedded length of winding}}{\text{total length of winding}} \times i_a{}^2 r_a \quad (58)$$

The value of a recommended by Arnold is the cylindrical surface of the core, plus the two end surfaces, plus half the lateral area of the walls of the ventilating ducts; or (Fig. 294)

$$a = \pi dl + \pi d_{aver}h(2 + n_v) \quad (59)$$

In the case of the end connections,

$$\left. \begin{aligned} w &= \frac{\text{free length of winding}}{\text{total length of winding}} \times i_a{}^2 r_a \\ a &= 2\pi d l_e \end{aligned} \right\} \quad (60)$$

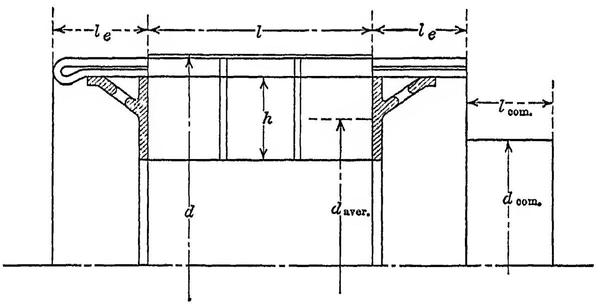


FIG. 294.—Dimensions of radiating surfaces.

In semi-enclosed machines the temperature rise is about 50 per cent. greater than that given by the above formula, and in enclosed machines it is about twice as great as given by the formula.

188. Heating of the Field Coils.[1]—The field coils are heated not only by the i^2r losses that occur in them but also by the losses in the pole faces caused by eddy currents, and by radiation from the armature. The heat is dissipated in three ways: by convection in the surrounding air; by conduction through the pole cores and yoke; and by direct radiation. The temperature inside the coil varies from point to point in a manner depending upon the depth of the winding and upon the nature of the insulation, being highest near the middle of the cross-section of the coil and lowest on the exposed surface. Impregnated coils run cooler than ordinary coils, for the insulating compound is a better

[1] For the results of elaborate studies of field coil heating refer to articles by Neu, Levine and Havill, Electrical World, Vol. XXXVIII, p. 56, 1901; and by Ott, London Electrician, 1907, p. 805.

heat conductor than the air it replaces. Measurement of the rise of temperature by the increase of resistance gives the *average* rise of temperature of the winding as a whole, the *maximum* rise at the hottest point being from 12 to 20 per cent. greater than the average rise. The average rise of temperature of the entire winding is from 40 to 60 per cent. greater than the average rise of temperature of the exposed surface; the latter is determined by taking the mean of thermometer readings at the middle and ends of the exposed cylindrical surface.

Formulas for computing the rise of temperature of field coils are of the form

$$\theta - \theta_1 = C \frac{\text{watts lost in coil}}{\text{radiating surface of coil}} \tag{61}$$

Different writers assign various values to the constant C, depending upon the selection of what constitutes the radiating surface. Obviously, C means the rise of temperature due to a radiation of 1 watt per unit area. If the radiating surface is expressed in square inches and is taken to mean the outer cylindrical surface exclusive of the exposed end, the value of C for open type machines, and under standstill conditions, is from 70 to 80, with an average of 75. The value of C decreases with increasing peripheral velocity of the armature, due to fanning action, by approximately 5 per cent. per 1000 feet per minute, or

$$C = 75(1 - 0.00005v) \tag{62}$$

where v is peripheral velocity in feet per min. This is an average value, the decrease of C being somewhat greater if the coils are short, because of the cooling effect of the yoke; and somewhat less if the coils are long. In machines of the protected type C is approximately 50 per cent. greater than the above value, and in enclosed machines from two to three times greater than given by equation (62).

Field coils of the ventilated type of construction are made of concentric parts with an open space of about ½ inch between them. The greater surface presented to the air by reason of this construction permits of increased radiation; however, the internal surfaces of the ducts are not as effective as an equal area on the outside. For a given temperature rise the ventilated coil will

radiate about 50 per cent. more watts per sq. in. than an ordinary coil; or, what amounts to the same thing, C may be taken as equal to 50.

189. Heating of the Commutator.—The commutator is heated by the losses due to brush friction, P_{bf}, and by the flow of the current across the contact resistance between commutator and brushes. The rise of temperature can be computed from the formula

$$\theta - \theta_1 = 20\frac{W}{A}\frac{1}{1 + 0.00051v} \qquad (63)$$

where

W = total loss at the commutator

A = $\pi d_{com} l_{com}$

v = peripheral velocity of commutator in feet per minute.

190. Rating of Enclosed Motors.—If a motor of the open type is converted into one of the enclosed type, it is clear that its rating must be reduced to avoid excessive temperature rise. Experience shows that a reduction in horse-power rating of about 30 per cent., accompanied by an increase of speed of 20 per cent. will give a temperature rise within standard limits. The reduction in horse-power rating decreases the current and consequently the i^2r losses, and the increase of speed permits a reduction of the flux per pole, thereby lowering the excitation loss and the core loss. The core loss decreases notwithstanding the increase of speed, for the effect of reduced flux density more than outweighs the effect of increased frequency of the magnetic reversals (see Fig. 290); the core loss varies nearly as the square of the flux density, and approximately as the first power of the speed since the hysteresis loss is always greater than the eddy current loss.

PROBLEMS

1. A 220-volt shunt motor has an armature resistance (including brush contact resistance) of 0.44 ohm and a shunt field resistance of 169 ohms. When running without load the armature current is 1.5 amp. and the speed is 1175 r.p.m. Find the conventional efficiency, the efficiency of conversion and the mechanical efficiency when the armature current is 25 amp. What is the horse-power output with this armature current, and what is the corresponding speed of the motor, assuming constant flux?

2. Solve Problem 1 on the assumption that the armature resistance of 0.44

ohm does not include the brush contact resistance, and using the A.I.E.E. rule for brush drop.

3. Find the efficiency of the above motor when the armature current has values of 5, 10, 15, 25, and 35 amp., and plot a curve showing the relation between efficiency and horse-power output. Use the A.I.E.E. rule for brush drop.

4. At what value of horse-power output will the motor of Problem 1 develop maximum efficiency, and what is the value of the maximum efficiency?

5. If the rated output of the motor of the above problems is 7.5 h.p., what will be the armature current at ¼, ½, ¾, and 1¼ times rated load, allowing for brush drop? Plot a curve showing the relation between armature current and per cent. of rated load.

6. Compute the all-day efficiency of the motor of the preceding problems if it operates at ¼ load for 3 hr., at ½ load for 2 hr , at ¾ load for 1½ hr., at full load for 3 hr., and at 1¼ times full load for ½ hr., the working day being 10 hr.

7. If it is required to design a shunt motor that shall develop a full-load efficiency of 85 per cent., and in which the maximum efficiency shall occur at three-fourths of full load, what must be the variable and fixed losses expressed in per cent. of full-load rating? What will be the maximum efficiency?

8. If the machine of Problem 1 is operated as a shunt generator with a terminal voltage of 220 volts and an armature current of 25 amp., what will be its conventional efficiency, its efficiency of conversion and its electrical efficiency, assuming that the constant loss is the same as in the case of motor action, allowing for brush drop as in Problem 2?

9. Assuming that the data of Problem 1 apply when the machine has been standing idle for a considerable period in a room whose temperature is 25° C., what is the average rise of temperature of the field winding if, after several hours run under full load, the field current is found to be 1.09 amp.? If there is no demagnetizing effect due to armature current, what will be the speed, assuming that the magnetization curve is represented by Froelich's equation (Chapter VI) such that an exciting current of 0.65 amp. produces two-thirds as much flux as an exciting current of 1.3 amp.?

10. If the motor of Problem 1 is to be designed with four poles, the pole faces being approximately square, what should be the diameter and axial length of the armature core, assuming an average air-gap flux density of about 45,000 lines per sq. in., 500 ampere-conductors per in. of periphery, and a ratio of pole arc to pole pitch of 0.7?

CHAPTER XI

BOOSTERS AND BALANCERS. TRAIN LIGHTING SYSTEMS

191. Boosters.—A *booster* is a dynamo-electric machine whose armature is connected in series with a circuit, its generated e.m.f. being added to or subtracted from that of the circuit, depending upon the polarity of its excitation. Boosters may be driven by any form of prime mover, but are generally direct-connected to a motor taking current from constant potential mains.

192. The Series Booster.—An obvious use for a booster is to raise the voltage of a generator, or of a section of the bus-bars of a central station, by an amount sufficient to compensate the ohmic drop in a feeder supplying a distant load, in case the load is of such character as to require the same voltage as receiving devices at or near the source of supply. Since the line drop is directly proportional to the current, the voltage of the booster should also be proportional to the current; in other words, the booster should have an external characteristic consisting of a straight line through the origin. It is impossible to exactly realize this form of characteristic without auxiliary devices, but it may be approximated sufficiently closely for practical purposes by designing the booster as a series-wound generator with flux densities well within the point of magnetic saturation. The hysteresis effect illustrated in Fig. 106, p. 152, is especially objectionable in boosters, and should be reduced to a minimum. Further, if the excitation is of such character that the main flux is subject to wide variations, the magnetic circuit must be laminated throughout in order that eddy currents set up by a change in the flux may not be of sufficient magnitude to retard the change of flux and so make the machine sluggish in its action.

The compensation, by means of a series booster, of the drop

of potential in a circuit due to its ohmic resistance is equivalent to a complete cancellation of the resistance of the circuit. Ordinarily, if the resistance of a circuit is to be reduced, the reduction would be made by an increase of the cross-section and therefore of the weight and cost of the line. Up to a certain point, which may readily be computed for a given set of conditions, it will be cheaper to save energy by adding copper than to install a booster equipment; beyond that point the booster will be more economical.

The apparent cancellation of the resistance of a circuit by means of a series booster is sometimes utilized in electric railways employing a ground return to mitigate the *electrolysis* of underground structures such as water and gas mains, telephone cables, and the like. The return circuit of the ordinary street railway system consists of the track and the surrounding earth, the current dividing between these paths in the inverse ratio of their resistances. Even with well-bonded tracks a considerable flow of current may take place through the earth along paths of low resistance afforded by underground metallic structures, resulting in damage wherever stray currents leave these paths to return through moist earth to the track or to the grounded bus at the power house. It is becoming standard practice to minimize the danger of electrolysis in such systems by installing insulated *negative feeders* or cables which connect points along the track directly to the negative bus of the generating station, thereby draining the track current away from the stray paths. These negative feeders are clearly the more effective the lower their resistance. If a series booster is now connected in such a feeder so that its e.m.f. acts in the direction from the track to the negative bus, the equivalent resistance of the feeder may be reduced nearly to zero, and most of the current will return to the station by way of the feeder. Boosters used in this way are called *negative* or *track-return* boosters.

193. The Shunt Booster.—In constant-potential systems in which the load changes gradually, but covers a range from a very small to a considerable value, it is common practice to use a storage battery in parallel with the bus-bars. The battery may then be used to carry the entire load at times of light load, and in parallel with the generator at the time of peak load. At

other times the battery takes charging current from the generator, thus insuring a fairly uniform load on the generator during its working period, with consequent economy in cost of fuel. In a system of this kind a so-called *shunt booster* is used to force charging current into the battery against the latter's counter e.m.f., the connections being shown in Fig. 295. The field winding of the booster is connected across the main bus-bars, never across its own armature, hence the machine is really separately excited. The booster armature is in series with the battery during the charging period, and is called upon to supply a relatively small e.m.f., hence the above connection of the field winding. The booster voltage is manually controlled by means of the field rheostat, adjustment being made when the readings of the am-

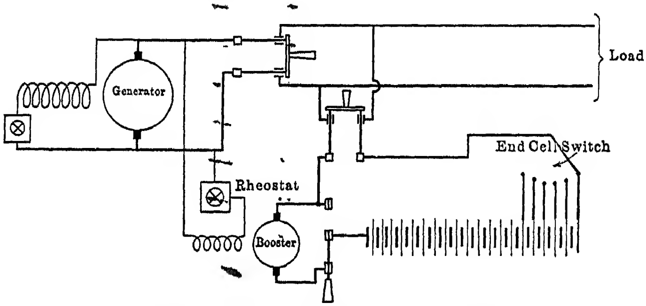


Fig. 295.—Connections of shunt booster.

meters indicate that it is necessary. By inserting a reversing switch in the field circuit, or by using a reversing rheostat, the booster e.m.f. may be added to that of the battery, thereby assisting the battery to discharge if its voltage is low, or if the demand for current is unusually heavy.

A lead storage battery when discharged to the permissible limit gives 1.8 volts per cell, and when fully charged requires an impressed voltage of 2.65 volts per cell to give it the "overcharge" that is periodically required to keep it in good condition. If, therefore, the voltage of the system is V, the total number of cells required is $V/1.8$ to provide for the contingency that the battery alone, when nearly exhausted, may be used to carry the load. At the end of a prolonged charge the battery voltage will then have risen to $2.65 \times (V/1.8)$, hence the booster must be

capable of generating $V\left(\frac{2.65}{1.8} - 1\right) = 0.47\,V$ volts; thus, in a 110-volt system, the maximum booster voltage will be 52 volts. The design of the booster will then be completely determined when the maximum discharge rate of the battery is known.

The capacity of the motor that drives the booster need be only from two-thirds to three-fourths of the volt-ampere capacity of the booster for the reason that when the latter delivers its maximum current the voltage is low, and when the voltage is highest, during the periods of overcharge, the current must be considerably reduced. The normal (eight-hour) discharge rate of a lead battery is defined as that current which, flowing uniformly for eight hours, will reduce the battery voltage to the minimum value of 1.8 volts per cell; the current during overcharge should be not greater than one-half of the eight-hour rate.

In Fig. 295 the cells shown at the right-hand end of the battery are the *end-cells* which are cut in and out of circuit by means of an end-cell switch. Their purpose is to adjust the battery voltage to the line requirements to compensate for the changes in voltage due to varying conditions of charge and discharge. Thus, in a 110-volt system, the number of cells required will be 110/1.8 = 61 when fully discharged; but when a fully charged battery begins to discharge its terminal voltage is 2.15 volts per cell, therefore requiring $\frac{110}{2.15} = 51$ cells. Consequently in such a system 61 cells would be installed, 10 of them as end-cells. The number of end-cells may be reduced if the booster field is provided with a reversing switch, for in that case the booster e.m.f. can be made to oppose that of the battery to a sufficient extent to bring the terminal voltage to the proper value.

194. The Constant-current or Non-reversible Booster.—In isolated plants supplying a lamp load and a fluctuating motor load, as in hotels and office buildings, it is necessary to maintain a constant lamp voltage, and it is permissible or even desirable to allow the voltage of the power circuit to fall when there is a heavy rush of current, as on starting an elevator. Fig. 296 represents a type of installation occasionally used in such a case. The shunt field winding of the booster, *f*, is connected across the constant potential lighting bus and its magnetizing effect is opposed by that of

the series winding, S, as indicated by the arrows. The excitation due to f is normally the greater of the two, and the resultant differential excitation produces a booster voltage that acts in the same direction as the generator, and which is from 10 to 15 volts under normal load conditions. At normal load the adjustments are such that the battery neither charges nor discharges, in other words, the sum of the voltages of generator and booster equals the open-circuit voltage of the battery. The entire lighting and power load is then carried by the main generator. If the motor load is suddenly increased, there is an initial tendency to draw the increased current from the generator, but this results in an increased excitation of the series winding of the booster and a reduction of its generated e.m.f.; the original condition of balanced voltage at the battery terminals is therefore disturbed, and the battery discharges and relieves the generator of the current in

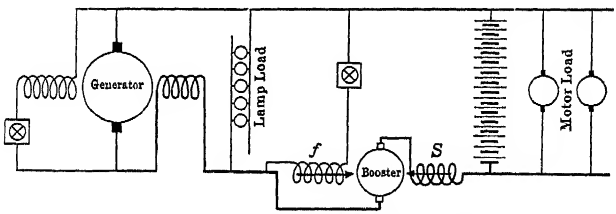


FIG. 296.—Constant-current or non-reversible booster.

excess of the normal amount. Conversely, a decrease of the motor load results in a momentary weakening of the series excitation of the booster and a charging current therefore flows into the battery. The current through the armature and series field of the booster is therefore not constant, as the term constant-current booster might imply, but it is substantially so, the total variation of a few per cent. being no more than is sufficient to cause the battery to take up the fluctuations of current above and below the average value. When a storage battery charges or discharges, its terminal voltage rises or falls, respectively, by an amount very nearly proportional to the current; thus, if the current is equal to the eight-hour rate, the change of voltage is 0.05 volts per cell, and at the one-hour rate[1]

[1] If the capacity of a storage battery is C_8 amp-hr. when discharged at the 8-hr. rate (see p. 420), its capacity is greatly decreased if it is discharged at

(equivalent to four times the current at the eight-hour rate), the variation is 0.2 to 0.21 volts per cell, provided the battery is initially fully charged. The function of the booster is then to produce a change of voltage at the battery terminals corresponding to the charge or discharge rate demanded by the load. For example, assume that the voltage of the motor circuit is 230, requiring, say, 115 cells when charged to a normal voltage of 2 volts per cell. If the load calls for a supply of current equivalent to the eight-hour discharge rate of the battery, over and above the normal supply of I_{aver} amperes, the booster voltage must be lowered by $115 \times 0.05 = 5.75$ volts. This can be accomplished by so proportioning the series winding that an increase of the current through it from I_{aver} to I_{aver} $(1 + p)$ will produce the necessary change in field excitation, where $p \times 100$ is the prescribed percentage variation of booster current. The linear variation of booster voltage of course requires that the magnetic circuit be worked on the straight part of the magnetization curve (see Fig. 102, p. 149). Change of battery voltage with varying conditions of charge can be compensated by manual regulation of a rheostat in the shunt field of the booster; but in the type of service to which the non-reversible (and other automatic) boosters are adapted, the fluctuations of load causing alternate charge and discharge are so rapid that the general condition of the

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greater rates. Thus, if the current is such that the voltage per cell falls to 1.8 volts in 1 hr., the current is said to be the 1-hr. rate, and the capacity falls to ½ C_8 amp-hr. The reduced capacity is due to the fact that the high rates of discharge produce chemical changes of great velocity in a thin surface film of the active material, thereby preventing the electrolyte from penetrating to fresh material. The relation between discharge rate (n) and the corresponding capacity in amp-hr. (C_n) is given approximately by the formula

$$C_n = \frac{C_8}{2}\sqrt[3]{n}$$

(See data in Storage Battery Engineering, by Lamar Lyndon, 3d ed., p. 98, and Foster's Electrical Engineers' Pocketbook, 7th ed., 1913, p. 875.)

If i_8 is the current corresponding to the 8-hr. rate, and i_n the current corresponding to the n-hr. rate, it is clear that $C_8 = 8i_8$ and $C_n = ni_n$; whence, substituting the above approximate relation between C_n and C_8, it follows that

$$i_n = \frac{4i_8}{\sqrt[3]{n^2}} \text{ (nearly)}$$

battery changes very little. The non-reversible booster is suited to systems in which the average motor load is small and the fluctuations are considerable.

195. Reversible Booster.—In systems in which it is not desirable that battery discharge shall be accompanied by a drop of voltage of the power circuit, as in a railway system having a large average load, the *reversible* booster shown diagrammatically in Fig. 297 is sometimes used. It differs from the non-reversible booster in that the current through its armature is not uni-directional, though in both types the shunt and series field windings are differentially connected. The object of the booster is to hold the load on the generator at a constant value equal to the average load on the system, leaving the battery to take up the fluctuations. It is adapted to systems in which the average load is large compared with the range of the fluctuations.

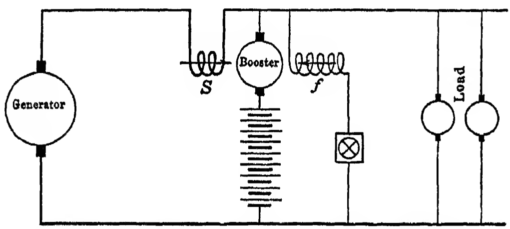


FIG. 297.—Differential or reversible booster.

Referring to Fig. 297, the battery is so designed that its open circuit voltage is equal to that of the system; consequently, when the load on the system has its normal (average) value, the battery must neither charge nor discharge, and the current through the shunt field of the booster must be adjusted so that its magnetizing effect exactly neutralizes that of the series winding S. With increased demand on the line there is a slight increase of current through S, and a resultant magnetization of the booster in such a direction that the generated e.m.f. acts in the same direction as the battery; discharge of the battery then takes place. The e.m.f. generated in the booster armature must therefore be equal to the drop of battery voltage that corresponds to the discharge rate demanded by the load, plus the ohmic drop in the armature of the booster itself. On the other hand, if the load

falls below normal the shunt winding overpowers the series winding, and the booster voltage is added to that of the generator, with the result that a charging current flows into the battery. The capacity of the booster is determined by the fact that maximum current and maximum e.m.f. occur simultaneously.

Although the open-circuit voltage of the battery is nominally equal to that of the generator and of the system, its actual voltage may vary over a considerable range, depending upon the state of the battery charge. To compensate these changes the excitation of the shunt field must be adjusted by hand regulation of a rheostat in series with the shunt winding.

196. Auxiliary Control of Boosters.—Both the reversible and the non-reversible boosters described in Articles 194 and 195 have the disadvantage that a given change of current in the series coil of the differential winding produces a definite voltage, without regard to the fact that the change of battery voltage corresponding to each rate of charge or discharge varies with the condition of the battery; that is to say, a given change of current in the series winding of the booster will not always automatically result in the desired rate of charge or discharge. Moreover, the heavy current that must be handled by the series winding requires a conductor of large cross-section and a machine frame of excessive dimensions and weight per kilowatt of capacity. To obviate these difficulties there have been developed several automatic systems that regulate the battery by external means, and in which the booster has a simple shunt winding. These systems have practically superseded the types of differential boosters described above.

197. The Hubbard Counter E.M.F. System (Controlled by the Gould Storage Battery Co.) is shown diagrammatically in Fig. 298. The field coil f of the booster B is in series with the armature of a small motor-driven exciter E, the field of the latter being in turn excited by the main generator current, or fractional part thereof. The adjustments are so made that when the load has its average value the exciter E produces an e.m.f. equal and opposite to that of the line. There is, therefore, no current through the booster field winding, no e.m.f. is generated in the booster armature, and the battery, which has a normal voltage equal to that of the line, neither charges nor

discharges. An increase of load above the average value results in an increase of the current through the series coil of the exciter, the generated e.m.f. of the latter then exceeds the line voltage, and a flow of current is established through the booster field winding in the proper direction to generate in the booster armature an e.m.f. that assists the battery to discharge.

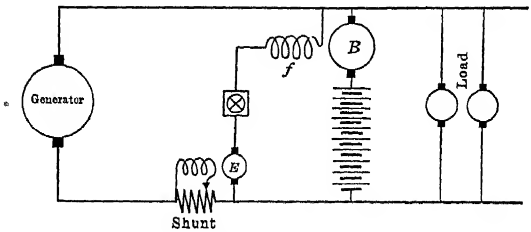


FIG. 298.—Hubbard counter e.m.f. system of booster regulation.

Conversely, a decrease of load weakens the field of the exciter, the polarity of the booster reverses, and the battery then takes a charging current.

198. The Entz System (Electric Storage Battery Co.) of external control for installations of large capacity is illustrated in Fig. 299. The main output of the station passes through a coil S consisting of a few turns of heavy strap copper, and produces an

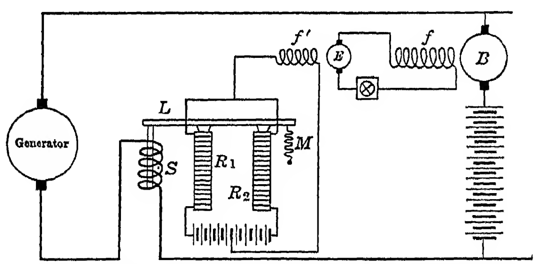


FIG. 299.—Automatic booster regulation, Entz system.

electromagnetic pull on a core attached to one end of a pivoted lever L. When the station output has its average value the pull of the electromagnet is balanced by a spring M in such a manner that the lever presses upon the piles of carbon plates, R_1 and R_2, with forces that make the resistances of the two piles equal to each other. The carbon piles are connected to each other at

the top and to one terminal of the field winding f' of a small motor-driven exciter E; at their lower ends the carbon piles are connected to the terminals of a small auxiliary storage battery, the middle point of which is connected to the other terminal of f'. The armature of the exciter E supplies current to the field winding, f, of the booster B. So long as the resistances of R_1 and R_2 are equal to each other and all the cells of the auxiliary battery are equally charged, there will be no difference of potential between the terminals of f'; consequently there will be no e.m.f. generated in either the exciter or the booster. Under these conditions the main battery, which has a normal voltage equal to that of the line, will neither charge nor discharge. If the load current increases there will be a tendency to increase the generator current through S, and the pressure on R_1 will be increased; this causes a reduction of the resistance of R_1 and the auxiliary battery will send a current through f', thereby generating an e.m.f. in the exciter armature and energizing the field of the booster. The field windings of E and B are connected in such order that the booster voltage adds to that of the main battery and a discharge results. In case the load falls below its average value, the spring M overpowers the pull of S and the resistance of R_2 becomes less than that of R_1, producing a reversed flow of current through f' and, therefore, through f also, so that the booster voltage acts in the same direction as the line voltage and forces charging current into the main battery.

The combination of the resistances R_1 and R_2, the auxiliary battery and the exciter field winding f' is entirely similar to the circuits of a Wheatstone bridge. The two equal halves of the battery correspond to the ratio arms of the bridge, and R_1 and R_2 to the variable and unknown resistances; the field winding f' is the equivalent of the galvanometer. The polarity of f' is affected by the same causes that make the galvanometer in the bridge circuit deflect one way or the other as the resistances of the bridge arms are varied.

In installations of small capacity the exciter and the auxiliary battery can be dispensed with; the use of the auxiliary battery is not absolutely necessary in any case, for the main battery, or a part of it, may be used directly. The purpose of the auxiliary battery is to avoid imposing unequal loads upon individual cells

of the main battery. If the exciter is omitted, the connections shown leading to f' are transferred to f, but this can be done only when the capacity of the booster and the magnitude of its field current are small; the size of the carbon piles is limited by the fact that the practically constant current through S can produce only a narrow range of pressure variation on the carbon plates, hence the unbalancing of the bridge circuit can produce only moderate current through circuit f' (or f).

199. The Bijur System (General Storage Battery Co.) of external control, illustrated in Fig. 300, also utilizes the principle of the Wheatstone bridge, or potentiometer circuit. The equal ratio arms R_1 and R_2, connected in series across the line, are each provided with a series of taps connected to a set of contact points of graduated lengths, as at P_1 and P_2, which dip in or out of the small troughs of mercury, Hg, as the lever L is tipped one way or the other by the control magnet S or by the restraining spring. The battery is designed to have a normal open-circuit voltage equal to that of the line, consequently it will neither charge nor discharge when the system is carrying its average load if the control apparatus is adjusted so that under these conditions the lever is horizontal; for the booster field f is then connected to points which have no difference of potential between them, with the result that the booster remains unexcited. An increase of load causes a slightly increased flow of current through S and the resulting counter-clockwise movement of the lever short-circuits more and more of the resistance R_1 as the movement proceeds; this disturbs the balanced condition of the circuit through f and a current will flow through it in such a direction that the generated e.m.f. of the booster causes the battery to discharge. A decrease of load will make the pull of the spring overpower that of the coil S, the lever turns in the clockwise

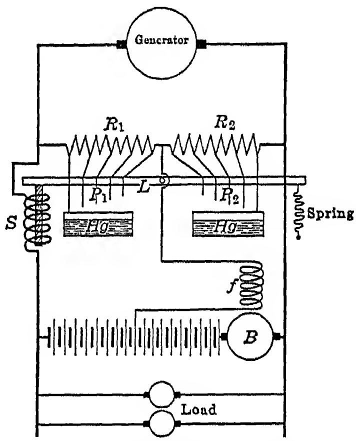


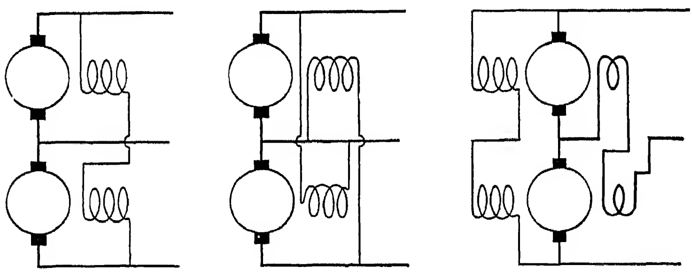
FIG. 300.—Automatic booster regulation, Bijur system.

direction, short-circuiting more or less of R_2, and the current through f reverses, causing a reversal of the booster e.m.f. and a flow of charging current into the battery.

The Bijur system differs from other systems of external control in that the latter involve a variation of generator current proportional to the battery charge or discharge, whereas the former causes a response from the battery to the desired extent with a fixed variation of generator current. This follows from the fact that the magnet S and the restraining spring are so proportioned that with a given current through S the pulls due to them balance each other at all points within the range of motion of the lever. The result of this condition of neutral equilibrium is that a change of current that unbalances the forces by an amount just sufficient to overcome the friction of the moving parts will produce a continuous movement of the lever. The excitation of the booster will then go on increasing in the proper direction to relieve the generator of all but the initial variation.

200. Balancers.—Fig. 301 (*a*, *b* and *c*) represents three possible methods of connecting a balancer set for the purpose of maintaining equality, or approximate equality, between the voltages on the two sides of a three-wire system (see Art. 123, Chap. VI). If, with the connections shown in Fig. 301*a*, the load becomes unbalanced, the voltage on the more heavily loaded side will fall while that on the more lightly loaded side will rise. Under these conditions the unit on the heavily loaded side will act as a generator, thereby checking the extent of the voltage drop, while the other unit will act as a motor and so limit the rise of voltage on that side; but the drop in speed of the balancer, due to the load on the motor element, will prevent the generator element from assuming a sufficient part of the unbalanced load to maintain the potential of the neutral as nearly constant as would be the case were the speed to remain constant. A partial compensation of this shift of the neutral may be effected by the system of field connections shown in Fig. 301*b*; in this case the drop in voltage on the heavily loaded side will weaken the field of the motor, thus tending to increase its speed, while at the same time the rise in voltage on the lightly loaded side will strengthen the field of the generator element, thereby tending to still further balance the voltage on the two sides of the system; but the

balance cannot be perfect for the reason that the automatic response of the balancer depends for its inception upon an actual unbalancing of the voltage. Perfect regulation is however possible if the units comprising the balancer are compound-wound as in Fig. 301*c*, where the series windings are connected in such a manner that the current in the neutral excites the generator cumulatively, while in the motor it acts differentially. The voltage on the heavily loaded side is therefore kept up by the combined effect of increased excitation and increased speed; but whereas in the system of Fig. 301*b*, this automatic action was dependent upon an unbalanced *voltage*, in the system of Fig. 301c it depends upon the unbalanced *current*, and it is therefore possible to adjust the compounding to maintain perfect equality of voltage on both sides of the neutral.



a *b* *c*

FIG. 301.—Connections of balancer set, three-wire system.

201. Train Lighting.[1]—The condition to be satisfied by any system of train lighting is that the lamp voltage shall be maintained at a constant value independently of the number of lamps in use and independently of the speed and direction of motion of the train. In the case of steam railroads three methods of electric lighting are in use:

1. The *straight storage* system, in which each car is equipped with its own storage battery.

2. The *head-end* system, in which a single constant-voltage generator placed in the baggage car or on the locomotive supplies current to the entire train.

3. The *axle-lighting* system, in which a small generator, mounted under each car, is driven directly from the axle.

[1] See Trans. A.I.E.E., Vol. XXI, 1903, pp. 129–227.

The straight storage system was used in the earliest installations of electric lighting on steam railroads. It has the disadvantage that the gradual exhaustion of the battery results in inferior illumination toward the end of long runs. The batteries must be charged at terminal or division points, or else be replaced by fully charged batteries.

In the head-end system a single compound-wound generator is driven by a turbine taking steam from the locomotive; in some cases the turbo-generator unit is installed in the baggage car, in others it is mounted on the locomotive. The complete equipment must include storage batteries, generally one for each car, in order that the lights may be operated when the train is parted, as during switching, and at low train speed. The standard (lead) battery equipment for Pullman sleepers consists of 16 cells, corresponding to a nominal lamp voltage of 30 volts. The variation of battery voltage between the extremes of full discharge and full charge requires the use of an automatic regulator in order to maintain constant voltage at the lamps.

In the axle-lighting system the maintenance of constant voltage is complicated by the fact that the speed of a generator positively driven from the car axle will not only vary through wide limits, but the machine must be capable of operating in either direction. Generators of the ordinary types do not possess inherent operating characteristics suitable for such service, and to make a machine of ordinary type conform to the requirements, more or less elaborate regulating devices must be used. Naturally, axle-driven generators must be used in connection with storage batteries in order that the lights may not go out when the train is stationary or when the speed is so low that the generator voltage is less than the normal lamp voltage.

The design of generators for automobile lighting is similar to that of axle-driven machines for train lighting except that there is no need to provide for reversal of the direction of rotation. This follows from the fact that in the former case the generator is driven from the engine, which always runs in the same direction.

202. Voltage Regulation in Train Lighting Systems.—To prevent objectionable variation of the candle-power of the lamps, automatic regulation must be provided to compensate

for the variation of battery voltage between the extremes of full charge and full discharge, and, in the case of axle-driven generators, to overcome voltage variations due to change of speed. The various methods of regulation may be classified as either *mechanical* or *electrical* (or electromagnetic). In some systems the maintenance of constant voltage also involves regulation for constant current output from the generator, hence the latter, when in use, delivers constant power; such regulation is not entirely satisfactory, for it takes no account of the fact that the charging current of a lead battery should "taper," that is, become gradually less, as the battery approaches the fully charged condition. It is possible to arrange the regulatory devices in such a manner that the voltage and current output of the generator are controlled by the battery voltage, or else to make the generator control the line voltage and therefore also that of the battery.

Under the heading of *mechanical* methods of regulation may be included those axle-lighting systems in which the generator voltage is controlled by the slipping of the driving belt, as in the Stone generator, or by a slipping clutch. In these systems the speed of the generator is maintained constant when the load increases above a definite predetermined load which causes slipping to occur. In the Stone system the generator is provided with an automatic device, consisting of a rocker arm on the shaft, for reversing the polarity of the generator terminals when the direction of rotation is reversed; there is also an automatic, centrifugally operated switch arranged to establish the connection between the generator and the battery when the speed and generator voltage have reached predetermined pick-up values, and to break the connection when the speed is below the assigned limit.

Under *electrical* or *electromagnetic* methods of regulation may be grouped all systems in which voltage control is obtained (*a*) by the automatic variation of resistance in the lamp circuit or in the exciting circuit of the generator; or (*b*) by the utilization of the armature reaction of the generator to secure the desired characteristics. Examples of these methods are given in the following articles.

203. Resistance Regulation.—Fig. 302 illustrates diagram-

matically a type of automatic regulator which operates by varying the resistance of a pile of carbon disks connected in the main lamp circuit. If the battery voltage V_B rises above normal, as during charging, the lamp voltage V_L tends to increase also. This causes an increased flow of current through the solenoid S_1, and the movement of its plunger increases the pressure on the carbon pile r, thereby reducing its resistance and permitting an increased flow of current through solenoid S_2. The motion of the plunger of S_2 then releases the pressure on the carbon pile R, increasing its resistance to a sufficient extent to absorb the greater part of the increase of V_B as an ohmic drop in R. Since the response of the solenoids S_1 and S_2 is dependent upon a variation of V_L, the lamp voltage cannot be held absolutely constant, but the variation will be small; the use of the solenoid S_1 and pile r increases the sensitiveness of the response of S_2 to a change in V_L. The lamp regulator is used in conjunction with a generator regulator described in Art. 204.

204. Generator Field Regulation.—Fig. 303 illustrates a method of regulating the generator voltage by the variation of a

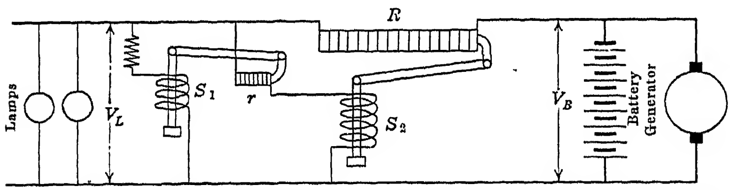


FIG. 302.—Voltage regulation by means of resistance in main line (Gould system).

resistance R in its field circuit. The carbon pile R is acted upon by the two solenoids V and B, the former responding to changes of the generator voltage due to change of speed, and the latter to variations of the battery current. For example, assuming that the contact K is closed and that the generator is charging the battery, any increase of speed will tend to increase both the generator voltage and the charging current. As the charging current increases, the upward pull of solenoid B relieves the pressure normally exerted upon R by the weight of the plunger of B, thus increasing the field resistance of the generator and lowering its voltage. To prevent excessive overcharge of the battery due to high generator speed, the plunger of solenoid V is ar-

ranged so that the increased line voltage causes it to relieve the pressure on the right-hand side of *R*, thereby increasing the field resistance and lowering the generator voltage.

The automatic switch *K* is closed, and the connection between

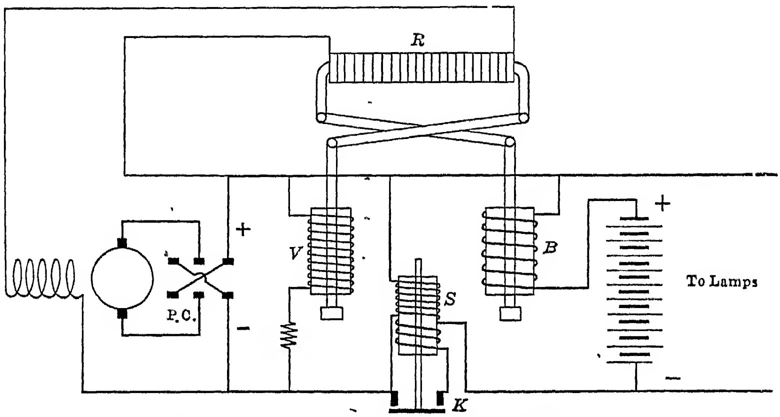


FIG. 303.—Voltage regulation by means of field rheostat (Gould system).

generator and battery is established when the speed of the generator is sufficiently high to generate a voltage capable of actuating the solenoid *S*; the generator current, flowing through the series winding of switch *K*, reinforces the pull of the shunt winding *S*. When the speed falls below this pick-up speed, the battery

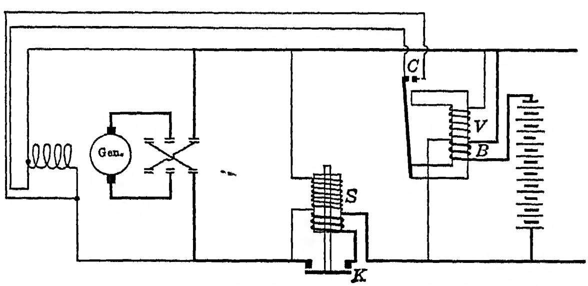


FIG. 304.—Voltage regulation using vibrating contact.

voltage overpowers that of the generator and a reverse current flows through the series winding of *K*, with the result that the net force acting upon the plunger is not sufficient to hold the contacts closed against the gravitational pull. The entire load is

then carried by the battery alone. The pole-changer is represented diagrammatically at *PC*.

Another method, somewhat similar to that of Fig. 303, but involving a vibrating contact analogous to that of the Tirrill regulator, is illustrated in Fig. 304. The automatic switch *K* operates in the same manner as in Fig. 303, but the solenoids *B* and *V* act on the same magnetic circuit and open and close the contact *C*. Thus, if the battery charging current exceeds the safe limit, coil *B* closes contact *C* and momentarily short-circuits the field winding of the generator, thus reducing the generator voltage. Coil *V* operates similarly if the generator voltage rises too high because of high rotative speed.

205. Field and Line Regulation.—Fig. 305 is a diagram of connections of a system of train lighting which, like the system

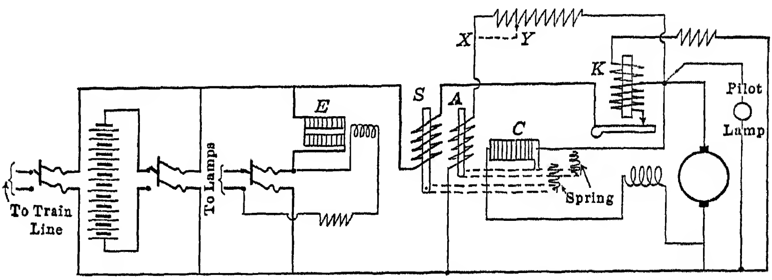


Fig. 305.—Combined field and line regulation. (Safety Car Heating & Lighting Co.)

described separately in Articles 203 and 204, includes independent regulation of generator voltage and of lamp voltage.

The resistance of the carbon pile *C*, in series with the shunt field winding of the generator, is controlled by the pressure exerted upon it by levers operated by the plungers of coils *S* and *A*. Coil *S* carries the entire generator current and is adjusted to hold the current at its full rated value. Coil *A*, shunted across the line, is set to hold the generator voltage at 39 volts on equipments having 16-cell lead batteries (2.45 volts per cell); and at 78 volts on "60-volt" equipments having 32 cells of lead batteries. If Edison batteries are used, these voltages are set at 43 volts and 86 volts, respectively, by opening the short-circuit *XY* on part of the resistance in series with the voltage coil *A*. The object of

limiting the generator voltage to 2.45 volts per cell of lead battery is to prevent excessive overcharging of the battery; when the battery is fully charged, the charging current will then automatically taper down to a safe value. The limitation of generator current imposed by coil S prevents overloading of the generator due to lamp load or to charging an exhausted battery.

The lamp voltage is controlled by the pair of carbon piles, E, in series with the lamps, the two piles being connected in parallel with each other. The pressure upon these piles is due to a system of levers and a toggle joint actuated by a coil connected across the lamp mains. The pull of this electromagnet is opposed by a spring, the design being such that the armature of the electromagnet will remain in any position within the limits of its travel when the lamp voltage is normal.

In this system the armatures of the magnets controlling the generator and the lamp circuit are provided with air dash-pots having graphite plungers. The effect of variation of temperature upon the voltage coils of the generator and lamp regulators is compensated by means of resistors, having zero temperature coefficients, placed in series with these coils.

The automatic switch K for establishing the connection between generator and battery at train speeds above the pick-up speed is similar to others already described. The shunt coil lifts the pivoted armature when the generator voltage equals the battery voltage, thus bringing into action the series coil, which assists the shunt coil in holding the switch tightly closed, and which accelerates the opening of the switch when the generator voltage falls below battery voltage.

The four brush arms of the generator are mounted on a rocker ring carried on ball bearings, the ring being free to rotate through 90 degrees between a pair of stops. When the machine is running in one direction, the friction of the brushes against the commutator holds the rocker ring against one of the stops and the brushes are then in the proper position for sparkless commutation. Reversal of the direction of rotation causes the rocker ring to be turned through 90 degrees against the other stop, thus preserving the original polarity of the generator.

Fig. 306 shows in diagrammatic form another system which includes a lamp regulator and a generator field regulator, F. The

automatic switch K is closed in response to the pull of the voltage coil when the generator speed and voltage have attained their proper values, thereby connecting the battery to the generator. The charging current, flowing through the series coil of the regulator F, tends to be maintained at constant value by the action of the carbon pile rheostat in circuit with the generator field. At the same time the ampere-hour meter, AHM, is running in the direction of charge, and when the battery is charged and the contact needle N has reached its point of contact, the resistance R is short-circuited; thereupon the switch S is energized, contact C is closed, and current flows through the shunt coil of the regulator F. The pull of the shunt coil

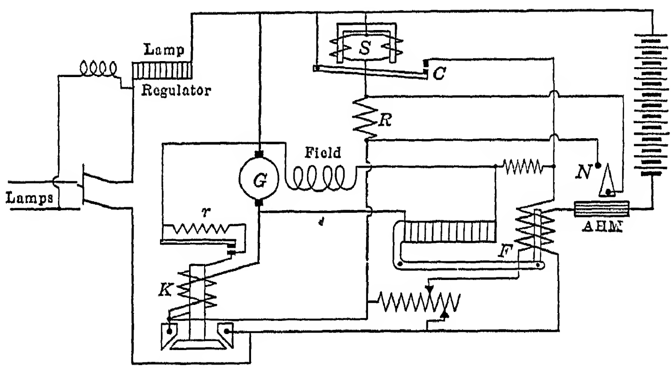


FIG. 306.—U. S. Lighting and Heating Company system of train lighting.

adds to that of the series coil, so that there results a sudden reduction of generator voltage and the battery is thereby "floated" on the line; that is, the generator supplies current directly to the load and the battery neither charges nor discharges.

206. Regulation by Means of Armature Reaction.—Regulation of generator voltage by making use of armature reaction under load conditions is exemplified in the Rosenberg train lighting generator (Art. 207) and in the third-brush type of generator used for automobile lighting (Art. 210). This type of regulation, since it is dependent upon the inherent characteristics of the generator, may be classed as electromagnetic.

207. The Rosenberg Train Lighting Generator.—The Rosenberg generator, first described[1] in 1905, embodies a number of interesting structural features and has operating characteristics that make it suitable for train-lighting service. Its distinctive properties are: (1) that it develops an e.m.f. the direction of which is independent of the direction of rotation, and (2) that it produces a current which, beyond a certain speed, remains practically constant no matter how much the speed is increased. The diagram of connections of a bipolar machine is shown in Fig. 307, but it will be understood that with suitable modifications the principle is applicable to multipolar machines. The

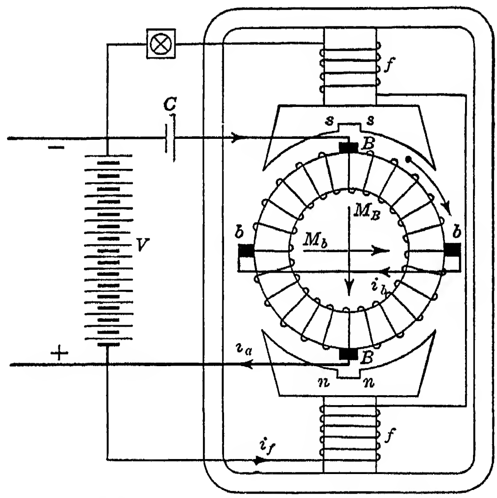


Fig. 307.—Diagram of Rosenberg train-lighting generator.

battery V, which must be used in connection with the generator if the latter is to function properly, supplies current to the lamps when the train is at rest and also to the shunt field winding ff, producing the polarity indicated by nn, ss. The axis of commutation of the brushes BB is in line with the axis of the poles, instead of being at right angles thereto as in the ordinary type of machine.

The brushes BB are connected to the battery terminals through

[1] Elektrotechnische Zeitschrift, 1905, p. 393.

an aluminum cell C which offers a very high resistance to the flow of current from the battery to the armature and only a very small resistance in the direction from the armature to the battery; this property of the aluminum cell prevents the discharge of the battery through the armature when the train is at rest or when running at a speed below that at which the generator picks up its load. In addition to the main brushes there is a pair of short-circuited auxiliary brushes, bb, placed at right angles to the polar axis, that is, in the same position as the main brushes of an ordinary generator.

Rotation of the armature through the magnetic field set up by ff will produce a flow of current through the short-circuited armature along axis bb, thereby creating a powerful cross-field, M_b, the lines of force of this field finding a path of low reluctance through the pole shoes. As is clear from the figure, clockwise rotation will result in a cross-field directed from left to right; the motion of the armature conductors through this cross-field then generates an e.m.f. and current along the BB axis in such a direction that the armature m.m.f., represented by the arrow M_B, opposes the excitation due to the field winding ff. In case the direction of rotation is reversed (that is, becomes counter-clockwise) the direction of the cross-field M_b also reverses, and the effect of this double reversal is to preserve the original polarity of the brushes BB. The fact that the armature m.m.f., M_B, opposes the excitation due to the field winding means that the field flux parallel to the BB axis is small, and this in turn prevents excessive current through the short-circuit bb. The machine differs widely from the ordinary generator in that what is usually the main field is of secondary importance with respect to the cross-field. The weak field in the BB axis obviates commutation difficulties that would otherwise arise due to the short-circuiting of winding elements under the middle of a pole face; such difficulty as might still exist is further overcome by notching the pole faces opposite the main brushes. It is clear that there is a definite limit beyond which the main current delivered by the brushes BB cannot increase, this limit being reached when the armature m.m.f., M_B, neutralizes the field excitation due to ff; for in that case there would be no e.m.f. and current in the bb axis, hence no e.m.f. in the main brush axis. It follows, there-

fore, that beyond a certain speed the machine will deliver a practically constant current. Any desired limit to the current may be set by adjusting the rheostat in the field circuit *ff*. The generator may be driven either by a belt from the car axle or by mounting the armature directly on the axle itself.

On the basis of the foregoing qualitative study of the physical phenomena occurring in the machine, Messrs. Kuhlman and Hahnemann[1] have developed the quantitative relations between the speed and current output of the machine operating as a generator. Thus let

n = speed of the armature in r.p.m.
i_f = constant exciting current in field winding *ff*
i_a = main current output
i_b = short-circuit current in axis *bb*
$n_f i_f$ = ampere-turns due to *ff*
$n_a i_a$ = effective ampere-turns of armature in axis *BB*
$n_b i_b$ = effective ampere-turns of armature in axis *bb*
V = terminal voltage of line, assumed constant
E_b = e.m.f. generated in short-circuit *bb*
Φ_B = field flux in axis *BB*
Φ_b = field flux in axis *bb*
r_a = armature resistance (including brushes)

If saturation of the magnetic circuit is neglected, so that the flux may be considered to be proportional to the m.m.f. that produces it, the following relations will hold

$$E_b = c_1 \Phi_B n \tag{1}$$

$$i_b = \frac{E_b}{r_a} = \frac{c_1}{r_a} \Phi_B n \tag{2}$$

$$\Phi_B = c_2(n_f i_f - n_a i_a) \tag{3}$$

$$\Phi_b = c_3 n_b i_b \tag{4}$$

$$V = c_1 \Phi_b n - i_a r_a \tag{5}$$

Substituting (2) and (3) in (4), there results

$$\Phi_b = c_1 c_2 c_3 \frac{n_b}{r_a} (n_f i_f - n_a i_a) n \tag{6}$$

[1] Elektrotechnische Zeitschrift, Vol. XXVI, 1905, p. 525.

and substituting this value of Φ_b in (5)

$$V = c_1^2 c_2 c_3 \frac{n_b}{r_a} (n_f i_f - n_a i_a) n^2 - i_a r_a \tag{7}$$

and

$$i_a = \frac{c_4 n_f i_f n^2 - V}{r_a + c_4 n_a n^2} = \frac{\dfrac{n_f i_f}{n_a} - \dfrac{V}{c_4 n_a} \dfrac{1}{n^2}}{1 + \dfrac{r_a}{c_4 n_a} \dfrac{1}{n^2}} \tag{8}$$

where

$$c_4 = c_1^2 c_2 c_3 \frac{n_b}{r_a}$$

From equation (8) the following conclusions may be drawn:

(*a*) If $n = 0$, $i_a = -\dfrac{V}{r_a}$,

which means, simply, that were it not for the aluminum cell C the armature, at standstill, would be a dead short-circuit on the line (or battery), the negative sign of i_a indicating a flow of current into the armature from the line.

(*b*) If $n = \infty$, $i_a = \dfrac{n_f i_f}{n_a}$ or $n_a i_a = n_f i_f$

which means that at infinite speed the armature m.m.f. (M_B) would exactly neutralize the field excitation. This condition therefore determines the limiting current output of the machine running as a generator, or

$$(i_a)_{max} = \frac{n_f i_f}{n_a}$$

This result also shows how the rheostat in the field circuit controls $(i_a)_{max}$ by fixing the value of i_f.

(*c*) If the machine is to act as a generator, i_a must be positive, hence

$$\frac{n_f i_f}{n_a} = (i_a)_{max} \geq \frac{V}{c_4 n_a} \frac{1}{n^2}$$

or

$$\frac{(i_a)_{max} r_a}{V} \geq \frac{r_a}{c_4 n_a} \frac{1}{n^2}$$

But the term $\dfrac{(i_a)_{max} r_a}{V}$ is the ratio of the maximum ohmic drop in

the armature to the line voltage, and since this ratio must be small from considerations of efficiency, it follows that $\frac{r_a}{c_4 n_a} \cdot \frac{1}{n^2}$ is still smaller in comparison with unity and that with increasing speed it rapidly approaches zero. Therefore the denominator of (8) may be considered equal to unity and the expression for i_a becomes, with only slight error,

$$i_a = (i_a)_{max} - \frac{V}{c_4 n_a} \cdot \frac{1}{n^2} \tag{9}$$

Equations (8) and (9) show that the current is zero when

$$n = n_0 = \sqrt{\frac{V}{c_4 n_a (i_a)_{max}}} = \sqrt{\frac{V}{c_4 n_f i_f}} \tag{10}$$

and that it rapidly approaches $(i_a)_{max}$ as a limit as the speed increases.

For example, suppose that the generator is to supply a maximum current of 50 amperes at a terminal voltage of 50 volts and that it is to pick up its load at a speed of 300 r.p.m. From (10),

$$300 = \sqrt{\frac{50}{c_4 n_a \times 50}}$$

$$c_4 n_a = \frac{1}{90{,}000}$$

and from (9)

$$i_a = 50 - \frac{4.5 \times 10^6}{n^2}$$

This is the equation of the curve shown in Fig. 308. The manner of variation of i_b, the current in the short-circuited path bb, is determined by combining equations (2), (3) and (9), resulting in the expression

$$i_b = \frac{V}{c_1 c_3 n_b} \cdot \frac{1}{n} \tag{11}$$

which represents an equilateral hyperbola. It is seen that i_b is a function of c_3 and this is dependent upon the reluctance in the path of the cross-field Φ_b. The curve showing i_b in Fig. 308 is based on the assumption that $i_b = 60$ amperes when $n = 300$, or $i_b = \frac{18{,}000}{n}$. The dashed portions of the curves of Fig. 308

computed from equations (9) and (11), correspond to negative values of i_a (indicating motor action), and while not entirely accurate because of the neglect of terms involving r_a in the pres-

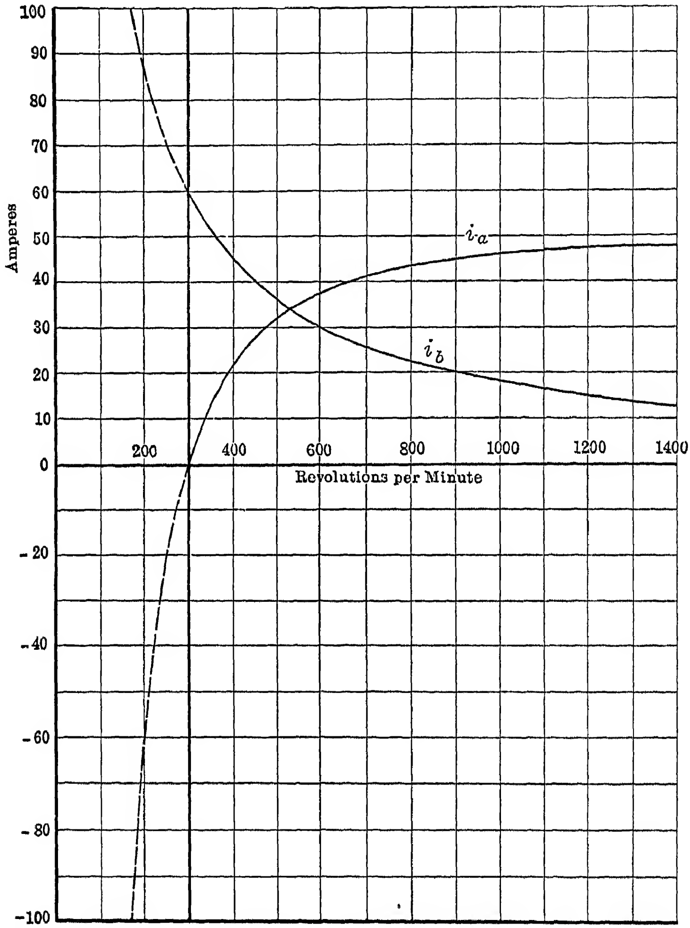


FIG. 308.—Relation between current and speed, Rosenberg generator.

ence of small values of n, they depart only slightly from the correct curves within the range shown in the diagram.

An examination of Fig. 307 will show that in two of the quadrants of the armature winding the currents i_a and i_b flow in the same direction in the conductors, and in the other two quadrants

they flow in opposite directions. The total current in the former case is

$$i_a + i_b = (i_a)_{max} - \frac{V}{c_4 n_a}\cdot\frac{1}{n^2} + \frac{V}{c_1 c_3 n_b}\cdot\frac{1}{n}$$

and reaches a maximum value when

$$\frac{d(i_a + i_b)}{dn} = \frac{2V}{c_4 n_a}\cdot\frac{1}{n^3} - \frac{V}{c_1 c_3 n_b}\cdot\frac{1}{n^2} = 0$$

or when $n = \frac{2c_1 c_3 n_b}{c_4 n_a}$. Substituting the values used above, the speed corresponding to this maximum current in the conductors is 500 r.p.m., and the currents themselves are $i_a = 32$ and $i_b = 36$.

208. Operation of Rosenberg Machine as a Motor.—The Rosenberg machine when supplied with current from an external

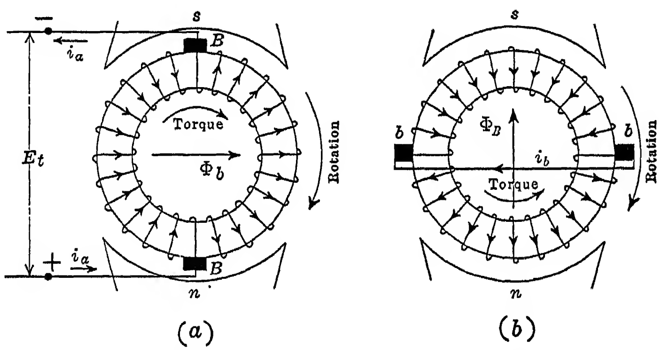


FIG. 309.—Relation between currents and fluxes in Rosenberg machine operating as motor.

source will operate as a motor, but it has no torque at standstill. The reason for this absence of starting torque is clear from Fig. 307, for the armature current and the field flux due to *ff* have their axes in the same direction and cannot, therefore, react upon each other; and there can be no cross-field to react upon the armature current until rotation through Φ_B produces current and flux in the *bb* axis. But if the armature is given a start in either direction it will continue to run in that direction. Thus, let Fig. 309 represent the same machine shown in Fig. 307 but taking current from, instead of supplying it to, the line, and let the starting impulse be in the clockwise direction. The figure is drawn in

two parts in order to show with greater clearness the effect of the two pairs of brushes; the arrows on the armature conductors of part (a) show the direction of flow of i_a, and those in (b) serve similarly for i_b. The direction of i_b is determined by applying Fleming's right-hand rule for generator action; had the initial rotation been counter-clockwise, the direction of i_b, and therefore also of Φ_b, would have been opposite to that shown. In either case the reaction between Φ_b and i_a produces a torque in the same direction as the initial rotation and, therefore, serves to accelerate the armature; and the torque due to the reaction between Φ_B and i_b always opposes the rotation. The resultant torque is the difference between these two opposing torques.

Analytically, the characteristics of this motor when supplied from constant potential mains are involved in the equations derived in the preceding article for the case of the generator. All that is necessary is to interpret negative values of i_a in those equations as current input to the motor, but some care should be used in applying equations (9) and (11), especially at low speeds, because the term involving r_a in equation (8) is not then negligible as has been assumed. Thus, if $r_a = 0.10$ ohm, corresponding to a maximum armature drop of 10 per cent. when the constants are those used in the foregoing discussion, the standstill current computed from (9) is $i_a = -\infty$, whereas the true value from (8) is $i_a = -500$. The range of speed through which motor action occurs is from $n = 0$ to $n = \sqrt{\dfrac{V}{c_4 n_f i_f}}$ (0 to 300 r.p.m., Fig. 308). Without going into further particulars it will be clear that the speed characteristic is similar to that of a cumulative compound motor.

209. A modification of the Rosenberg type of generator, together with a special method of voltage control, developed by the Electric Storage Battery Co., is illustrated in Fig. 310. Instead of connecting the shunt field winding across the machine terminals, as in Fig. 307, it is connected between opposite points of a Wheatstone bridge circuit (marked "control bridge" in Fig. 310). There is also a compensating winding, marked "series field," for the purpose of neutralizing the armature reaction due to the main generator current. Two of the bridge arms, marked R, consist of ordinary resistors, while the other two,

marked *IR*, have negative temperature coefficients. The junction points of the bridge not connected to the control field are connected directly across the line. A variation of generator voltage will then alter the difference of potential between the control field terminals, and the field current will change to a sufficient extent to readjust the generator voltage. If it is desired to give the battery an overcharge, the overcharge switch, which normally short-circuits the resistance R', is opened, the lamp circuit having been previously disconnected. This has the effect of reducing the voltage impressed on the bridge, in the same manner as though the generator voltage had itself decreased,

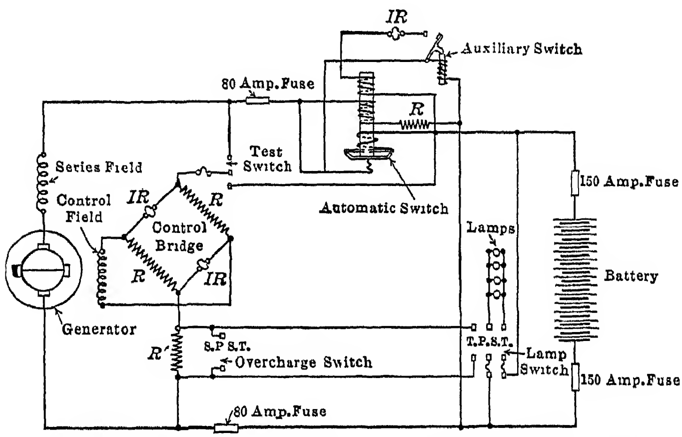


FIG. 310.—Rosenberg generator and control circuits, Electric Storage Battery Co.

hence the readjustment of bridge currents increases the main field excitation and raises the terminal voltage. The extent of the rise of voltage and, therefore, the magnitude of the charging current, is determined by the value of the resistance R'. It will be observed that the closing of the lamp circuit through the triple-pole switch short-circuits R', thereby reducing the generator voltage to the normal lamp voltage and preventing damage to the lamps because of high voltage during charging. It follows, therefore, that overcharging of the batteries must be accomplished during daylight runs.

The automatic switch, in addition to the usual shunt and series coils, has a third coil connected between the generator and the

battery. The pull due to the main shunt coil is insufficient to close the switch, or to keep it closed without the pull due either to the auxiliary coil or the series coil. The auxiliary coil, therefore, determines the closing of the switch by the difference between the voltages of generator and battery; and the switch will open when the current in the series coil drops to zero.

210. The Third-brush Generator.—In an English patent (No. 9364) issued to W. B. Sayers in 1896, there is disclosed a system of connections, illustrated in Fig. 311, designed to produce automatic compounding action in a constant-speed generator of

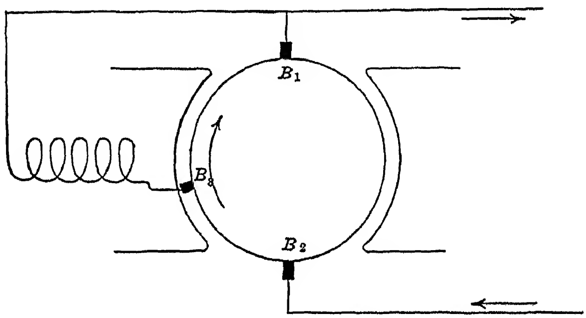


FIG. 311.—Diagram of connections of Sayers generator.

the shunt type by utilizing the reaction of the armature current upon the main magnetic field. The shunt winding, instead of being connected to the main brushes in the usual manner, is connected between an auxiliary brush, B_3, located about half way between the main brushes B_1 and B_2, and that one of the latter which will cause the shunt winding to subtend the trailing half of the pole faces. Under load conditions the armature reaction then increases the flux under the trailing half of the pole faces, thereby increasing the e.m.f. generated in that part of the armature winding included between the terminals of the shunt field winding, hence giving rise to increased excitation. The principles inherent in the Sayers generator are embodied in the third-brush type of generator now widely used as part of the electrical equipment of automobiles, but with the difference (1) that the terminals of the shunt winding subtend the *leading* half of the pole faces, and (2) that the machine must be

adapted to variable speed operation since it is driven through suitable gearing directly from the engine of the automobile. The third-brush generator presents a number of features of considerable technical interest; accordingly, there is given below a discussion of the physical reactions involved in its operation, and an approximate analytical derivation of its characteristics.

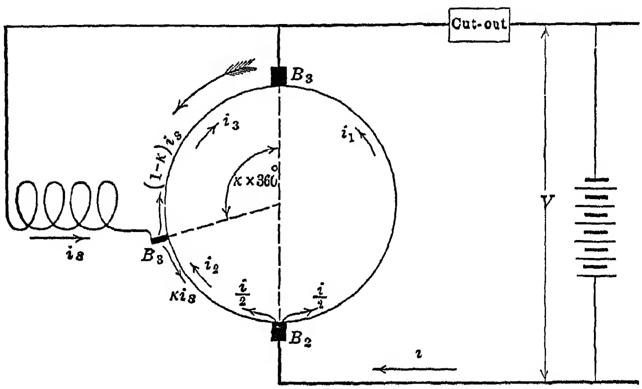


FIG. 312.—Diagram of connections of third-brush generator.

Referring to Fig. 312, it will be clear that the connections automatically set a limit to the magnitude of the current it will deliver, even though the speed be increased indefinitely, provided the battery voltage remains substantially constant. For if the machine is delivering current at some particular speed, an increase of speed will tend to increase both the generated e.m.f. and the current; but the increased armature current shifts the flux away from the leading pole tip because of the greater cross-magnetizing action, thereby reducing the e.m.f. generated in the armature between brushes B_1 and B_3 and consequently weakening the shunt excitation, or at any rate preventing an increase to the extent that would otherwise result from the increase of speed. This action continues with each increment of speed until the main field has become so weakened that a further increase of current causes a disproportionately large reduction of excitation, thereby not only preventing any further increase of current but actually causing it to fall off to smaller values as the speed is still further raised. Accordingly, it is to be expected

that the machine will develop a current that at first increases with rising speed and, after reaching a maximum value, falling off again.

The demagnetizing action due to the reaction of the main armature current is not the only effect of the kind that must be taken into account; it must also be considered that the auxiliary brush short-circuits an element of the armature winding that lies well under the pole face and in which there is generated an appreciable e.m.f. (see Fig. 313); this e.m.f. produces a considerable current in the short-circuited element, and the direction of this current in such that it sets up an additional demagnetizing action in the air-gap under the leading half of the pole faces, and so affecting the voltage in that part of the armature winding which supplies the excitation of the shunt winding.

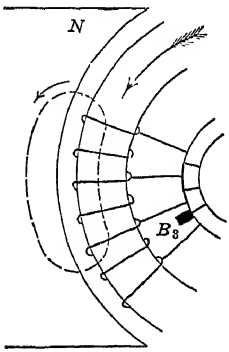


FIG 313 —Demagnetizing effect due to short-circuited coil.

It is clear that some type of automatic switch or cut-out must be connected between the generator and the battery to prevent the battery from discharging through the generator under standstill or low speed conditions. The cut-out usually employed is similar to those already described in connection with train lighting systems, and it does not operate to close the circuit until the speed has reached a value sufficient to develop in the generator an e.m.f. equal to that of the battery. It follows, therefore, that the speed-current characteristic may be expected to have the general form of Fig. 319, and that the effect of shifting the auxiliary brush will be to alter the points of pick-up and maximum current, as well as the magnitude of the latter.

ANALYTICAL THEORY

The following symbols recur frequently throughout the analysis and are tabulated below for convenient reference:

V = constant line voltage

E_a = total e.m.f. generated in the armature

i = line current

i_s = shunt field current
n = speed in r.p.m.
r_s = resistance of shunt field winding
r_a = armature resistance measured between main brushes and including brush contact resistance
r = resistance of short-circuit under brush B_3
n_s = shunt field turns per pair of poles
Z = number of armature conductors
Φ = flux per pole
d = diameter of armature
l = length of core
τ = pole-pitch
δ = air-gap
ψ = ratio of pole arc to pole-pitch
p = number of poles, taken as 2
a = number of parallel paths through armature, taken as 2
B_g = flux density in air-gap
k = factor determining position of brush B_3.

Before taking up the details of the analysis, it is desirable to have a general idea of the method of attack and of the approximations that must be introduced to keep the mathematical difficulties within reasonable bounds. In the first place, then, it will be assumed that saturation of the magnetic circuit may be neglected, in other words that flux may be taken as proportional to the m.m.f. that produces it. It follows then, that the flux per pole produced by a current i_s in the field winding, when the armature is without current, will be proportional to i_s, and that when the machine is under load the flux per pole will be proportional to the m.m.f. contributed by i_s diminished by the summation of all demagnetizing m.m.f.s due to the components of the armature current. The resultant e.m.f. generated between the main brushes may then be computed in the usual manner. Further, the e.m.f. generated between brushes B_1 and B_3, which is in turn responsible for the magnitude of the exciting current i_s, will be due to that part of the net flux per pole which lies between these brushes and which is cut by those conductors of the armature winding which occupy this particular belt. The final result to be attained is the determination of the relation between the speed of the generator and the corresponding magni-

tude of the line current flowing to the battery and to the external circuit in parallel therewith, this relation to involve only the constants of the machine.

It will be observed from the diagram of connections, Fig. 312, that the line current i shown entering brush B_2 can be thought of as dividing equally between the two identical paths leading to brush B_1, and that the shunt field current i_s entering brush B_3 may be considered to divide into two parts, ki_s and $(1 - k)i_s$ respectively, these two currents being inversely proportional to the resistances of the paths through which they flow. It follows, then, that currents i_1, i_2, and i_3 indicated in Fig. 312 are, respectively,

$$i_1 = \tfrac{1}{2}i + ki_s \tag{12}$$

$$i_2 = \tfrac{1}{2}i - ki_s \tag{13}$$

$$i_3 = \tfrac{1}{2}i + (1 - k)i_s \tag{14}$$

The main line current i flowing through the armature winding produces a cross-magnetizing m.m.f. distributed linearly over the armature periphery in the manner shown by the sloping line afa' of Fig. 314. Under the pole faces this m.m.f. may be assumed to produce a transverse field whose intensity at any point is taken to be proportional to the m.m.f. at that point; but between the poles, because of the high reluctance of the magnetic circuit in that region, the flux will be much less than proportional to the m.m.f. and will accordingly have a distribution represented by the saddle-shaped curve (see Art. 96, Chap. V). Ignoring the effect of saturation, the net result of this cross-magnetizing action is to increase the flux over the trailing half of the pole face by the same amount that it is reduced under the leading half, so that this action will not directly affect the magnitude of the total e.m.f. generated between the main brushes; but in the region between brushes B_1 and B_3, which is the part subtended by the terminals of the shunt winding, there is a net reduction of the flux responsible for the exciting current by an amount proportional to the difference between the two cross-hatched areas of Fig. 314, the effect of the portion of the flux between the poles being neglected.

To evaluate this reduction of flux between B_1 and B_3, consider the closed magnetic circuit indicated by the dashed line P,

which is drawn so as to include all of the armature conductors under a pole face; the m.m.f. acting around this circuit is $\frac{4\pi}{10}\psi\frac{Z}{2}\frac{i}{2}$, half of which, or $\frac{4\pi}{10}\psi\frac{Zi}{8}$, will be consumed in the air-gap at each pole tip, assuming that the reluctance of the iron part of the

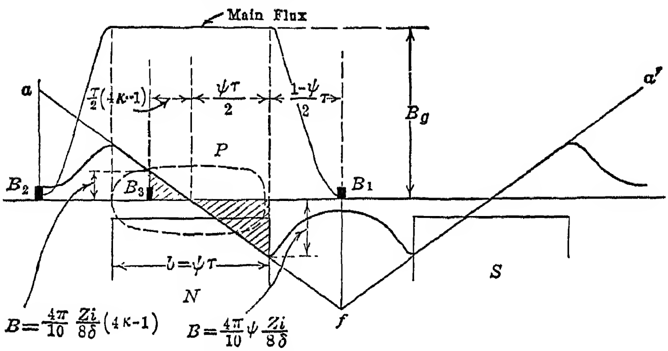


FIG. 314.—Distribution of m.m.f. due to current i.

path is negligible in comparison with that of the air-gap. If the length of the air-gap, corrected to take account of the effect of the slots, is δ cm., the field intensity at the pole tip will be given by $\frac{4\pi}{10}\psi\frac{Zi}{8\delta}$, hence the flux represented by the larger of the two shaded triangles will be given by the product of the mean flux density, $\frac{1}{2}\cdot\frac{4\pi}{10}\,\frac{\psi Zi}{8\delta}$ and the half area of the pole face $\frac{\psi\tau l}{2}$. The smaller triangle, being similar to the larger, its altitude is readily found to be $\frac{4\pi}{10}(4k-1)\frac{Zi}{8\delta}$ and its base $\frac{\tau}{2}(4k-1)$; the net reduction of flux in the zone B_1B_3 due to current i is then

$$\varphi'_d = \frac{4\pi}{10}\frac{Zi\tau l}{32\,\delta}[\psi^2 - (4k-1)^2] \tag{15}$$

The current $(1-k)i_s$ flowing through the kZ conductors between brushes B_1 and B_3 produces a peripheral distribution of m.m.f. of the form shown by the trapezoidal shaped diagram of Fig. 315, which fact will be made clear when it is considered that each conductor in this particular belt of the armature winding has a return conductor in the opposite quadrant, as indicated

in Fig. 316. The total m.m.f. acting around path Q, Fig. 316, is clearly $\frac{4\pi}{10}kZ(1 - k)i_s$, half of which, or $\frac{1}{2}\cdot\frac{4\pi}{10}kZ(1 - k)i_s$ is effective across the air-gap opposite brush B_3. But the remain-

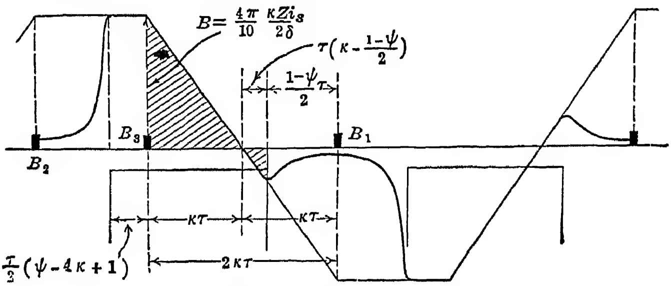


FIG 315.—Distribution of m m f due to current i_s.

ing part of the shunt exciting current, ki_s, flowing from brush B_3 to brush B_1 by way of the longer path through $(1 - k)Z$ conductors, produces an exactly similar though smaller m.m.f., as may be seen from Fig. 317; for it will be observed that the currents in the two layers of the first and third quadrants neutralize each other so far as magnetic effects are concerned, leaving for consideration only those in the second and fourth quadrants,

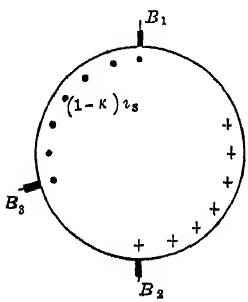


FIG. 316.

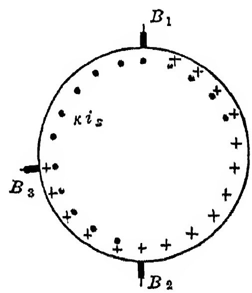


FIG. 317.

FIGS. 316 and 317.—Distribution of components of exciting current.

which are distributed exactly as in Fig. 316. Since the current in these conductors is ki_s, the m.m.f. around such a path as P, Fig. 316, is $\frac{4\pi}{10}kZ.ki_s$, which when added to the m.m.f. due to the belt of current of Fig. 316 gives a total m.m.f. around path P of $\frac{4\pi}{10}kZi_s$. In the region between brushes B_1 and B_3 the net

result of the effect of current i_s considered as a whole is a *magnetizing* one, and the amount of this magnetic flux is proportional to the difference between the areas of the two shaded triangles of Fig. 315. The altitude of the larger is $\frac{4\pi}{10}\frac{kZi_s}{2\delta}$ and its base is $k\tau$; since the base of the smaller is $\tau\left(k - \frac{1-\psi}{2}\right)$ its altitude, by similar triangles, is $\frac{4\pi}{10}\frac{Zi_s}{2\delta}\left(k - \frac{1-\psi}{2}\right)$ whence the magnitude of the magnetizing flux between B_1 and B_3 is

$$\varphi'_m = \frac{4\pi}{10}\frac{Zi_s\tau l}{16\delta}(1 - \psi)(\psi + 4k - 1) \qquad (16)$$

The net reduction of the flux between brushes B_1 and B_3 due to the combined effects of i and i_s is then given by the difference between (15) and (16), or it is

$$\varphi_d = \frac{4\pi}{10}\frac{Z\tau l}{32\delta}\left\{[\psi^2 - (4k - 1)^2]i - 2(1 - \psi)(\psi + 4k - 1)i_s\right\} \qquad (17)$$

But the flux between brushes B_1 and B_3 is actually reduced by an amount greater than that given by (17), on account of the demagnetizing effect of the short-circuit current under brush B_3. This short-circuit current, designated by i_{sc}, flows in the $\frac{Z}{2S}$ turns of the short-circuited coil, and produces an m.m.f. of $\frac{4\pi}{10}\frac{Z}{2S}\,i_{s.c.}$ distributed in the manner shown in Fig. 318. Within the zone B_1B_3 the value of the demagnetizing flux, represented by the doubly hatched area, is

$$\phi_{s.c.} = \frac{4\pi}{10}\frac{Z}{2S}\frac{\tau l}{4\delta}(\psi + 4k - 1)i_{s.c.} \qquad (18)$$

where $i_{s.c.}$ may be taken equal to the average e.m.f., $e_{s.c.}$, during the short-circuit period divided by the effective resistance r of the circuit including the coil and the brush contact; that is,

$$i_{s.c.} = \frac{e_{s.c.}}{r} \qquad (19)$$

But the value of $e_{s.c.}$ is determined by the general formula $e = Blv \times 10^{-8}$ where B is the flux density through which the conductors are moving. In the case under consideration the flux density opposite brush B_3 is the sum of B_g due to the main field;

of $\frac{4\pi}{10}\frac{Zi}{8\delta}(4k - 1)$ due to the cross field produced by i, as shown in Fig. 314; and of $\frac{4\pi}{10}\frac{kZi_s}{2\delta}$ due to i_s, as shown in Fig. 315. Consequently

$$e_{s.c.} = \left[B_g + \frac{4\pi}{10}\frac{Zi}{8\delta}(4k - 1) + \frac{4\pi}{10}\frac{kZi_s}{2\delta}\right] l \frac{Z}{S}\frac{\pi dn}{60 \times 10^8} \quad (20)$$

the factor $\frac{Z}{S}$ being introduced to account for the number of conductors in series in the short-circuited element; and $\frac{\pi dn}{60}$ is the peripheral velocity. Assuming that $B_g = ci_s$ where c is a constant, and observing that $\pi d = 2\tau$ equation (20) becomes

$$e_{s.c.} = \frac{2Z\tau ln}{S \times 60 \times 10^8}\left[\left(c + \frac{4\pi}{10}\frac{kZ}{2\delta}\right) i_s + \frac{4\pi}{10}\frac{Z(4k - 1)}{8\delta} i\right] \quad (21)$$

Substituting (19) and (21) in (18), and adding the result to (17), the total demagnetizing flux between brushes B_1 and B_3 due to all causes is

$$\Phi_D = \frac{4\pi}{10}\frac{Z\tau l}{32\delta}(\psi + 4k - 1)\left[(\psi - 4k + 1) + \frac{4\pi}{10}\left(\frac{Z}{S}\right)^2\frac{\tau l}{\delta r}\frac{4k - 1}{60.10^8}n\right]i$$
$$+ \frac{4\pi}{10}\frac{Z\tau l}{4\delta}(\psi + 4k - 1)\left[\frac{Z\tau ln}{S^2 r \times 60.10^8}\left(c + \frac{4\pi}{10}\frac{kZ}{2\delta}\right) - \frac{1 - \psi}{4}\right]i_s \quad (22)$$

Now the e.m.f. E_f generated between brushes B_1 and B_3 is due to that part of the main flux $B_g\psi\tau l$ which lies between them, diminished by the flux given by (22). The part of the main flux between B_1 and B_3 is

$$\phi_{B_1B_3} = \frac{\psi + 4k - 1}{2\psi}B_g\psi\tau l = \frac{\psi + 4k - 1}{2}c\tau l i_s \quad (23)$$

and the number of active conductors in this zone which cut $\Phi_{B_1B_3}$ to produce E_f is the $2k$th part of the total number on the armature. The e.m.f. generated between B_1 and B_3 is then $\frac{2k\varphi Zn}{60 \times 10^8}$, where φ is the difference between (23) and (22), and we have further that

$$i_s = \frac{E_f - 4kr_a i_3}{r_s} = \frac{E_f - 4kr_a\left[\frac{i}{2} + (1-k)i_s\right]}{r_s}$$

whence

$$i_s = \frac{E_f - 2kr_a i}{R} \tag{24}$$

where

$$R = r_s + 4k(1-k)r_a \tag{25}$$

On making the substitutions indicated, it is found that

$$i_s = i\frac{n(\gamma + \epsilon n) + 2kr_a}{n(\alpha - \beta n) - R} \tag{26}$$

where

$$\left.\begin{aligned}
\alpha &= \frac{kZ\delta l}{60.10^8}(\psi + 4k - 1)\left(c + \frac{4\pi}{10}\frac{Z}{2\delta}\frac{1-\psi}{4}\right)\\
\beta &= \frac{4\pi}{10}\frac{kZ^3\tau^2 l^2}{(60.10^8)^2}\frac{\psi + 4k - 1}{2S^2\delta r}\left(c + \frac{4\pi}{10}\frac{kZ}{2\delta}\right)\\
\gamma &= \frac{4\pi}{10}\frac{kZ^2\tau l}{60.10^8}\frac{\psi^2 - (4k-1)^2}{16\delta}\\
\epsilon &= \left(\frac{4\pi}{10}\right)^2\frac{kZ^4\tau^2 l^2}{(60.10^8)^2}\frac{\psi + 4k - 1}{16S^2\delta^2 r}(4k-1)
\end{aligned}\right\} \tag{27}$$

The total flux per pole, Φ, which is responsible for the e.m.f. E_a generated between the main brushes B_1 and B_2, may be considered as the algebraic sum of the flux $B_g\psi\tau l = c\psi\tau l i_s$ contributed by the field excitation, and of the fluxes, partly positive and partly negative, produced by the armature current. Referring to Fig. 314, it will be noted that the main line current i has only a cross-magnetizing effect upon the flux as a whole, the demagnetizing effect being nil if saturation is neglected; but from Fig. 315 it will be seen that there is a net magnetizing action due to i_s that is represented by the sum of φ_m' (equation (16)) and of the additional flux represented by the rectangle to the left of B_3, this extra flux being given by $\frac{4\pi}{10}\frac{kZi_s\tau l}{2\delta}\frac{\psi - 4k + 1}{2}$; adding this to (16), there is obtained

$$\varphi_m'' = \frac{4\pi}{10}\frac{Zi_s\tau l}{4\delta}\left[\frac{1-\psi}{4}(\psi + 4k - 1) + k(\psi - 4k + 1)\right] \tag{28}$$

Similarly, referring to Fig. 318, the short-circuited coil produces a demagnetizing flux φ_d'' represented by the difference between the two shaded rectangles, whence

$$\varphi_d'' = \frac{4\pi}{10}\frac{Z\tau l i_{s.c.}}{4S\delta}(4k - 1). \tag{29}$$

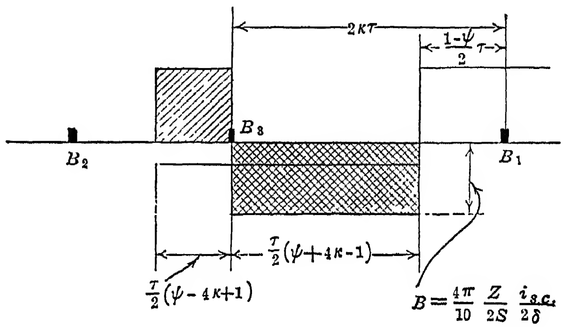


FIG. 318 —Distribution of m.m.f. due to $i_{s\,c}$.

We have then, that the net useful flux per pole is given by

$$\Phi = c\psi\tau l i_s + \varphi_m'' - \varphi_d'' \tag{30}$$

and

$$E_a = \frac{\Phi Z n}{60.10^8} = V + 2r_a i_1 = V + i r_a + 2k r_a i_s \tag{31}$$

Substituting (28), (29), and (30) in (31), and combining terms,

$$E_a = \zeta i_s n - \eta i_s n^2 - \lambda i n^2 = V + i r_a + 2k r_a i_s \tag{32}$$

where

$$\left.\begin{aligned}
\zeta &= \frac{Z\tau l}{60.10^8}\left\{c\psi + \frac{4\pi}{10}\frac{Z}{4\delta}\left[\frac{1-\psi}{4}(\psi+4k-1)+k(\psi-4k+1)\right]\right\}\\
\eta &= \frac{4\pi}{10}\frac{Z^3\tau^2 l^2}{(60.10^8)^2}\frac{4k-1}{2S^2\delta r}\left(c+\frac{4\pi}{10}\frac{kZ}{2\delta}\right)\\
\lambda &= \left(\frac{4\pi}{10}\right)^2\frac{Z^4\tau^2 l^2}{(60.10^8)^2}\frac{(4k-1)^2}{16S^2\delta^2 r}
\end{aligned}\right\} \tag{33}$$

Substituting the value of i_s from (26) in (32), it is found that

$$i = V\frac{-\beta n^2+\alpha n - R}{n^4(\beta\lambda-\epsilon\eta)-n^3(\alpha\lambda+\gamma\eta-\epsilon\zeta)+n^2(\gamma\zeta+\beta r_a+\lambda R-2kr_a(\eta+\epsilon)) + n[2kr_a(\zeta-\gamma)-r_a\alpha]+Rr_a-4k^2r_a^2} \tag{34}$$

Equation (34) is the desired relation between the line current and the speed, since it involves only these quantities in addition to the constants of the machine. It remains, therefore, to analyze this expression in order to develop its physical meaning.

In the first place, it is found on substituting values for the coefficients of n in (34) from the relations of (27) and (33) that the fourth degree term in n vanishes, the quantity $(\beta\lambda - \epsilon\eta)$ being identically zero. Writing equation (34) as

$$i = V\frac{y_n}{y_d}$$

where

$$y_n = -\beta n^2 + \alpha n - R \tag{35}$$

and

$$\begin{aligned} y_d = & -(\alpha\lambda + \gamma\eta - \epsilon\zeta)n^3 + (\gamma\zeta + \beta r_a + \lambda R - 2kr_a(\eta + \epsilon))n^2 \\ & + [2kr_a(\zeta - \gamma) - r_a\alpha]n + r_a(R - 4k^2r_a) \end{aligned} \tag{36}$$

it is clear that the variation of i with n may be studied by determining the graphs corresponding to (35) and (36), dividing the ordinates of the former by those of the latter, and multiplying the result by V.

Taking equation (35) first, it is evident that this represents a parabola of the form indicated by curve y_n in Fig. 319. When the speed is zero ($n = 0$), y_n reduces to

$$(y_n)_{n=0} = -R \tag{37}$$

from which it follows that the parabola intersects the axis of ordinates at a point distant R below the origin. Also, y_n becomes zero when

$$-\beta n^2 + \alpha n - R = 0$$

that is, when

$$n = \frac{\alpha \pm \sqrt{\alpha^2 - 4R\beta}}{2\beta}. \tag{38}$$

Consequently, if $\alpha^2 > 4R\beta$, both roots will be real and positive, and the parabola will intersect the axis of n at two points to the right of the origin, one at a speed

$$n_1 = \frac{\alpha - \sqrt{\alpha^2 - 4R\beta}}{2\beta} \tag{39}$$

the other at a speed

$$n_2 = \frac{\alpha + \sqrt{\alpha^2 - 4R\beta}}{2\beta} \tag{40}$$

Substituting the values of α and β from (27), the condition that $\alpha^2 > 4R\beta$ is found to be

$$r > \frac{8\pi}{10}\frac{ZR}{kS^2\delta}\frac{1}{\psi + 4k - 1}\frac{c + \frac{4\pi}{10}\frac{kZ}{2\delta}}{\left(c + \frac{4\pi}{10}\frac{kZ}{2\delta}\frac{1-\psi}{4}\right)^2} \tag{41}$$

Proceeding in like manner to analyze the function y_d, equation (36), it is seen that when $n = 0$

$$(y_d)_{n=o} = r_a(R - 4k^2r_a) \tag{42}$$

which is always a positive quantity, hence the cubic curve which is the graph of (36) intersects the axis of ordinates at a point distant $r_a(R - 4k^2r_a)$ above the origin. Further, the function y_d may be written

$$y_d = -An^3 + Bn^2 + Cn + D \tag{43}$$

where

$$\left.\begin{aligned} A &= \alpha\lambda + \gamma\eta - \epsilon\zeta \\ B &= \gamma\zeta + \beta r_a + \lambda R - 2kr_a(\eta + \epsilon) \\ C &= 2kr_a(\zeta - \gamma) - r_a\alpha \\ D &= r_a(R - 4k^2r_a) \end{aligned}\right\} \tag{44}$$

If the generator is to function properly, it is necessary that the graph of (43) be above the axis of n at all points between n_1 and n_2 determined by (39) and (40); in other words, that the function $y_d = 0$ shall have no real roots between these two values of the speed. Putting (43) equal to zero and dividing through by A, we have

$$n^3 - \frac{B}{A}n^2 - \frac{C}{A}n - \frac{D}{A} = 0;$$

if now there be substituted for n the expression

$$n = n' + \frac{B}{3A} \tag{45}$$

the term in n^2 drops out, and the resultant expression is

$$(n')^3 - n'\left(\frac{C}{A} + \frac{B^2}{3A^2}\right) - \frac{2}{27}\frac{B^3}{A^3} - \frac{BC}{A^2} - \frac{D}{A} = 0 \qquad (46)$$

This equation will have no real roots if the condition is satisfied that

$$\frac{1}{27}\left(\frac{C}{A} + \frac{B^2}{3A^2}\right)^3 < \frac{1}{4}\left(\frac{2B^3}{27A^3} + \frac{BC}{A^2} + \frac{D}{A}\right)^2 \qquad (47)$$

which reduces to

$$A^3C^3 < \frac{23}{108}A^2B^2C^2 + \frac{2}{81}AB^4C + A^2D\left(\frac{1}{4}A^2D + \frac{1}{27}B^3 + \frac{1}{2}ABC\right) \qquad (48)$$

Substitution of the actual values of A, B, C, and D leads to complicated expressions, but a first degree of approximation is readily obtained by observing that if the brush B_3 is nearly

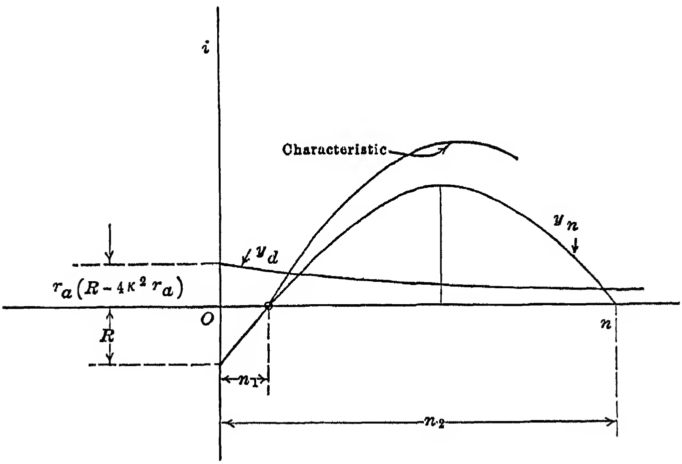


FIG. 319.—Characteristic curve of third-brush generator.

midway between the main brushes, as is usually the case in practice, the factor k is nearly ¼, hence terms involving $(4k - 1)$ approach zero. Reference to equations (27), (33), and (44) will show that under this condition the coefficient A approaches zero, while B and C remain positive in value. Therefore the condition represented by (48) is satisfied, and the cubic curve lies above the axis of n in the first quadrant.

It follows, then, that the general form of the characteristic showing the relation between i and n will have the form of Fig. 319, a result that is confirmed by experience. The curve crosses the axis of n at two points, one near the origin corresponding to the point of pick-up, at which the generator begins to charge the battery, the other at a point considerably to the right of the origin, the speed at this point being usually much greater than any operating speed.

It is also clear from the general equation (34), that the charging current is proportional to V, the assumed constant value of the battery voltage. As a matter of fact, while it has been tacitly assumed that V is likewise equal to the terminal voltage of the generator, the latter will exceed the battery voltage by an amount equal to the drop of potential in the leads connecting them; hence any increase in the resistance of these leads will cause the charging current to increase, provided the battery voltage remains constant, a conclusion which seems paradoxical, but which is nevertheless verified by test results.

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